CHAPTER TWENTY-NINE

MATHEMATICS, METROLOGY, AND PROFESSIONAL NUMERACY

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INTRODUCTION

Since the great decipherments of the 1930s and 1940s (Neugebauer 1935–37; Thureau-Dangin 1938; Neugebauer and Sachs 1945) Babylonia has had a well-deserved reputation as the home of the world's first 'true' mathematics, in which abstract ideas and techniques were explored and developed with no immediate practical end in mind. It is commonly understood that the base 60 systems of time measurement and angular degrees have their ultimate origins in Babylonia, and that 'Pythagoras' theorem' was known there a millennium before Pythagoras himself was supposed to have lived.

Most accounts of Babylonian mathematics describe the internal workings of the mathematics in great detail (e.g., Friberg 1990; Høyrup and Damerow 2001) but tell little of the reasons for its development, or anything about the people who wrote or thought about it and their reasons for doing so. However, internal textual and physical evidence from the tablets themselves, as well as museological and archaeological data, are increasingly enabling Babylonian mathematics to be understood as both a social and an intellectual activity, in relation to other scholarly pursuits and to professional scribal activity. It is important to distinguish between mathematics as an intellectual, supra-utilitarian end in itself, and professional numeracy as the routine application of mathematical skills by working scribes.

This chapter is a brief attempt at a social history of Babylonian mathematics and numeracy (see Robson forthcoming). After a short survey of their origins in early Mesopotamia, it examines the evidence for metrology and mathematics in Old Babylonian scribal schooling and for professional numeracy in second-millennium scribal culture. Very little evidence survives for the period 1600–750 BCE, but there is a wealth of material for mid-first-millennium Babylonia, typically neglected in the standard accounts of the subject. This chapter is doubtless flawed and incomplete, and may even be misguided, but I hope that at the very least it re-emphasises the 'Babylonian' in 'Babylonian mathematics'.

ORIGINS: NUMERACY AND MATHEMATICS IN EARLY MESOPOTAMIA

Professional numeracy predates literacy by several centuries in Babylonia. From at least the early fourth millennium bureaucrats and accountants at economic centres such as Uruk used tiny counters made of unbaked clay, shell, or river pebbles, along with other bureaucratic apparatus such as clay sealings and bevel-rim bowls. Over the course of the fourth millennium the scribes began to mark the external surface of the rough clay envelopes (*bullae*) in which they were stored with impressions of the counters contained inside them, before abandoning the storage process all together in favour of impressions alone. Some of the earliest tablets, then, are simply tallies of unknown objects, with no marking of what was being accounted for (Schmandt-Besserat 1992).

This need to identify the objects of accounting was, arguably, a prime motivation behind the invention of incised proto-cuneiform script. By the end of the Uruk period complex calculations were carried out: estimates were made of the different kinds of grain needed for brewing beer and making bread; harvest yields were recorded and summed over many years (Nissen *et al.* 1993). Several different number systems and metrologies were used, depending on the commodity accounted for. Trainee scribes practised area calculations and manipulating very large and very small numbers (Friberg 1997–8).

Over the course of the third millennium the visual distinction between impressed numbers and incised words was eroded as writing became increasingly cuneiform. But, as in almost all ancient and pre-modern societies, in early Mesopotamia writing was used to record numbers, not to manipulate them. Fingers and clay counters remained the main means of calculation long after the development of literacy.

Handfuls of school mathematical exercises and diagrams survive from ED III Shuruppag (Melville 2002) and from the Sargonic period, poorly executed in Sumerian on roughly made tablets (Foster and Robson 2004). Most concern the relationship between the sides and areas of squares, rectangles, and irregular quadrilaterals of conspicuously imaginary dimensions. They take the form of word problems, in which a question is posed and then either assigned to a named student or provided with a numerical answer (which is often erroneous).

Successive bureaucratic reforms gradually harmonised the separate metrologies, so that all discrete objects came to be counted in tens and sixties, all grains and liquids in one capacity system, etc., although there were always local variations both in relationships between metrological units and in their absolute values. The weight system, developed relatively late in the Early Dynastic period, is the only metrology to be fully sexagesimal; the others all retain elements of the proto-historic mixed-base systems of the Uruk period (Powell 1990).

However, the different metrological units – say length and area – were still not very well integrated. The scribes' solution was to convert all measures into sexagesimal multiples and fractions of a basic unit, and to use those numbers as a means of calculating, just as today one might convert length measurements in yards, feet and inches into multiples and fractions of metres in order to calculate an area from them. Clues in the structure of Sargonic school mathematics problems show the sexagesimal system already in use (Whiting 1984), while tiny informal calculations on the edge

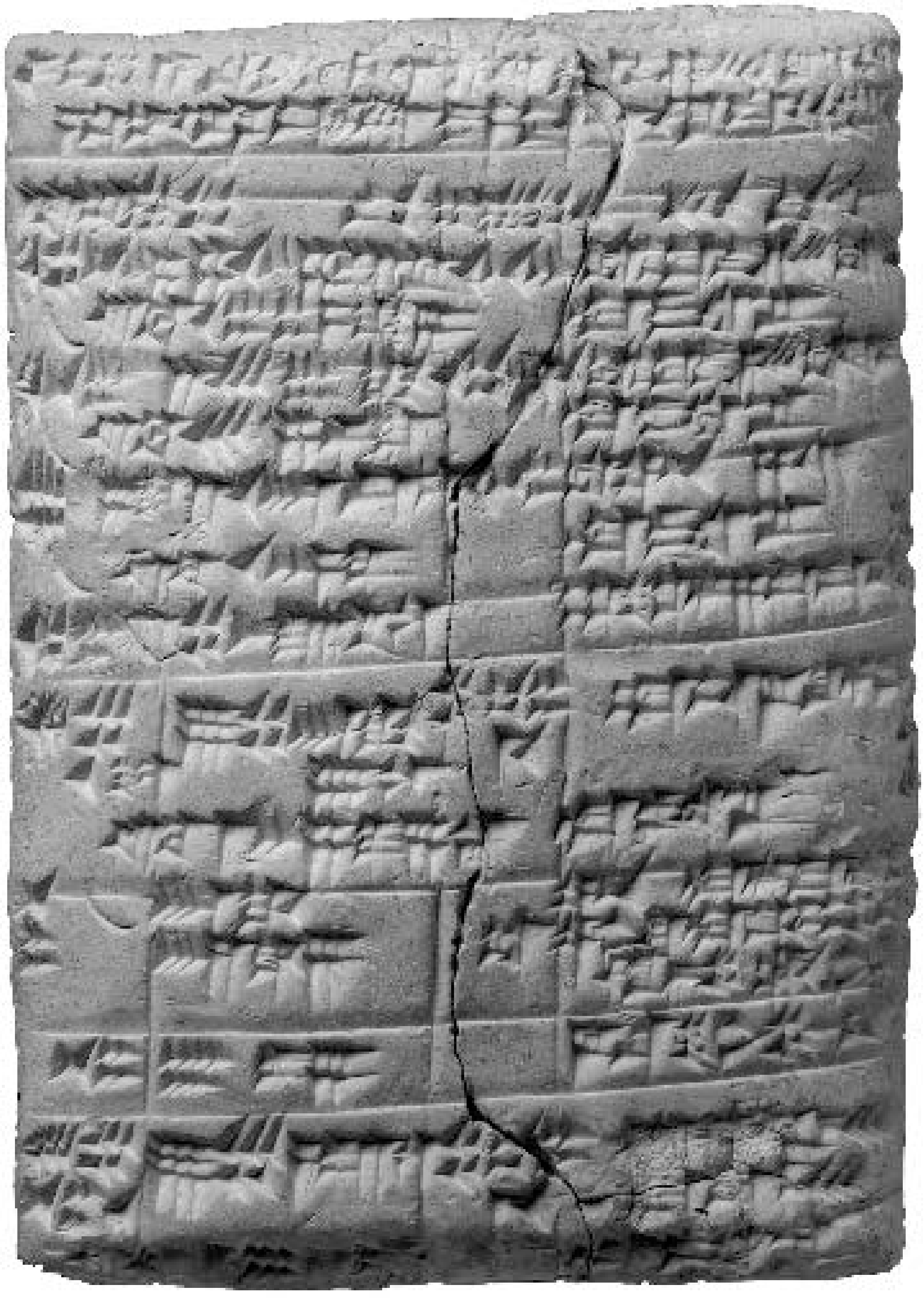

Figure 29.1 The obverse of Ashmolean 1922.277 (OECT 15: 7). An inspection of estimated yields (shukunnu) of fields in the towns of Nirda and Kurhianu, it was written in Hammurabi's thirty-fifth year. It has an introductory paragraph, headings, three levels of calculation (subtotals and totals), and interlinear notes. (Ashmolean Museum).

of Ur III institutional accounts show its adoption by professional scribes (Powell 1976). The earliest extant tables of reciprocals, which list pairs of numbers whose product is 60 – e.g., 2 and 30, 3 and 20, 4 and 15 – also date from the Ur III empire or early Old Babylonian period (Oelsner 2001). They are essential tools for sexagesimal arithmetic, as they neatly convert division by one number into multiplication by its reciprocal.

OLD BABYLONIAN MATHEMATICS AND NUMERACY

Metrology

By the beginning of the second millennium all the essential building blocks of Old Babylonian mathematics were already in place: the textual genres – tables, word problems, rough work and diagrams – and the arithmetical tools – the SPVS, reciprocals, and standardised constants for calculations and metrological conversions. However, the changes that had taken place in mathematics by the eighteenth century BCE were both quantitative and qualitative. In line with other products of scribal training, the sheer volume of surviving tablets is overwhelming: many thousands of highly standardised arithmetical and metrological tables, and hundreds of word problems, mostly in Akkadian, testing increasingly abstract and complex mathematical knowledge. The majority of the published sources are unprovenanced, having reached museum collections through uncontrolled excavation or the antiquities market in the late nineteenth and early twentieth centuries. However, archaeologically contextualised finds from Ur, Nippur, and Sippar have recently enabled close descriptions of the role of mathematics in the scribal curricula of particular schools (Friberg 2000; Robson 2001b; Tanret 2002).

House F in Nippur has produced by far the most detailed evidence, if only because of the vast number of tablets excavated there (Robson 2002). This tiny house, about 100 metres south-east of Enlil's temple complex E-kur, was used as a school early in the reign of Samsu-iluna, after which some 1,400 fragments of tablets were used as building material to repair the walls and floor of the building. Three tablet recycling boxes (Sumerian pú-im-ma) containing a mixture of fresh clay and mashed up old tablets show that the tablets had not been brought in from elsewhere. Half were elementary school exercises, and half extracts from Sumerian literary works. Both sets of tablets yield important information on mathematical pedagogy.

Mathematical learning began for the handful of students in House F in the second phase of the curriculum, once they had mastered the basic cuneiform syllabary. The six-tablet series of thematically grouped nouns – known anachronistically as OB Ur_5-ra, but more correctly as $^{\check{g}i\check{s}}$taskarin, 'tamarisk', after its first line – contains sequences listing wooden boats and measuring vats of different capacities, as well as the names of the different parts of weighing scales. Later in the same series come the stone weights themselves and measuring-reeds of different lengths. Thus students were first introduced to weights and measures and their notation in the context of metrological equipment, not as an abstract system (albeit always in descending order of size).

In House F systematic learning of metrological facts took place in phase three, along with the rote memorisation of other exercises on the more complex features of cuneiform writing. This time the metrological units were written out in ascending order: first the capacities from ⅓ sila to 3,600 gur (ca. 0.3–65,000,000 litres); then weights from ½ grain to 60 talents (0.05 g–1,800 kg); then areas and volumes from ½ sar to 7,200 bur (12 m^2–47,000 ha), and finally lengths from one finger to 60 leagues (17 mm–650 km). The entire series, fully written, contained several hundred entries, although certain sections could be omitted or abbreviated. It could be formatted as a list, with each entry containing the standard notation for the measures only, or as a table, where the standard writings were equated with values in the sexagesimal system. Further practice in writing metrological units, particularly areas, capacities, and weights, came in the fourth curricular phase, when students learned how to write legal contracts for sales, loans, and inheritances.

Arithmetic

Arithmetic itself was concentrated in the third phase of the House F curriculum, alongside the metrology. Again the students memorised a long sequence of facts, this time through copying and writing standard tables of reciprocals and multiplications. There were about 40 tables in the sequence after the reciprocals, in descending order from 50 to 1;15.[1] Each table had entries for multiplicands 1–20, 30, 40, and 50. When the students first learned and copied each table they tended to write them in whole sentences: 25 a-rá 1 25/a-rá 2 50 ('25 steps of 1 is 25, <25> steps of 2 is 50'), but when recalling longer sequences of tables in descending order abbreviated the entries to just the essential numbers: 1 25/2 50. Enough tablets of both kinds survive to demonstrate that although the House F teacher presented students with the entire series of multiplication tables to learn, in fact the students tended to rehearse only the first quarter of it in their longer writing exercises.

Active engagement with mathematics came only on entering the advanced phase of learning, which focused heavily on Sumerian literature. About 24 literary works were copied frequently in House F. Some, such as *The Farmer's Instructions*, have some sort of mathematical content; others convey strong messages about the role of mathematics in the scribal profession. Competent scribes use their numeracy and literacy in order to uphold justice, as Girini-isag spells out in criticising his junior colleague Enki-manshum:

> You wrote a tablet, but you cannot grasp its meaning. You wrote a letter, but that is the limit for you! Go to divide a plot, and you are not able to divide the plot; go to apportion a field, and you cannot even hold the tape and rod properly; the field pegs you are unable to place; you cannot figure out its shape, so that when wronged men have a quarrel you are not able to bring peace but you allow brother to attack brother. Among the scribes you (alone) are unfit for the clay. What are you fit for? Can anybody tell us?[2]

Girini-isag's point is that accurate land surveys are needed for legal reasons – inheritance, sales, harvest contracts, for instance. If the scribe cannot provide his services effectively he will unwittingly cause disputes or prevent them from being settled peacefully. Thus mathematical skills are at the heart of upholding justice. This point is also made in the curricular hymns to kings such as Ishme-Dagan and Lipit-Eshtar, who attribute their skills in numerate justice to the goddess Nisaba, patron of scribes, who 'generously bestowed upon {them} the measuring rod, the surveyor's gleaming line, the yardstick, and the tablets which confer wisdom'.[3]

On one or two literary tablets from House F students also carried out arithmetical calculations, showing all the intermediate steps as well as the final answer. At Ur students made calculations on the same tablets as Sumerian proverbs (Robson 1999: 245–72). Many of them can be linked to known types of word problems – most of which, unfortunately, have no known archaeological context. We must therefore leave House F behind to explore the more sophisticated aspects of Old Babylonian mathematics.

Mathematical word problems

The OB corpus of mathematical problems splits into two roughly equal halves: on the one hand, what might loosely be called concrete algebra (Høyrup 2002); and on the other, a sort of quantity surveying (Robson 1999; Friberg 2001). It is too crude and anachronistic to label these halves 'pure' and 'applied'; there are also significant overlaps between them.

Old Babylonian 'algebra' was for many years translated unproblematically into modern symbols, so that a question such as 'A reciprocal exceeds its reciprocal by 7. What are the reciprocal and its reciprocal?' could be represented as $x - 60/x = 7$ and the prose instructions for solving it understood as manipulations of that equation to yield $x = 12$ (YBC 6967: Neugebauer and Sachs 1945: text Ua). However, Jens Høyrup's pioneering discourse analysis (Høyrup 1990; 2002) made it clear that all 'algebraic' procedures should be understood as the manipulation of lines and areas. In this case, by visualising the unknown reciprocals as the sides of rectangle of area 60 the rectangle can be turned into an L-shaped figure, and the original lengths found by completing the square:

> A reciprocal exceeds its reciprocal by 7 {Figure 29.2a}. What are the reciprocal and its reciprocal? You: break in two the 7 by which the reciprocal exceeds its reciprocal so that 3;30 (will come up) {Figure 29.2b}. Combine 3;30 and 3;30 so that 12;15 (will come up). Add 1 00, the area, to the 12;15 which came up for you so that 1 12;15 (will come up) {Figure 29.2c}. What squares 1 12;15? 8;30. Draw 8;30 and 8;30, its counterpart, and then take away 3;30, the holding-square, from one; add to one. One is 12, the other is 5 {Figure 29.2d}. The reciprocal is 12, its reciprocal is 5.

FIGURE FROM m/s p. 738: 29.2

Figure 29.2 The geometrical manipulations implicit in YBC 6967.

No diagrams such as those shown in Figure 29.2 actually survive. Either they were ephemera drawn in the dust or instantly recycled clay, or they were simply imaginative, conceptual tools that never took written form. Nevertheless, their real or imagined existence fits features of the mathematical text that could not be comfortably explained by interpretation through symbolic algebra.

Fully half the known corpus of Old Babylonian mathematical problems uses the management of building work and agricultural labour as a pretext for setting exercises in line geometry or geometric algebra. Carrying bricks, building earthen walls, and repairing canals and associated earth works are among the commonest problem-setting scenarios. Some are illustrated with diagrams of plane or three-dimensional figures. Although many use terminology and technical constants that are also known from earlier administrative practice the majority of the problems are highly unrealistic, both in the measurements of the objects described and in the nature of the questions posed. Quantity surveying could also be used as a pretext for developing complex problems on geometrical algebra. For instance, problem 19 of YBC 4657 (Neugebauer and Sachs 1945: text G) asks, 'A trench has an area of 7½ sar; the volume is 45. Add the length and width and (it is) 6½ rods. What are the length, width, and its depth?' A professional surveyor would never find himself knowing the parameters given in the question without also knowing the measurements that the question asks for.

YBC 6967 is a single tablet; YBC 4657 is one of a series of four, of which three survive. Most collections of mathematical problems are more or less thematic, and their structuration is often explicitly pedagogical, with problems getting progressively harder. The numerical values in the collections tend to stay constant, though; teachers kept separate lists of different parameters that could be given to students in their actual exercises. Plimpton 322, the famous table of 'Pythagorean' numbers, is one such parameter list (Robson 2001a). Some of the calculations from OB Ur are witnesses to a single problem assigned with different numerical values to six pupils – or to the same student six times.

Professional numeracy

Although scribal students sometimes signed their names at the bottom of their tablets, it has not yet been possible to trace any individual's career from the school house to life as a working scribe; nor has anyone attempted to chart the relationship between school mathematics and the needs of the professionally numerate. OB school mathematics went way beyond practical necessity; rather, it – like the messages of curricular Sumerian literature – instilled confidence, pride, and a sense of professional identity into the young scribes.

Institutional accountants needed, at the most basic level, only to be able to count, add and subtract, and to record numbers, weights and capacities accurately. Additions and subtractions were carried out mentally or by means of counters; such calculations are never found on tablets, either from school or work. A scribe writing daily records of commodities coming in and out of storage might never need more advanced numeracy. Overseers and surveyors needed multiplication, division, and standardised constants for a multitude of tasks such as calculating areas of fields or volumes of canal excavations, and for estimating harvest yields or the number of labourers needed. I have never seen an institutional account that called for any more complex arithmetical method, such as finding non-standard reciprocals or square roots (both favourite school exercises).

That said, the complexity of some OB accounts is astonishing. Ur III documents were all formatted as rather cumbersome lists in which qualitative and quantitative data are mixed in sentence-like entries. During the early OB period scribes developed the tabular account, as an efficient way of recording, storing, and sorting data. Tabulation enabled the horizontal separation of different categories of quantitative information and the easy addition of quantitative data, along a vertical axis. At the same time, data could be sorted by criteria such as responsible officials, destination, or date of transaction. Headings obviated the necessity to repeat descriptive information. Columns of derived data – additions, subtractions, multiplications – enabled calculations to be performed along both horizontal and vertical axes for the purposes of double-checking. At the same time, the columnar format could be ignored where necessary to provide note-like explanatory interpolations. Tables were truly powerful information-processing tools, cognitively distinct from well-organised lists, but they remained a minority preference for Old Babylonian scribes. It was in Kassite Nippur that tabular accounts had their greatest heyday (Robson 2004a).

FIRST-MILLENNIUM MATHEMATICS AND NUMERACY

Metrology in scribal schools

There are currently very few sources for mathematics, or for any other scholarly activity, in Babylonia between around 1600 and 750 BCE. For the succeeding five centuries there is a rich variety of evidence, much of it archaeologically contextualised, for the learning, teaching, and practical use of mathematical skills.

All metrologies had changed drastically since Old Babylonian times, but it was the areal system that had been most radically overhauled. In 'reed measure', the standard unit of area was no longer the sar, or square rod (ca. 36 m^2), but the square reed, 7 × 7 cubits (ca. 12.25 m^2). It was subdivided into the areal cubit = 7 × 1 linear cubits, and the areal finger = 7 cubits × 1 finger). Alternatively, in 'seed measure', areas were considered to be proportional to one of several fixed capacity measures and thus expressed in terms of the capacity of seed needed to sow them.

In Neo-Babylonian times, a formal elementary curriculum that seems to have varied little from city to city is attested at Ur and the cities of northern Babylonia (Gesche 2000). It had changed significantly since Old Babylonian times: metrological lists occupied a marginal position in this curriculum, being found on around five per cent of published school tablets. These particular tablets all have the same format: there is a long extract from one of half a dozen standard lexical lists on the front, and many shorter pieces from a variety of texts on the back. Lists of capacities, weights, and lengths are attested in roughly equal numbers. Most are long extracts or entire lists, but some consist of two or three lines repeated over and over again. There are no tables, either giving sexagesimal values of metrological units, as in the Old Babylonian period, or describing relations between metrological systems, as in other first-millennium contexts. Lists of squares of integers are the only other type of mathematical exercise attested in the first-millennium school curriculum (Robson 2004b).

Metrological lists were found among school tablets excavated by an Iraqi team in Babylon in the late 1970s. They had been re-used as packing materials under the floors of two temples near the north-east corner of the precinct of E-temen-anki, Marduk's ziggurat in Babylon. The larger temple was dedicated to Nabû of *harû* (the name of his primary temple in Borsippa); the smaller temple belonged to the minor goddess Ashratum 'Lady of the steppe'. Almost all the tablets originally bore colophons naming their student authors and dedicating them to the god Nabû, either in his primary aspect or as 'Nabû of accounts'. They may well have been brought to the temple as votive offerings, having been written somewhere else (Cavigneaux 1981).

On present evidence, the mathematical elements of formal elementary scribal education in the mid-first millennium BCE consisted *only* of metrological lists – lengths, capacities, areas – and tables of squares. For more sophisticated mathematical activities we must look to professional scholars in Babylon and Uruk.

Mathematics for *āšipus*

The Shangu-Ninurta family of *āšipus* (exorcists, or incantation priests) occupied a courtyard house in eastern Uruk in the fifth century BCE. Three rooms and a courtyard have survived. Before the family left, they had carefully buried much of their household library – about 180 tablets – and whatever archival tablets they did not want to take with them, in clay jars within the house. The approximate proportions of the library's scholarly contents were:

- 30 per cent medical (physiognomic and diagnostic omens; medical prescriptions and incantations);
- 20 per cent other incantations, rituals, and magic;
- 19 per cent hymns, literature, and lexical lists;
- 12 per cent observed and induced omens (*Enuma Anu Ellil*, terrestrial omens, extispicy, etc.);
- 12 per cent astronomy, astrology, and mathematics; and
- 7 per cent unidentified (Hunger 1976; von Weiher 1983–98).[4]

Fifty tablets have colophons on, recording information such as the owner and scribe of the tablet, its source, and its place within a scholarly series. Seven of the family's tablets are mathematical, written or owned by one Shamash-iddin, and his son Rimut-Anu. Their mathematics is predominantly concerned with reciprocals, lengths, and areas, with a secondary interest in time-keeping.

Two large tablets, one of which appears to be a continuation of the other, contain a sequence of about 50 mathematical rules and problems about the 'seed' and 'reed' measures of area (Friberg *et al.* 1990; Friberg 1997). The second one has a catch-line before the colophon, '*Seed and reeds*. Finished', which suggests that they comprised a standard series. The colophon itself, which states that the tablet is a 'copy of a wooden writing board, written and checked against its original', is a salutary reminder that much first-millennium scholarship was written on perishable media. The very last problem of this second tablet is about finding the square area that can be paved with standard square baked bricks ⅔ cubit (ca. 35 cm) long:

> 9 hundred baked bricks of ⅔ cubits each. I enlarged an animal yard. What is the square side of the animal yard? You go 0;40 steps for each brick. You take 30 each of 15 00. You go 30 steps of 0;40: 20. The square side of the animal yard is 20 cubits.

The solution is simply to find the number of bricks that would make up the edge of the square area, by finding the square root of the total number (happily a square integer), then to multiply by the length of each brick. Although the problem itself is reminiscent of OB mathematics, the technical terminology and method of solution are both radically different.

Three tablets from Shamash-iddin's house bear metrological tables and reciprocal tables. Two contain various tables relating length and both systems of area measure to each other and to the sexagesimal system, and to various time-keeping schemes. One begins with a list of numerical writings for the major gods. The third tablet is a common first-millennium table of many-place reciprocal pairs from 1 to 4. It appears to have been calculated, at least in part, not simply copied from another exemplar, as witnessed by two rough tablets with nine calculations of regular reciprocals taken from near the beginning of the table. The original number is multiplied repeatedly by simple factors of 60 until it is reduced to 1; then 1 is multiplied up again by those same simple factors in reverse: once more, a favourite OB school problem is solved using a new method (Friberg 1999).

Mathematics for *kalûs*

The latest dateable mathematical cuneiform tablets were written by members of the Sîn-leqi-unninni family in Seleucid Uruk. The Sîn-leqi-unninnis are the best known of all the scribal families of Late Babylonian Uruk, partly because their eponymous ancestor was considered to be the author of the famous Epic of Gilgamesh, and partly because they have left a vast amount of documentary evidence (Beaulieu 2000). While most of their tablets come from uncontrolled excavation at Uruk (Thureau-Dangin 1922), several were found in the Resh temple of Anu in the city centre (van Dijk and Mayer 1980). Anu-belshunu and his son Anu-aba-uter, who worked there as *kalûs*, or lamentation priests, wrote and owned a large number of mathematical astronomical tablets, mostly ephemeredes, or tables of predictions. Unlike other scribal families, they put their name to very few copies of the standard series of omens or medical compilations, but concentrated instead on the *kalûtu*, the standard series of incantations and rituals associated with their profession as lamentation priests working for Anu's temple Resh in Uruk city centre. Their scholarly tablets almost all reflect their professional concerns: the mathematical prediction of ominous celestial events, and the correct performance of ritual reactions to them (Pearce and Doty 2000).

Anu-belshunu made a copy of the 'Esangila tablet', for a member of another scribal family, in the 220s BCE. This text, known also from the vicinity of Babylon, uses the measurements of the great courts and the ziggurat Etemenanki near Marduk's temple Esangila as a pretext for some simple metrological exercises in seed and reeds, the different kinds of area measure (AO 6555: George 1993: 109–19, 414–34).

Anu-aba-uter's compilation of mathematical problems, which must have been written in the 180s BCE, is the latest datable mathematical cuneiform tablet known.

Few of its 17 problems, allowing for changes in writing habits, would look out of place within the OB mathematical corpus. Problems 1–2, for instance, concern the sum of arithmetical series, 9–13 the capacity of a cube whose sides are known, and 14–17 regular reciprocal pairs whose sums are known, solved by a standard OB cut-and-paste method. The only innovatory methods of solution are for problems 3–8, about triangles, squares, and the diagonals of rectangles – which are not exactly innovatory mathematical subjects in themselves. Problem 6 – to find the area of a rectangle given the length, width and diagonal – is not attested at all in the OB corpus but is found in a contemporary mathematical compilation from Babylon (AO 6484: Neugebauer 1935–37: I 96–107).

ADMINISTRATIVE METROLOGY AND NUMERACY

The mathematical obsession with 'seed and reeds' suggests that much pedagogical effort was expended in teaching conversion between the two area systems, but in fact numerate professionals seem to have had entirely separate uses for them. For instance, some 70 house plans and agricultural land surveys survive from the reign of Darius I, perhaps drawn up for taxation purposes. They seem to have been housed in a central archive with agricultural land surveys, although their original archaeological context is now lost (Nemet-Nejat 1982).

The field plans are typically not drawn to scale, as the dimensions of the boundaries are recorded textually, along with the area in seed measure. This was calculated by the traditional 'surveyors' formula', by which the lengths of opposite sides were averaged and then those averages multiplied together and converted from square reeds to seed. The cardinal directions of the field boundaries are written on the edges of the tablet, along with damaged details of the neighbouring properties. Calculations were still carried out in the sexagesimal place value system, even when the preferred recording format used partly decimalised absolute value.

Contemporary house plans, also drawn up by professional surveyors, look very similar, but use the 'reed measure' system of area metrology. 'Reed measure's' dependence on the number seven, which is not a factor of 60, may even have been a deliberate move to professionalise and restrict access to urban land measurement. Yet analysis of the actual calculations involved in house mensuration shows that the surveyors used several simple strategies to lessen the burden of calculation and conversion between sexagesimal and metrological systems. Nevertheless, it was an arithmetically fiddly operation which must have been learned on the job: as we have seen, institutionalised schooling would have prepared surveyors only to measure and write the numerals and metrological systems, not to convert between, and multiply with them.

In the Hellenistic period, legal documents recording the sale of prebends, or rights to shares in temple income, show a fascinating move away from sexagesimal numeration towards the Greco-Egyptian notation of fractions as sums of unit fractions (1/*n*) (Cocquerillat 1965). For instance, in 190 BCE the *kalû* Anu-belshunu bought a Temple Enterer's prebend of 'one-sixth plus one-ninth of a day [on the 1st] day, 24th day, and 30th day – a total of one-sixth plus one-ninth of one day on those days – and one-third of a day on the 27th day' (HSM 913.2.181: Wallenfels 1998). The scribe, one Shamash-etir, could easily have expressed the first fraction sexagesimally as 0;16 40 [= 5/18] of a day if he wanted: he was the chief priest of the Resh temple and was later to tutor Anu-belshunu's son Anu-aba-uter extensively in mathematical astronomy. But in this context he – and all other scribes of prebendary contracts – made a contextual, social choice against the sexagesimal system and in favour of Greek-style expression. Shamash-etir's notation hints that his priestly circle's legal contracts and scholarly writing were now a tiny cuneiform island in a sea of alphabetic Greek and Aramaic; it is no wonder that his protégé Anu-aba-uter is the last Babylonian mathematician known to us.

CONCLUSIONS

Babylonian mathematics underwent many changes in the millennia of its history, but those changes cannot be fully understood – or sometimes even identified – if viewed from a mathematical standpoint alone. Terminology, methods, metrologies, notations all adapted to changing social needs and interests as well as intellectual ones, while shaping and challenging the ideas of the individuals and groups who created and used them. It is often assumed that Otto Neugebauer left nothing more to be said about Babylonian mathematics. On the contrary: its very Babylonian-ness is only now beginning to be explored.

NOTES

1 Sexagesimal numbers are transcribed with spaces separating the sexagesimal places and a semicolon marking the boundary between integers and fractions. For example, 1 12;15 represents 1 × 60 + 12 + 15/60, or 72¼.
2 *Eduba* dialogue 3, lines 19–27: Vanstiphout (1997: 589).
3 Lipit-Eshtar hymn B, lines 23–24: Black *et al.* (1998–: 2.5.5.2).
4 The other tablets from the house belonged to later inhabitants, most notably Iqisha of the Ekur-zakir family (later fourth century BCE).

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