E. Robson: Mathematics in Ancient Iraq

CHAPTER ONE

Scope, Methods, Sources

The mathematics of ancient Iraq, attested from the last three millennia BCE, was written on clay tablets in the Sumerian and Akkadian languages using the cuneiform script, often with numbers in the sexagesimal place value system (§1.2). There have been many styles of interpretation since the discovery and decipherment of that mathematics in the late nineteenth and early twentieth centuries CE (§1.1), but this book advocates a combination of close attention to textual and linguistic detail, as well as material and archaeological evidence, to situate ancient mathematics within the socio-intellectual worlds of the individuals and communities who produced and consumed it (§1.3).

1.1 THE SUBJECT: ANCIENT IRAQ AND ITS MATHEMATICS

Iraq—Sumer—Babylonia—Mesopotamia: under any or all of these names almost every general textbook on the history of mathematics assigns the origins of 'pure' mathematics to the distant past of the land between the Tigris and Euphrates rivers. Here, over five thousand years ago, the first systematic accounting techniques were developed, using clay counters to represent fixed quantities of traded and stored goods in the world's earliest cities (§2.2). Here too, in the early second millennium BCE, the world's first positional system of numerical notation—the famous sexagesimal place value system—was widely used (§4.2). The earliest widespread evidence for 'pure' mathematics comes from the same place and time, including a very accurate approximation to the square root of 2, an early form of abstract algebra, and the knowledge, if not proof, of 'Pythagoras' theorem' defining the relationship between the sides of a right-angled triangle (§4.3). The best-known mathematical artefact from this time, the cuneiform tablet Plimpton 322, has been widely discussed and admired, and claims have been made for its function that range from number theory to trigonometry to astronomy. Most of the evidence for mathematical astronomy, however, comes from the later first millennium BCE (§8.2), from which it is clear that Babylonian astronomical observations, calculational models, and the sexagesimal place value system all had a deep impact on the later development of Old World astronomy, in particular through the person and works of Ptolemy. It is hardly surprising, then, that ever since its discovery a century ago the mathematics of ancient Iraq has claimed an important role in the history of early mathematics. Seen as the first flowering of 'proper' mathematics, it has been hailed as the cradle from which classical Greek mathematics, and therefore the Western tradition, grew. But, as laid out over the course of this book, the mathematical culture of ancient Iraq was much richer, more complex, more diverse, and more human than the standard narratives allow.

The mathematical culture of ancient Iraq was by no means confined to the borders of the nation state as it is constructed today. The name al-'Iraq (Arabic 'the river shore') is first attested about a century after the Muslim conquests of the early seventh century CE,[1] while the lines on modern maps which delimit the territory of Iraq are the outcome of the division of the collapsing Ottoman empire amongst European powers at the end of the First World War. The mathematics of pre-Islamic Iraq, as it has been preserved, was written on small clay tablets in cuneiform writing. Because, as argued here, mathematics was an integral and powerful component of cuneiform culture, for present purposes it will be a useful first approximation to say that cuneiform culture and mathematical culture were more or less co-extensive. The core heartland of the cuneiform world was the very flat alluvial plain between Baghdad and the Gulf coast through which the Tigris and Euphrates flow (figure 1.1). It was known variously in antiquity as Sumer and Akkad, Babylonia, Karduniaš, or simply The Land. The Land's natural resources were primarily organic: reeds, small riverine trees, and other plant matter, but most importantly the earth itself. Alluvial clay was the all-purpose raw material par excellence, from which almost anything from sickle blades to monumental buildings could be manufactured. Equally, when judiciously managed the soil was prodigiously fertile, producing an abundance of arable crops (primarily barley), as well as grazing lands for herds (sheep and goats but also cattle). Even the wildest of marshlands were home to a rich variety of birds and fish and the all-purpose reeds, second only to clay in their utility. What the south lacked, however, were the trappings of luxury: no structural timber but only date-palms and tamarisks, no stone for building or ornamentation other than small outcrops of soft, dull limestone, and no precious or semi-precious stones at all, let alone any metals, base or precious. All these had to be imported from the mountains to the north, east, and west, in exchange for arable and animal products.

The centre of power shifted north at times, to northern Iraq and Syria east of the Euphrates, known in ancient times as Assyria, Subartu, Mitanni, or the land of Aššur. Life here was very different: rainfall could be counted on for wheat-based agriculture, building stone was abundant, and mountainous sources of timber and metal ores relatively close to hand. Conversely, the dates, tamarisks, and reeds of the south were absent here, as were the marshes with their rich flora and fauna. Overland trade routes ran in all directions, linking northern Iraq with the wider world.[2]

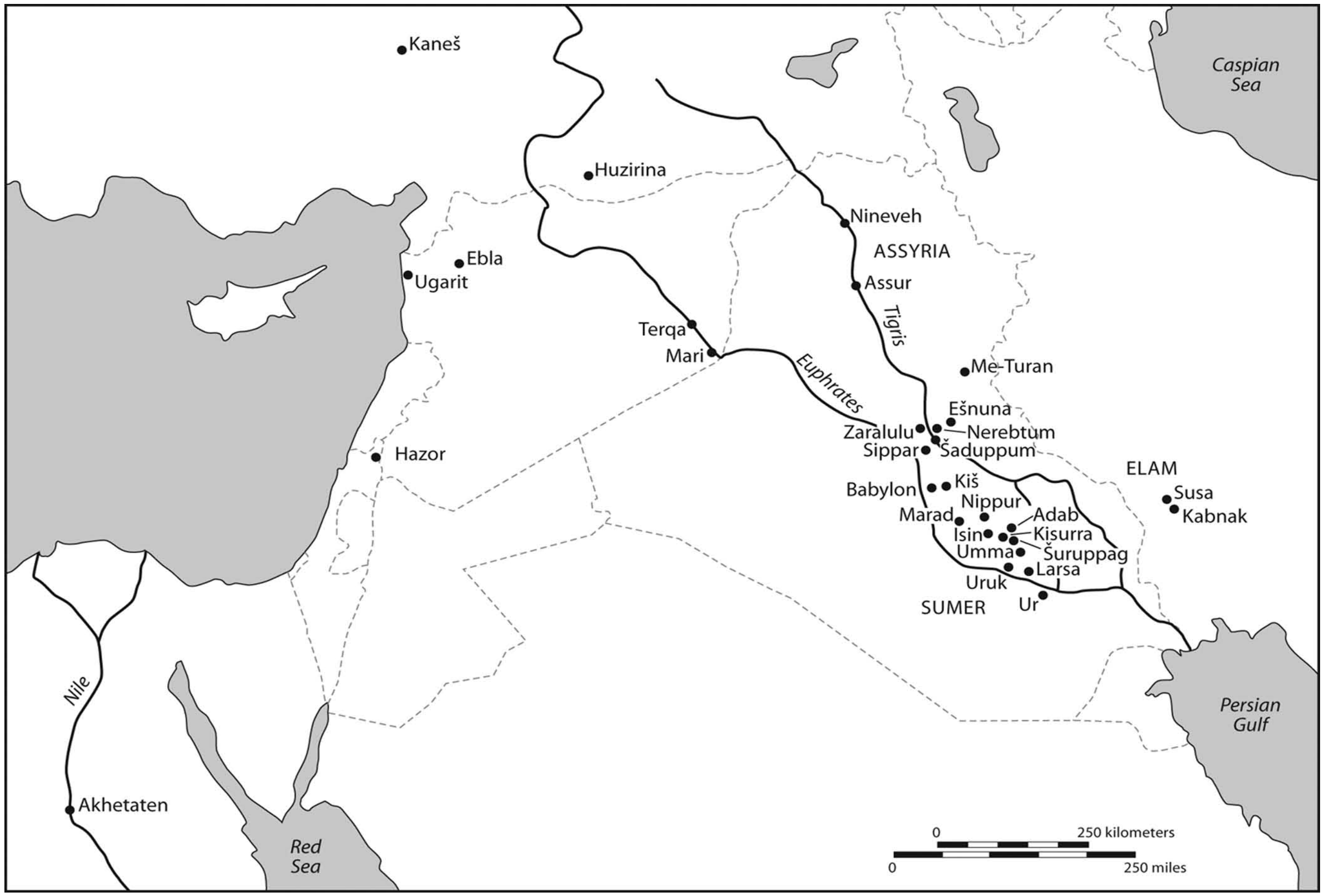

Figure 1.1 Map of the ancient Middle East, showing the major findspots of mathematical cuneiform tablets.

The fluid peripheries over which these territories had at times direct political control or more often cultural influence expanded and contracted greatly over time. At its maximum extent cuneiform culture encompassed most of what we today call the Middle East: the modern-day states of Turkey, Lebanon, Syria, Israel and the Palestinian areas, Jordan, Egypt, and Iran. Chronologically, cuneiform spans over three thousand years, from the emergence of cities, states, and bureaucracies in the late fourth millennium BCE to the gradual decline of indigenous ways of thought under the Persian, Seleucid, and Parthian empires at around the beginning of the common era. The history of mathematics in cuneiform covers this same long stretch and a similarly wide spread (table 1.1).

The lost world of the ancient Middle East was rediscovered by Europeans in the mid-nineteenth century (table 1.2). Decades before the advent of controlled, stratigraphic archaeology, the great cities of Assyria and Babylonia, previously known only through garbled references in classical literature and the Bible, were excavated with more enthusiasm than skill, yielding vast quantities of cuneiform tablets and *objets d'art* for Western museums.[3] The complexities of cuneiform writing were unravelled during the course of the century too, leading to the decipherment of the two main languages of ancient Iraq: Akkadian, a Semitic precursor of Hebrew and Arabic; and Sumerian, which appeared to have no surviving relatives at all.

In the years before the First World War, as scholars became more confident in their interpretational abilities, the first mathematical cuneiform texts were published.[4] Written in highly abbreviated and technical language, and using the base 60 place value system, they were at first almost impossible to interpret. Over the succeeding decades François Thureau-Dangin and Otto Neugebauer led the race for decipherment, culminating in the publication of their rival monumental editions, *Textes mathématiques babyloniens* and *Mathematische Keilschrifttexte,* in the late 1930s.[5] By necessity, scholarly work was at that time confined to interpreting the mathematical techniques found in the tablets, for there was very little cultural or historical context into which to place them. For the most part the tablets themselves had no archaeological context at all, or at best could be attributed to a named city and a time-span of few centuries in the early second millennium BCE. The final reports of the huge and well-documented excavations of those decades were years away from publication and nor, yet, were there any reliable dictionaries of Akkadian or Sumerian.

After the hiatus of the Second World War, it was business as usual for the historians of cuneiform mathematics. Otto Neugebauer and Abraham Sachs's *Mathematical cuneiform texts* of 1945 followed the paradigm of the pre-war publications, as did Evert Bruins and Marguerite Rutten's *Textes mathématiques de Suse* of 1961.[6] Neugebauer had become such a towering figure that his methodology was followed by his successors in the

TABLE 1.1
Overview of Mathematics in Ancient Iraq

Dates	*Political History*	*Mathematical Developments*
Fourth millennium BCE	Urbanisation Uruk period, c. 3200–3000	Pre-literate accounting Commodity-specific number systems
Third millennium BCE	Early Dynastic period (city states), c. 3000–2350 First territorial empires of Akkad and Ur, c. 2350–2000	Literate numeracy Sophisticated balanced accounting Sexagesimal place value system
Second millennium BCE	Babylonian kingdom, c. 1850–1600 Amarna age, c. 1400	Old Babylonian (OB) pedagogical mathematics: geometry, algebra, quantity surveying Spread of cuneiform culture and numeracy across the Middle East
First millennium BCE	Assyrian empire, c. 900–600 Neo-Babylonian empire, c. 600–540 Persian empire, c. 540–330 Seleucid empire, c. 330–125	Beginnings of systematic observational astronomy Reformulation of cuneiform mathematics Mathematical astronomy
First millennium CE	Parthian empire, c. 125 BCE–225 CE Sasanian empire, c. 225–640 Abbasid empire, c. 750–1100	Last dated cuneiform tablet, 75 CE Astronomical activity continues Baghdadi ‘House of Wisdom’: al-Khwarizmi, decimal numbers and algebra
Second millennium CE	Mongol invasions, c. 1250 Ottoman empire, c. 1535–1920 Modern Iraq, c. 1920–	(Iraq a political and intellectual backwater) Rediscovery of ancient Iraq and cuneiform culture

TABLE 1.2
The Rediscovery of Cuneiform Mathematics

Date	*Event*
1534–1918	Iraq under Ottoman rule
Before 1800	Travellers' tales of ancient Babylonia and Assyria
1819	Tiny display case of undeciphered cuneiform at British Museum
1842	Anglo-French rediscovery of ancient Assyria; priority disputes
1857	Assyrian cuneiform officially deciphered at Royal Asiatic Society in London
1871	Discovery of Babylonian flood story in British Museum
1877	Discovery of Sumerian language and civilisation; no mention in Bible
1880–	Mass recovery of cuneiform tablets in Babylonia
1889–	First decipherments of cuneiform astronomy and sexagesimal place value system
1900	First Old Babylonian (OB) mathematical problems published
1903–	Progress in understanding sexagesimal numeration and tables
1916	First decipherment of OB mathematical problem
1920	Formation of modern Iraqi state
1927	Neugebauer's first publication on OB mathematics
1927–39	Neugebauer and Thureau-Dangin's 'golden age' of decipherment
1945	Neugebauer and Sachs's final major publication on OB mathematics
1955	Neugebauer and Sachs publish mathematical astronomy
1956	First volume of *Chicago Assyrian Dictionary* published (finished in 2008)
1968	Ba'athist coup in Iraq
1976–	Increased interest in third-millennium mathematics
1984	First volume of *Pennsylvania Sumerian Dictionary* published (now online)
1990	Høyrup's discourse analysis of OB maths; war stops excavation
1996–	Developing web technologies for decipherment tools and primary publication
2003–	Iraq War and aftermath result in major archaeological looting and the virtual collapse of the State Board of Antiquities and Heritage

discipline, though often without his linguistic abilities. Cuneiformists put mathematical tablets aside as 'something for Neugebauer' even though he had stopped working on Babylonian mathematics in the late 1940s. Since there was almost no further output from the cuneiformists, historians of mathematics treated the corpus as complete. In the early 1950s the great Iraqi Assyriologist Taha Baqir published a dozen mathematical tablets from his excavations of small settlements near Baghdad, but virtually the only other editor of new material was Bruins, who tended to place short articles in the small-circulation Iraqi journal *Sumer* (as did Baqir) or in *Janus*, which he himself owned and edited.[7] All attempts at review or criticism met with such vitriolic attacks that he effectively created a monopoly on the subject.

Meanwhile, since Neugebauer's heyday, other aspects of the study of ancient Iraq had moved on apace. The massive excavations of the pre-war period, and the more targeted digs of the 1950s onwards, were being published and synthesised. The monumental *Chicago Assyrian dictionary* gradually worked its way through the lexicon of Akkadian, volume by volume. The chronology, political history, socio-economic conditions, and literary, cultural, religious, and intellectual environments of Mesopotamia were the subjects of rigorous, if not always accessible, scholarship. In the course of the 1970s and '80s attention turned to much earlier mathematical practices, as scholars led by Marvin Powell and Jöran Friberg found and analysed the numeration, metrology, and arithmetic of the third millennium BCE, from sites as far apart as Ebla in eastern Syria and Susa in southwestern Iran.[8] Denise Schmandt-Besserat began to formulate her mould-breaking theories of the origins of numeracy and literacy in the tiny calculi of unbaked clay that she had identified in prehistoric archaeological assemblages all over the Middle East.[9]

Nevertheless, it would be no exaggeration to say that between them, Neugebauer's renown for scholarly excellence and Bruins's reputation for personal venom seriously stifled the field of Babylonian mathematics until their deaths in 1990. It is perhaps no coincidence that 'Algebra and naïve geometry', Jens Høyrup's seminal work on the language of Old Babylonian algebra, was also published in that year, signalling a paradigm shift away from the history of Mesopotamian mathematics as the study of calculational techniques and their 'domestication' into modern symbolic algebra. Høyrup's work was in effect a discourse analysis of Mesopotamian mathematics: a close scrutiny of the actual Akkadian words used, and their relationship to each other. In this way he completely revolutionised our understanding of ancient 'algebra', showing it to be based on a very concrete conception of number as measured line and area.[10] An interdisciplinary project based in Berlin developed further important new methodologies in the early 1990s, leading to the computer-aided decipherment of the complex metrologies in the proto-literate temple accounts from late fourth-millennium Uruk which had resisted satisfactory interpretation for over eighty years.[11] Uruk also provided new sources from the other end of the chronological spectrum, as Friberg published mathematical tablets from the latter half of the first millennium BCE.[12]

In the past decade, large numbers of new mathematical tablets have come to light, both from excavation and from renewed study of old publications and large museum collections, and are now attested from almost every period of cuneiform culture. The published corpus now comprises over 950 tablets (table B.22). Still the largest body of evidence, though, is the pedagogical mathematics—exercises set and solved, metrological and mathematical tables copied and recopied—from the early second millennium BCE or Old Babylonian period. This currently accounts for over 80 percent of the published sources, not far short of 780 tablets. There are fewer than sixty known mathematical tablets from the whole of the third millennium, on the other hand (about 6 percent), and just over twice that number from the millennium and a half after 1500 BCE (some 13 percent). Thus the main focus of attention is still therefore on the large body of Old Babylonian material.

With some exceptions, the new generation of scholarship has taken a long time to filter through to the wider historical community. Cuneiformists have been put off by technical mathematics, historians of mathematics by technical Assyriology. Thus mathematics tends to be ignored in general histories of the ancient Near East, and even though it has an inviolable place at the beginning of every maths history textbook, the examples found there are for the most part still derived from a few out-of-date general works. Neugebauer's *The exact sciences in antiquity*, first published in 1951, was justly influential, but Van Der Waerden's derivative *Science awakening* (first English edition 1954) and later *Geometry and algebra in ancient civilizations* (1983) are both deeply Eurocentric and diffusionist. All in all it is time for a new look, from a new perspective—which is what this book sets out to do.

1.2 THE ARTEFACTS: ASSYRIOLOGICAL AND MATHEMATICAL ANALYSIS

Perhaps the most important methodological thread running through this book is that although mathematics is most immediately the product of individuals, those individuals are shaped and constrained by the society in which they live, think, and write. In order to understand the mathematics of a particular people as richly as possible, historians need to contextualise it. This approach is especially important for comprehending the mathematics of ancient Iraq, where anonymous tradition was prized over named authorship and we are more often than not completely unable to identify the work or influence of individuals within the written tradition. But context, crucial though it is, has to be paired with scrupulous attention to the mathematical, linguistic, and artefactual details of the tablets themselves. In order to demonstrate this, on the following pages a typical example, in the standard style of primary publication, is used to explain the basics of the media, script, numeration, and language of the sources, and to exemplify the usual methods of decipherment, interpretation, and publication. The final section demonstrates some of the different ways in which contextualisation can add layers of meaning to the interpretation of individual objects.

The primary publication of a cuneiform tablet should normally comprise at least a hand-copy (scale drawing) and transliteration, and often a photograph and translation as well. The sample tablet, 2N-T 30 (figure 1.2), has been partially published twice before: once as a rather blurry photograph, and once as a transliteration and translation based on that photograph. The hand-copy presented here is also based on that photograph, and on personal inspection of the tablet in Baghdad in March 2001.[13]

1 45	0;00 01 45
1 45	0;00 01 45
⅓ kuš$_3$ ½ šu-si-ta-am$_3$ ib$_2$-[si$_8$]	A square is ⅓ cubit, ½ finger on each side.
a-šag$_4$-bi en-nam	What is its area?
a-šag$_4$-bi	Its area
9 še igi-5-ğal$_2^!$ še-kam	is 9 grains and a 5th of a grain.

As the photograph shows, the text is not on a flat writing surface like paper or papyrus, but on a small cushioned-shaped tablet of levigated clay (that is, clay that has been cleaned of all foreign particles so that it is pure and smooth), measuring about 7.5 cm square by 2.5 cm thick at its maximum extent. Clay tablets varied in size and shape according to place and time of manufacture, and according to what was to be written on them; they could be as small as a postage stamp or as large as a laptop computer, but more usually were about the size of a pocket calculator or a mobile phone (though often rather thicker). Scribes were adept at fashioning tablets to the right size for their texts, making the front side, or *obverse*, much flatter than the back, or *reverse*. Some specialised genres of document traditionally required particular types of tablet, as in the case of 2N-T 30: it is square, with text only in the bottom right and top left corners; the rest of the tablet (including the reverse) is blank. Tablets were more usually rectangular, with the writing parallel to the short side and covering the whole surface of the clay. When the scribe reached the bottom of the obverse, instead of turning the tablet through its vertical axis (as we would turn the pages of a book),

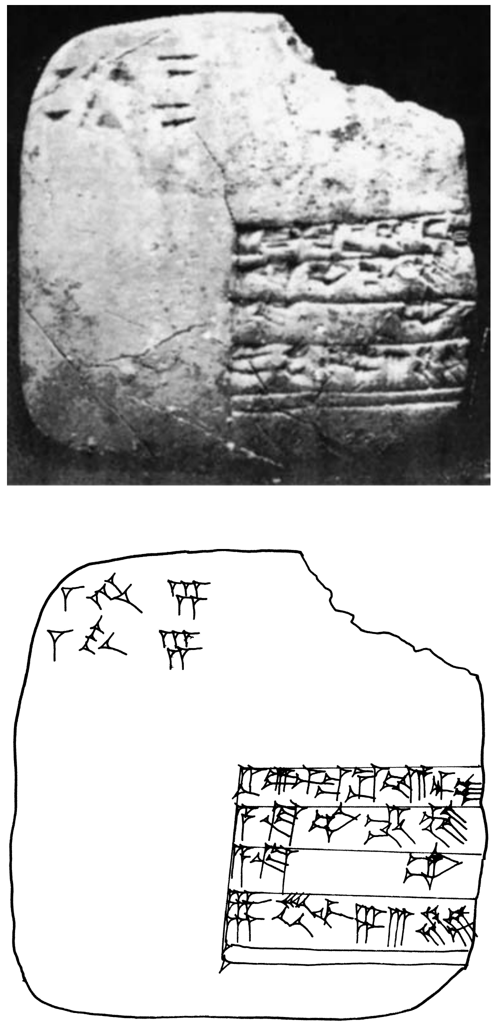

Figure 1.2 A mathematical exercise on an Old Babylonian cuneiform tablet. (2N-T 30 = IM 57828. Photograph: F. R. Steele 1951, pl. 7, courtesy of the University of Pennsylvania Museum of Archaeology and Anthropology; drawing by the author.)

he would flip it horizontally, so that the text continued uninterrupted over the lower edge and onto the reverse.[14] The text would thus finish at the top edge of the tablet, next to where it had started. Scribes could also use the left edge of the tablet to add a few extra lines or a summary of the document's contents. Larger tablets were ruled into columns, and here the same conventions applied: on the obverse the columns were ordered left to right and then the final column ran directly down the tablet and onto the other side, becoming the first column on the reverse of the tablet. The remaining columns were then ordered right to left, ending up in the bottom left corner of the reverse, right next to the top of the obverse.

Tablets were rarely baked in antiquity, unless there was some special reason for doing so, or it happened by accident in a fire. Tablets in museum collections tend to be baked as part of the conservation process, where they are also catalogued and mended. Tablets were often broken in antiquity—this one has lost its top right corner—and where possible have to be pieced together like a three-dimensional jigsaw puzzle or a broken cup, sometimes from fragments that have ended up in different collections scattered around the world. Often, though, missing pieces are gone forever, crumbled into dust. This tablet has two catalogue numbers, one from excavation and one from the museum that now houses it. The excavation number, '2N-T 30', signifies that it was the thirtieth tablet (T 30) to be found in the second archaeological season at Nippur (2N) in 1948, run jointly by the Universities of Chicago and Pennsylvania. This is the designation used in the archaeological field notes, enabling its original findspot and context to be traced; we shall return to this later. The museum number, 'IM 57828', indicates that it is now in the Iraq Museum in Baghdad (IM) and was the 57,828th artefact to be registered there. Like almost all museum numbers, it gives no information at all about the tablet's origins.

The writing on the tablet is a script composed of wedge-shaped impressions now known as cuneiform (figure 1.3). It runs horizontally from left to right (whatever the direction of the columns it is in) and where there are line rulings (as on the bottom right of this tablet) the signs hang down from the lines rather than sitting on them. As can be seen from the top left of the tablet, though, the lines were not always ruled.

The appearance and structure of cuneiform changed significantly over the course of its history, but some common key features remained throughout. In the earliest written documents, from the late fourth millennium BCE, there were two distinct sign types. Numbers were *impressed* into the surface of the clay, while other signs were *incised* into stylistic representations. Most of the non-numerical signs were *logograms* and *ideograms*; that is, they represented whole words or idea. A small subset acted as *determinatives*, classifying the words they were attached to as belonging to a particular category, such as vessels, wooden objects, or place-names. Determinatives did not represent parts of speech but were aids to reading the words with which they were associated. Many of the non-numerical signs were *pictographic*; that is, they looked like the objects they represented, but others were more abstract shapes. Each sense unit was grouped into *cases*, rectangular areas ruled onto the surface of the tablet.

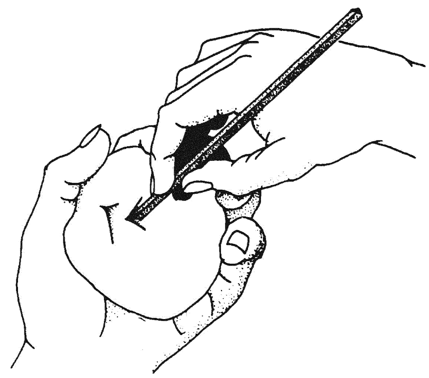

Figure 1.3 Writing cuneiform on a clay tablet with a stylus. (Drawing by the author.)

The early writing system was very limited in scope and function, but over the first half of the third millennium BCE it gradually acquired more power and flexibility by using the same signs to represent not just an idea or a thing, but the sound of that word in the Sumerian, and later Akkadian, language. Thus, as writing became language-specific and acquired the ability to record *syllables* it became increasingly important to arrange the signs into lines on the surface of the tablet, following the order of the spoken language. At the same time, the signs themselves lost their curviform and pictographic visual qualities, becoming increasingly cuneiform and visually abstract. However, signs retained their ideographic significance even as they acquired new syllabic meanings. In other words, they became *multivalent*; that is, a single cuneiform sign could have as many as twenty different meanings or *values* depending on the context in which it was used. For instance (table 1.3), the sign DUG functioned as a determinative in front of the names of pottery vessels; as a logogram with the meaning 'pot' (Sumerian *dug* = Akkadian *karpatum*) or 'cup' (Sumerian *lud* = Akkadian *luṭṭum*); and as an Akkadian syllabogram with values *dug, duk, duq*, *tuk, tuq* and *lud, lut, luṭ*. Conversely, the writing system also encompassed *homovalency*; that is, the potential for different signs to represent the same syllables; some of these, however, were at least partially contextually determined. For instance, there are three other signs with the value 'dug': one is just a syllable, while the others are logograms for the adjective 'good' and the verb 'to speak'. The total repertoire of cuneiform signs is around six hundred, but not all of those signs, or all possible values of a sign, were in use at any one time or in any one genre of text.[15]

In modern transliteration, determinatives are shown in superscript (there are none in 2N-T 30), and Sumerian syllables in normal font. Akkadian

TABLE 1.3
Four Different Cuneiform Signs that Take the Value *Dug*

Late IV mill.	*III mill.*	*II mill.*	*I mill.*	*Determinative Values*	*Logographic Values*	*Main Syllabic Values*
				dug 'pottery vessel'	dug = *karpatum* 'pot'; lud = *luṭṭum* 'cup'	*dug, duk, duq, tuk, tuq* and *lud, lut, luṭ*
—				—	du$_{12}$ = *zamārum* 'to play (a musical instrument)'; tuku = *rašûm* 'to acquire'	*tug, tuk, tuq, duk, raš*
				—	dug$_3$ = *ṭâbum* 'to be good, sweet'; ḫi = *balālum* 'to mix'	*ḫi, ḫe, ṭa*
				—	dug$_4$ = *qabûm* 'to speak'; gu$_3$ = *rigmum* 'noise'; inim = *awatum* 'word'; ka = *pûm* 'mouth'; kir$_4$ = *būṣum* 'hyena'; kiri$_3$ = *appum* 'nose'; zu$_2$ = *šinnum* 'tooth'; zuḫ = *šarāqum* 'to steal'	*ka, qa*

syllables are written in italics, and logograms in small capitals. (But in translations, this book shows all untranslated words—such as metrological units—in italics, whether they are in Sumerian or Akkadian.) Homovalent signs are distinguished by subscript numbers, as for instance kuš$_3$ or si$_8$. Signs that are missing because the tablet has broken away are restored in square brackets: [si$_8$] or in half brackets ⌈ǧeš⌉ if the surface is damaged and the sign not clearly legible; uncertain signs can also be signalled by a superscript question mark: še$^?$ and erroneously formed signs indicated with an exclamation mark: ǧal$_2^!$. Sometimes scribes omitted signs; editors restore them inside angle brackets: <ti>. When a scribe has erroneously written an extra sign it is marked in double angle brackets: «na». Modern editor's glosses are given in parentheses: (it is), with comments in italics: (*sic*).

Cuneiform was used to record many different languages of the ancient Middle East, just as the Roman alphabet today is not reserved for any one language or group of languages. Only two were used extensively for mathematics, however: Sumerian and Akkadian. Sumerian was probably the first language in the world to be written down and maybe because of that it appears to be a language isolate: that is, it is genetically related to no other known language, living or dead. It seems to have died out as a mother tongue during the course of the late third and early second millennium BCE, but continued to be used as a literary and scholarly language (with a status similar to Latin's in the Middle Ages) until the turn of the common era. It does, of course, have features in common with many languages of the world. In the writing system four vowels can be distinguished—a, e, i, u—and fifteen consonants b, d, g, ĝ, ḫ, k, l, m, n, p, r, s, š, t, and z (where ḫ is approximately equivalent to kh or Scottish ch in 'loch', ĝ to ng in 'ring', and š to sh in 'shoe'). Words are formed by *agglutinating* strings of grammatical prefixes and suffixes to a lexical stem, as can happen to some extent in English, for instance with the stem 'do': un-do-ing. The main feature of the Sumerian case system is *ergativity*: grammatical distinction between the subjects of transitive and intransitive verbs. Compare in English the intransitive 'the man walked' with transitive 'the man walked the dog'; in Sumerian the second 'man' is in the ergative case, 'the man(-erg.) walked the dog'. Its gender system is a simple dichotomy, people/others, and its word order is verb-final. Instead of tense, Sumerian uses aspect, distinguishing between completed and incomplete action. Thus to completely Sumerianise 'the man walked the dog', we would have 'man(-erg.-pers.) dog(-other) (he-it-)walk(-compl.). It is perhaps not surprising that there is still much argument about the details of how Sumerian works and exactly how it should be translated.[16]

Akkadian, by contrast, is very well understood. It is a Semitic language, indirectly related to Hebrew and Arabic. Like Sumerian it was written with the four vowels a, e, i, u and all the consonants of Sumerian (except ĝ) as well as q, ṣ (emphatic s, like ts in 'its'), ṭ (emphatic t), and a glottal stop, transliterated: '. Like other Semitic languages, Akkadian works on the principle of *roots* composed of three consonants, which carry the lexical meaning of words. For instance, the root *mḫr* carries the sense of equality and opposition, while *kpp* signifies curvature. Words are constructed by *inflection*, that is, by the addition of standard patterns of prefixes, suffixes, and infixes in and around the root which bear the grammatical meaning (tense, aspect, person, case, etc.). Thus *maḫārum* and *kapāpum* are both verbal infinitives 'to oppose, to be equal', and 'to curve'. The first-person present tense is *amaḫḫar* 'I oppose', the simple past *amḫur* 'I opposed'. Nouns, adjectives, and adverbs are all derivable from these same roots, so that the reflexive noun *mitḫartum* ‘thing that is equal and opposite to itself’ means a square and *kippatum* ‘curving thing’ is a circle.[17]

The mixed cuneiform writing system enabled polysyllabic Akkadian words to be represented with a logogram, usually the sign for the equivalent Sumerian stem; indeed some signs, especially weights and measures, were almost never written any other way. So while it *appears* that 2N-T 30 is written in Sumerian, it may equally be in highly logographic Akkadian: it is impossible to tell. And just as it is sometimes difficult to judge the language a scribe was *writing* in, we cannot even begin to determine what language he might have been speaking or thinking in. The death of Sumerian as a written and/or spoken language is still a hotly debated issue, but it must have taken place over the last third of the third millennium and/or the first half of the second.[18]

The tablet 2N-T 30 uses two different systems of numeration. The numbers on their own in the top left corner are written in the *sexagesimal place value system* (SPVS), while those in the main text belong to various *absolute value systems* that were used for counting, weighing, and measuring. The earliest known written records, the temple account documents from late fourth-millennium Uruk, used a dozen or so very concrete numeration systems that were not only absolute in value but were also determined by the commodity being counted or measured (table A.1 and see chapter 2). Over the course of the third millennium this bewildering variety of metrologies was gradually rationalised to five: length; area, volume, and bricks; liquid capacity; weight; and the cardinal (counting) numbers (table A.3 and see chapter 3). Although the number sixty became increasingly prominent, all systems except the weights retained a variety of bases (comparable to more recent pre-decimal Imperial systems) and all used signs for metrological units and/or different notations for different places. For instance, the length of the square in 2N-T 30 is ⅓ cubit and ½ finger. A cubit was approximately 0.5 m long, and comprised 30 fingers—so the square is 10½ fingers long, c. 17.5 cm.

But the interface between the different systems was never perfect: for instance, there was an area unit equal to 1 square rod, but none equal to 1 square finger or 1 square cubit. In other words, the exercise on 2N-T 30 is not a simple matter of squaring the length in the units given: it is first necessary to express that length in terms of rods, where the two systems meet nicely. Then the length can be squared, to give an area expressed in (very small) fractions of a square rod (c. 36 m^2) or, more appropriately, integer numbers of smaller area units. The area system borrowed the sexagesimal divisions of the mina weight for its small units, so that a sixtieth part of a square rod is called a shekel (c. 0.6 m^2, or 2.4 square cubits), and a 180th part of a shekel is known as a grain (c. 33 cm^2, or 12 square fingers).

The area in 2 N-T 30 is almost too small for the area system to handle, so the scribe has approximated it as 9⅕ grains.

The function of the sexagesimal place value system (SPVS), which came into being during the last centuries of the third millennium (§3.4), was thus to ease movement between one metrological system and another. Lengths, for instance, that were expressed in sexagesimal fractions of the rod instead of a combination of rods, cubits, and fingers, could be much more easily multiplied into areas expressed first in terms of square rods, and then converted to more appropriate units if necessary. The SPVS, in other words, was only a calculational device: it was *never* used to record measurements or counts. That is why it remained a purely positional system, never developing any means of marking exactly how large or small any number was, such as the positional zero or some sort of boundary marker between integers and fractions. In other words, these deficiencies were not the outcome of an unfortunate failure to grasp the concept of zero, but rather because neither zeros nor sexagesimal places were necessary within the body of calculations. For the duration of the calculation, then, the absolute value of the numbers being manipulated is irrelevant; only their relative value matters—as long as the final result can be correctly given in absolute terms.

This is apparent on the top left corner of 2N-T 30, where the scribe has expressed 10½ fingers as a sexagesimal fraction of a rod, namely as 1 45. In modern transliteration sexagesimal places are separated by a space or comma, rather like the reading on a digital clock. But in translation it is often useful to show the absolute value of a sexagesimal number if known, so we can write 0;00 01 45, where the semicolon is a 'sexagesimal point' marking the boundary between whole and fractional parts of the number. In cuneiform no spaces are left between sexagesimal places, because there are separate ciphers for ten signs and units, but like modern decimal notation it is a *place value* system. That is, relative size is marked by the order in which the figures are written, in descending order from left to right. For instance, 22 45 (= 1365) could never be confused with 45 22 (= 2722). On 2N-T 30, the number 1 45 has been written twice, because it is to be squared, but the resulting 3 03 45—better, 0;00 00 03 03 45 square rods—has not been recorded. Nevertheless, it is clear from the final answer that the scribe got the right result: he must have multiplied up by 60 (shekels in a square rod) and then by 180 (grains in a shekel). The exact sexagesimal answer is 9;11 15 grains, which the scribe had to approximate as 9⅕ grains (i.e., 9;12), because the absolute systems did not use sexagesimal fractions but only the simple unit parts $1/n$.

In sum, a good idea of the mathematical sense of 2N-T 30 can be gained from its internal characteristics alone: its contents could be summarised simply as $0;00\ 01\ 45^2 = 0;00\ 00\ 03\ 03\ 45$. But that would leave many questions unanswered: Who wrote the tablet, and under what circumstances? Who, if anyone, did he intend should read it? Why didn't he record the sexagesimal result but only the version converted into area measure? What else did he write? What else did he do with his life? Such questions, once one starts to ask them, are potentially endless, but the tablet alone cannot answer them: we have look beyond, to its textual, archaeological, and socio-historical context if we want to know more.

1.3 THE CONTEXTS: TEXTUALITY, MATERIALITY, AND SOCIAL HISTORY

In the early 1990s Jens Høyrup pioneered the study of syntax and technical terminology of Old Babylonian mathematical language, with two aims in view.[19] First, a close reading of the discourse of mathematics reveals, however approximately, the conceptual processes behind mathematical operations—which translation into modern symbolic algebra can never do. This line of enquiry is most profitable on more verbose examples than 2N-T 30, particularly those that include instructions on how to solve mathematical problems. Many examples are presented in later chapters. Such work can be carried out, and largely has been, on modern alphabetic transliterations of the ancient texts with little regard to the medium in which they were originally recorded. But the idea of close examination can also apply to non-linguistic features: the disposition of text and numerals on the tablet, the presence and presentation of diagrams, the use of blank space, and so forth. In the case of 2N-T 30 one might be interested in the spatial separation of word problem and solution from the numerical calculation, as well as the fact that there are no traces on the tablet of the scribe's actual working or sexagesimal answer.

Second, looking at clusters of linguistic and lexical features enables unprovenanced tablets to be grouped and separated along chronological and geographical lines. This approach can also be supported by tracing the distribution of problem types and numerical examples within the known corpus and comparing the techniques of calculation used to solve them. Doing that for 2N-T 30 yields six other Old Babylonian tablets very like it (but with different numerical parameters) and eight tablets containing just the squaring calculation without the setting and solution expressed metrologically (table 1.4 and figure 1.4).

The large majority of the tablets are from Nippur, like 2N-T 30, but it is difficult to tell whether that is historically significant or is more an outcome of the history of excavation and publication. However, it probably is noteworthy that all the Nippur tablets are square, while those from Ur are both round (technically speaking Type R and Type IV respectively: see

Table 1.4
Old Babylonian Calculations of Squares

Tablet Number	*Provenance*	*Length of Square*	*Sexagesimal Calculation*	*Tablet Description*	*Publication*
NCBT 1913	Unknown	<58 rods, 4 cubits>	$58\ 20^2 =$ $56\ 42\ 46\ 40$	Round. The problem is not set metrologically, but the answer is given in both SPVS and area units. Horizontal line at the end; erasures.	Neugebauer and Sachs 1945, 10
NBC 8082	Unknown	1 rod, 4 cubits	$1\ 20^2 = 1\ 46\ 40$	Square with rounded corners; lower right corner missing. Numbers auxiliary to the calculation are written down the left-hand side of the tablet, separated from the rest by a vertical line.	Neugebauer and Sachs 1945, 10 (no copy); Nemet-Nejat 2002, no. 13
2N-T 472 = UM 55-21-76	Nippur	1 cubit, 2 fingers	$[5\ 20^2 = 28\ 36\ 40]$	Square with rounded corners; upper half missing.	Neugebauer and Sachs 1984 (no copy); Robson 2000a, no. 7
CBS 11318	Nippur	1 cubit	$5^2 = 25$	Square with rounded corners; lower left corner missing.	Neugebauer and Sachs 1984
2N-T 116 = IM 57846	Nippur	⅔ cubit, 9 fingers	$[9\ 40^2 = 1\ 33\ 26\ 40]$	Tablet shape unknown.	Neugebauer and Sachs 1984 (no copy)

2N-T 30 = IM 57828	Nippur	⅓ cubit, ½ finger	1 45^2 = <3 03 45>	Square with rounded corners.	F. R. Steele 1951, pl. 7; Neugebauer and Sachs 1984
UM 29-15-192	Nippur	2 fingers	20^2 = 6 40	Square with rounded corners.	Neugebauer and Sachs 1984
UM 29-16-401	Nippur	—	44 26 40^2 = 32 55 18 31 06 [40]	Square with sharp corners? (Now all broken.) Horizontal line above and below answer.	(See table B.10)
3N-T 611 = A 30279	Nippur	—	16 40^2 = 4 37 46 40	Square with rounded corners.	(See table B.10)
UET 6/2 211	Ur	—	16 40^2 = 4 37 46 40	Round.	Robson 1999, 251
CBS 3551	Nippur	—	7 36^2 = 57 47 (*sic*, for 45) 36	Square with sharp corners. Erasures. Horizontal line below answer.	(See table B.10)
N 3971	Nippur	—	7 05^2 = 50 10 25	Square with rounded corners. Erasures.	Robson 1999, 275
CBS 7265	Nippur	—	5 15^2 = 27 33 45	Rectangular, portrait orientation.	(See table B.10)
HS 232	Nippur	—	4 50^2 = $23^{!}$ 21 40	Square with sharp corners? Horizontal line below answer.	Friberg 1983, 82
UET 6/2 321	Ur	—	1 33 45^2 = <2 26 29 03 45>	Round. The answer has been erased.	Robson 1999, 251

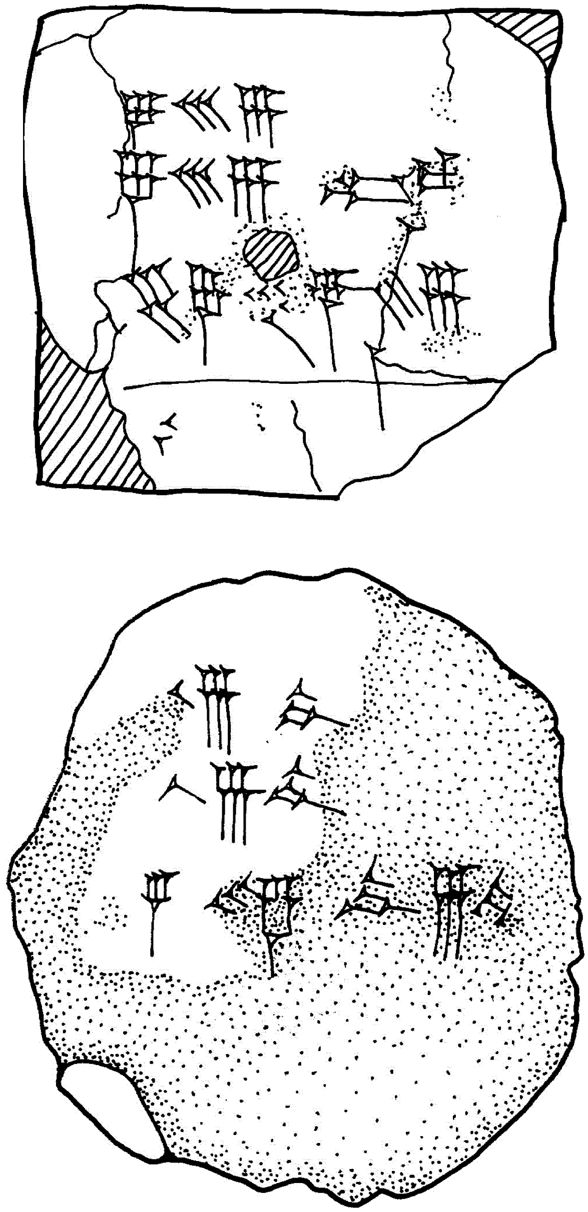

Figure 1.4 Two Old Babylonian exercises in finding the area of a square. (CBS 3551 and UET 6/2 211. Robson 2000a, no. 4; 1999, 521 fig. A.5.7.)

table 4.5). None of them has anything written on the back. Further, there is a striking similarity in textual layouts: all the Nippur tablets which set the problem in metrological terms write it in a box in the bottom right corner, just like 2N-T 30. Even more importantly, every single one of the sexagesimal calculations is laid out in the same way, with the length written twice in vertical alignment and the answer immediately underneath. Only the sharp-cornered square tablets mark it with a horizontal ruling, and 2N-T 30 is the only one of its kind with no sexagesimal solution. Some show traces of erased signs that suggest the remains of scratch calculations.

A further search would yield a list of mathematical problems about squares with the parameters now missing, and a compilation of simple geometrical problems about squares with diagrams but no answers.[20] A large tablet from Susa also catalogues (amongst other things) thirty statements and answers of the squaring problem, with lengths running systematically from 1 finger to 4 rods, just like the entries in a metrological list (see §4.2).[21] The hunt could go on, but enough evidence has already accumulated to show that elementary exercises in finding the area of a square were common in their own right, quite apart from squarings carried out in the course of solving other sorts of problems. That strongly suggests that the purpose of at least some cuneiform mathematical activity was pedagogical.

This discussion has already touched on the fact that the shape of tablets, as well as what is written on them, can be important. Once historians start to consider the material culture of mathematical cuneiform *tablets* as well as the features of ancient mathematical *texts,* then they become increasingly sensitive to their identities as archaeological artefacts with precise findspots and belonging to complex cultural assemblages. If mathematical tablets come from a recorded archaeological context, then they can be related to the tablets and other objects found with them as well as to the findspot environment itself.

Controlled stratigraphic excavation has been carried out to great effect in Iraq for over a century now. In the years before the First World War, the German teams led by Robert Koldewey and Walter Andrae at Assur, Babylon, and elsewhere set impeccable standards which have been followed, more or less, by most academic archaeological teams from all over the world ever since. However, scrupulous recording of the context of artefacts quite simply generated too much data for pre-computerised analyses to manage, however much painstaking and time-consuming work was put into their publication (it is not uncommon for final reports to appear decades after the last trench was scraped). Tablets in particular have often become separated from their findspot information. It is only in the last few years with the advent of mass-produced relational database programmes that large finds of epigraphic material have been satisfactorily analysed contextually. The tablet 2N-T 30 constitutes a good example. It was excavated in 1948 from an archaeological site in the ancient city of Nippur in southern Iraq, now in the middle of desert but once on a major artery of the Euphrates. In the early days of exploration, expeditions were sent out to Iraq with the express goal of recovering as many artefacts as possible for their sponsoring institutions. The University of Pennsylvania organised several large-scale trips to Nippur in the 1880s and '90s, bringing back tens of thousands of tablets for its museum and depositing many more in the Imperial Ottoman Museum of Istanbul (for at that time Iraq was still a province of the Ottoman empire). Modern archaeology no longer operates like that; instead, it focusses on small areas which are carefully chosen with the aim of answering specific research questions. After the Second World War, the Universities of Philadelphia and Chicago jointly initiated a new series of excavations at Nippur, which ran on and off until the Gulf War brought all archaeology in Iraq to a halt. One of their early aims was to understand better the area where their Victorian precursors had found a spectacularly large trove of Sumerian literary and scholarly tablets—not least because there had been a major controversy over whether the tablets constituted a putative temple library or not, an issue which could not be resolved at the time because the process of excavation had never been recorded.[22] The later excavators chose, in 1947, to open a trench in one of the old diggings, on a mound the Victorians had dubbed 'Tablet Hill', in the twin hopes of recovering more tablets of the same kind and of learning more about their origins. Area TB, as they called it (T for Tablet Hill, B because it was their second trench on the mound) measured just 30 by 40 m. When in the second season they reached Old Babylonian levels, they found not the monumental walls of temple architecture but well-built and spacious houses, densely grouped together. The dwelling they labelled House B had been rebuilt many times over the course of its useful life. In the second layer from the surface they found fifty-three tablets in the central courtyard and four of its six rooms (loci 10, 12, 17, 31, 45 in figure 1.5) as well as the remains of domestic pottery. Tablet 2N-T 30 was in room 12.[23]

Unfortunately, although the excavation was published in the early 1960s, and has been reanalysed since, there has been no systematic study of the tablets found there. Partly this is a result of the division of finds between Philadelphia, Chicago, and the Iraq Museum in Baghdad, but it is more a symptom of the fact that tablets in general have not tended to be treated as archaeological artefacts. Nevertheless, there are enough data in the original excavation notes and in an unpublished catalogue of the 2N-T tablets to enable the assemblage of tablets found in House B to be reconstructed. It turns out that 2N-T 30 is just one of eight mathematical tablets found there (table 1.5), most of which are elementary calculations of squares or regular reciprocal pairs (for which see §4.3) while the other two are extracts

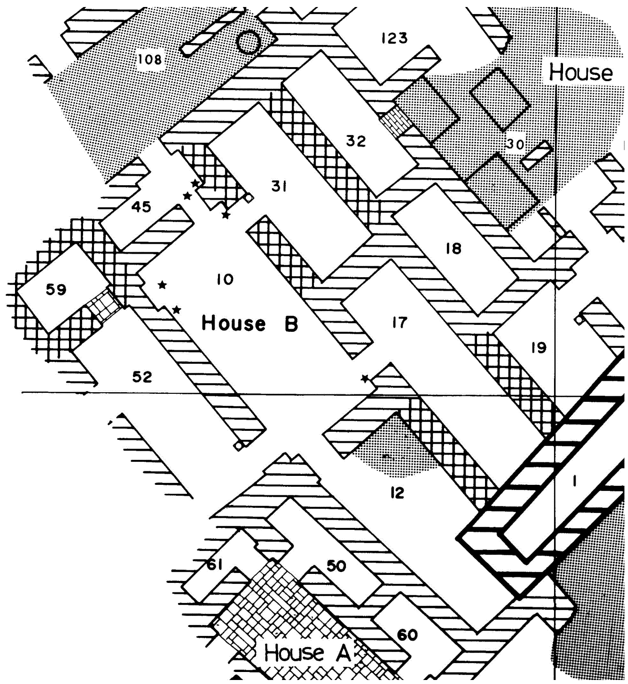

Figure 1.5 House B, Area TB, Level II.1, in Old Babylonian Nippur, excavated in 1948. Mathematical tablets were found in rooms 10, 12, and 45. (Stone 1987, pl. 30, courtesy of The Oriental Institute of The University of Chicago.)

from standard pedagogical lists (for which see §4.2). Two of the calculations are written on the same tablets as Sumerian proverbs from a standard scholastic compilation now known as Sumerian Proverbs Collection 2, which mostly set out appropriate behaviours for scribes (see further §4.4). Extracts from that proverb collection are found on nine further tablets, and elementary Sumerian literary compositions on nine more. All but five of the remaining tablets contain standard lists of cuneiform signs, Sumerian words, or other well-attested items in the curricular repertoire of Old Babylonian Nippur.[24] The curricular context of mathematics is examined in more detail in chapter 4, but a reasonable provisional conclusion would be that somebody wrote 2N-T 30 as part of a general scribal education that was quite standardised across the city, and that House B was probably either his own home or his teacher's.

Situating individual objects, and thereby sub-corpora of mathematics, within an archival and physical environment thus enables consideration of the *people* involved in creating, learning, and transmitting mathematical ideas, and to relate those ideas to the wider Mesopotamian world. The

Table 1.5
Mathematical Tablets Found in House B of Area TB, Old Babylonian Nippur

Excavation Number	*Museum Number*	*Findspot*	*Description*	*Publication*
2N-T 27	IM 57826	Room 12, level II-1	Calculation of regular reciprocal pair (7 55 33 20 ~ 7 34 55 18 45); erased exercise on reverse; rectangular tablet, c. 5.5 × 7 cm	Unpublished
2N-T 30	IM 57828	Room 12, level II-1	Problem and calculation of square; square tablet, c. 6.5 × 6.5 cm	(See table 1.4)
2N-T 35	IM unknown	Room 12, level II-1	Fragment of metrological list or table, c. 4 × 4 cm	Unpublished
2N-T 115	IM 57845	Room 45, level II-1	Calculation of regular reciprocal pair (9 28 53 [20] ~ 6 19 [41 15]); illegible exercise; multiplication table (times 1 40) on reverse; fragment c. 5.5 × 4.5 cm	Neugebauer and Sachs 1984
2N-T 116	IM 57846	Room 45, level II-1	Calculation of square; c. 6.5 × 7.5 cm	(See table 1.4)
2N-T 131	IM 57850	Room 10, level II-1	Fragment of table of squares, c. 5.5 × 3 cm	Unpublished
2N-T 496	IM 58966	Room 10, level II-2	Calculation of regular reciprocal pair (16 40 ~ 3 36) and Sumerian Proverb 2.42; rectangular tablet, c. 8.5 × 8 cm	Al-Fouadi 1979, no. 134; Alster 1997, 304; Robson 2000a, 22–3
2N-T 500	A 29985	Room 10, level II-1	Calculation of regular reciprocal pair (17 46 40 ~ 3 22 30) and Sumerian Proverb 2.52; rectangular tablet, c. 8 × 7.5 cm	Gordon 1959, pl. 70; Alster 1997, 55; Robson 2000a, 21–2

socio-historical context of ancient Iraqi mathematics can be split into three concentric spheres: the inner zone, closest to the mathematics itself, is the scribal *school* in which arithmetic and metrology, calculational techniques, and mathematical concepts were recorded as a by-product of the educational process. As House B illustrates, schools were not necessarily large institutions but could simply be homes in which somebody taught young family members how to read, write, and count. Who was mathematically educated, and to what degree? Who taught them, and how? The answers to these questions are only partially answerable at the moment, and differ depending on the time and place, but attempts will be made chapter by chapter for all the periods covered in this book.

Beyond school is the sphere of *work*: the domain of the professionally numerate scribal administrator who used quantitative methods of managing large institutions and, in later periods, of the scholar who used patterned and predictive approaches to the natural and supernatural worlds. Tracking the continuities and disjunctions between work and school mathematics can enable us to estimate the extent to which mathematical training equipped scribes for their working lives. This is a potentially enormous topic, so for manageability's sake the book focuses on the domains of land and labour management, though it could equally well have highlighted livestock, agriculture, manufacturing, or construction. Equally, other areas of professionally literate intellectual culture are analysed for traces of mathematical thinking in literature, divination, and of course astronomy.

The outer sphere extends beyond the literate to various aspects of Mesopotamian *material culture*: an ethno-mathematical approach can, for instance, identify the external constraints on mathematical thinking, or use the detailed insights of the mathematical texts to reveal the conceptualisation of number, space, shape, symmetry, and the like in other aspects of ancient life, thereby encompassing the lives and thoughts people with no professional mathematical training. The theory and methods of ethnomathematics were developed by scholars such as Ubiratan D'Ambrosio and Marcia Ascher for the study of cultures without writing;[25] but an ethnomathematical approach is just as applicable to the areas of literate societies that are beyond the reach of written mathematics. It proves a particularly useful tool for understanding ancient Iraq, in which only a tiny minority of professional urbanites could read and write.

But the ancient world was not just composed of social institutions: it was populated by individuals with families, friends, and colleagues. So this book also spotlights individual creators, transmitters, and users of mathematical thoughts and ideas. That does not mean 'great thinkers' along the lines of Euclid or Archimedes or Ptolemy, but rather school teachers and pupils, bureaucrats and accountants, courtiers and scholars. Some of this evidence takes the form of names on tablets, but even anonymous writings can be grouped by handwriting, stylistic idiosyncrasies, and subject matter. Even when a tablet's origins are unknown, its orthography (spelling conventions), palaeography (handwriting), layout, shape, size, and other design features can all give important indications of the time and place of composition as well as clues about its authorship and function.

Such prosopographic work is necessary primarily because large numbers of tablets have no known context, having been acquired on the antiquities market by dealers, museums, and collectors. Before the creation of the modern Iraqi state and the drafting of strong antiquities laws in the 1920s[26] even the big Western museums employed agents to undertake opportunistic or systematic searches for 'texts' which, they were optimistic enough to suppose, could 'speak for themselves' without recourse to archaeological context. More recently, since the Gulf War of 1991 and again since the Iraq War of 2003, many more cuneiform tablets have flooded the international antiquities market. Some can still be found on sale today, despite increasingly stringent international legislation. This glut was caused first by the looting of provincial museums after the Gulf War, later fuelled by the Iraqi economic crisis of the 1990s, and more recently provoked by the invasion and insurgency since 2003. International sanctions against Iraq followed by gross post-war mismanagement led both to the virtual collapse of the State Board of Antiquities and Heritage, which has responsibility for the protection of some ten thousand identified archaeological sites nationwide, and to sudden and desperate poverty across larges swathes of a previously prosperous population. Now, as in former times, the perpetrators have been both impoverished local inhabitants who exploit the tells for profit like any other natural resource of the area, and naive or unscrupulous purchasers whose paramount goal has been the recovery of spectacular objects for public display or private hoarding. Almost every cuneiform tablet on sale today, whether in a local antiques shop, through a major auction house, or on the web, is stolen property. In most countries it is technically a criminal offence to trade them. But legality apart, the example of 2N-T 30 has shown that the archaeological context of a cuneiform tablet is as crucial to the holistic decipherment of that tablet as the writing it carries. While historians have to do the best they can with the context-free tablets that came out of the ground in the bad old days, anyone at all committed to the sensitive understanding of the past should resist the temptation to buy. Cuneiform tablets and other archaeological artefacts must be allowed to remain undisturbed until they can be excavated and studied with the care and attention that they and the people who wrote them deserve.