The Long Career of a Favorite Figure: The *apsamikku* in Neo-Babylonian Mathematics

Eleanor Robson
University of Cambridge

Introduction

The *apsamikkum*, (GANA$_2$) GEŠTUG$_2$.ZÀ.MÍ, or concave square, is a well-known geometrical figure in Old Babylonian (OB) mathematics (see figure 15.1).[1] It features in eleven entries from five coefficient lists of technical constants; two, possibly four, illustrated statements of problems; and in two problems with worked solutions.[2] From this evidence it has been deduced that the *apsamikkum* was composed of four identical quarter-arcs, as if four identical circles were placed tangent to each other. Its main components are the external arc (SAG, *pūtum*) of length 1; the diagonal (DAL, *tallum*), running corner-to-corner, of length 1;20; and the short transversal (*pirkum*), running across the narrowest part of the figure, of length 0;33 20. Its area is 0;26 40, or $\frac{4}{9}$, of the arc squared.[3]

The exact etymology of the Akkadian word remains contentious, though the most plausible remains Goetze's suggested derivation from Sumerian AB ZÀ.MÍ, 'window of the lyre,' clearly related to the Sumerogram GEŠTUG$_2$.ZÀ.MÍ, literally 'ear of the lyre,' both presumably referring to a sound hole.[4] As Anne Kilmer has pointed out, the figure is also highly reminiscent of the shape of the bovid's noses on lyres from the Early Dynastic Royal Cemetery of Ur, suggesting an etymology ÁB, 'cow,' rather than AB, 'window.'[5] However, the logogram ÁB.ZÀ.MÍ is attested only in first-millennium liver omens,[6] which

1. Figure 15.1 is reproduced from Eleanor Robson, *Mesopotamian Mathematics, 2100–1600* B.C.*: Technical Constants in Bureaucracy and Education*, Oxford Editions of Cuneiform Texts, vol. 14 (Oxford: Clarendon Press, 1999), 53, fig. 3.13.

2. See the Appendix for translations and publication details of all these occurrences.

3. Robson, *Mesopotamian Mathematics*, 17–21.

4. Albrecht Goetze, "A Mathematical Compendium from Tell Harmal," *Sumer* 7 (1951): 139.

5. Anne Kilmer, "Sumerian and Akkadian Names for Designs and Geometrical Shapes," in *Investigating Artistic Environments in the Ancient Near East*, ed. Ann Gunter (New York: Smithsonian Institution, 1990), 88–89.

6. From Aššurbanipal's library at Nineveh: BE *ina* EGIR NÍG.TAB GÍR *šá* 2 30 UZU

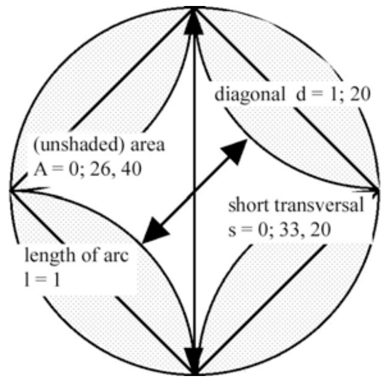


Figure 15.1: The Old Babylonian *apsamikkum*

implies a rather late folk etymology. Nevertheless the suggestion remains an attractive one. There are also Neo-Assyrian descriptions of the constellations Ursa Major and Cancer as *áp-sà-ma-ak-ku*,[7] but without accompanying illustrations it is impossible to know how these were visualized.

Concave squares are also ubiquitous in Mesopotamian visual culture, from Halaf pottery of the sixth millennium B.C. to images of Neo-Assyrian textile

---

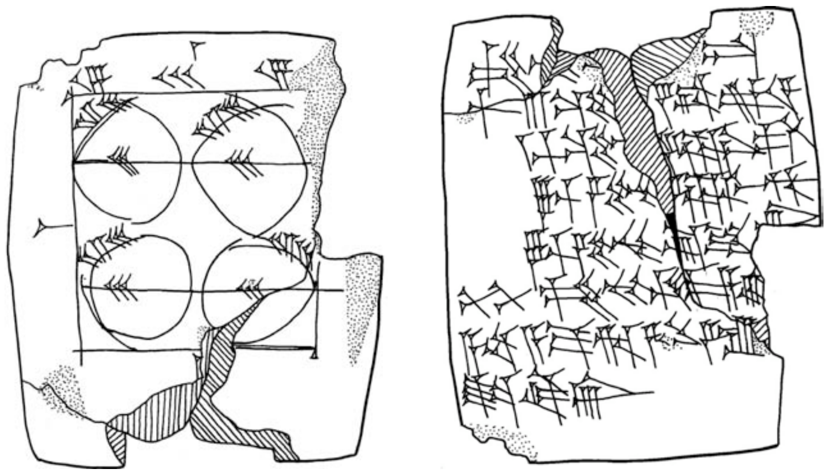
GIM ÁB.ZÀ.MÍ-*ma* RA-*iṣ*, 'If there is a piece of flesh flattened like an *apsamikku* in the rear of the Dyeing Vat to the left of the Path. . . .' *Pān tākalti* Tablet 2, lines 75, 76, 83 and 84, Aššurbanipal colophon type 1, published in Ulla Koch-Westenholz, *Babylonian Liver Omens: The Chapters* Manzāzu, Padānu, *and* Pān tākalti *of the Babylonian Extispicy Series, Mainly from Aššurbanipal's Library*, CNI Publications, vol. 25 (Copenhagen: The Carsten Niebuhr Institute of Near Eastern Studies), 308–09. For the handcopy, see Reginald Campbell Thompson, *Cuneiform Texts from Babylonian Tablets, etc. in the British Museum*, vol. 20 (London: Longmans, et al., 1904), pl. 33. BE *ina* ŠUB AŠ.TA UZU GIM ÁB.ZÀ.MÍ, 'If in the Throne Base there is a piece of flesh like an *apsamikku*. . . .' *Pān tākalti* Tablet 9, line 49, published in Koch-Westenholz, *Babylonian Liver Omens*, 364; handcopy on pl. XXII (Sm 373).

From Seleucid Uruk: BE SILIM GIM ÁB.ZÀ.MÍ-*ma* RA-*iṣ*, 'If the Wellbeing is flattened like an *apsamikku*, ...' *Pān tākalti* Tablet 6, line 86, published in Koch-Westenholz, *Babylonian Liver Omens*, 352. For the handcopy, see Françqis Thureau-Dangin, *Tablettes d'Uruk à l'usage des prêtres du temple d'Anu au temps des Séleucides*, Textes Cunéiformes de Louvre, vol. 6 (Paris: Librarie Orientaliste Paul Geunther, 1922), pl. IX, ln. 35'. BE ŠÀ.NIGIN GIM ÁB.ZÀ.MÍ, 'If the coils are like an *apsamikku*, ...' *Šumma tīrānu* Tablet 3, line 27 in Albert Clay, *Babylonian Records in the Library of J. Pierpont Morgan*, vol. 4 (New Haven: Yale University Press, 1923), 32. Both tablets were written in year 99 of the Seleucid Era for Nidinti-Anu, son of Anu-bēlšunu, descendant of Ekur-zākir, *āšipu* of Anu and Antu.

7. $^{\text{mul}}$AL.LUL *áp*-⌜*sà*⌝-[*ma*]-⌜*ak*⌝-[*ku*], 'Cancer is an *apsamikku*;' $^{\text{mul}}$MAR.GÍD.DA *áp-sà-ma-ak-ku*, 'Ursa Major is an *apsamikku*': VAT 9428 13, r4 from Assur, translated by Ernst Weidner, "Eine Beschreibung des Sternhimmels aus Assur," *Archiv für Orientforschung* 4 (1927): 73–85.

designs.[8] One is also identifiable in TSŠ 77, an Old Babylonian school tablet from Kisurra (itself perhaps a mathematical exercise).[9] It is not surprising, then, to discover an *apsamikku(m)* in Neo-Babylonian (NB) guise on a mathematical tablet in the British Museum, on a tablet explicitly "written for the scribe [...] to see." I present a discussion of that tablet here in honor of Alice Slotsky's work on Neo- and Late Babylonian scientific materials in the British Museum, and trust that she and her scholarship will remain as elegant, popular and productive as the *apsamikku(m)* for many years to come.[10]

BM 47431, a Neo-Babylonian Mathematical Exercise



(a) Obverse (b) Reverse

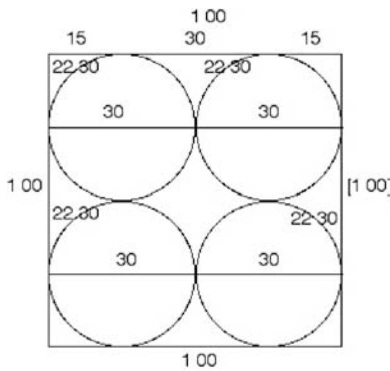
Figure 15.2: BM 47431

8. Kilmer, "Sumerian and Akkadian Names."

9. Marvin Powell, "The Antecedents of Old Babylonian Place Notation and the Early History of Babylonian Mathematics," *Historia Mathematica* 3 (1976): 431, fig. 2. Manfred Krebernik, "Neues zu den Fara-Texten," *Nouvelles assyriologiques brèves et utilitaires* (2006): 15.

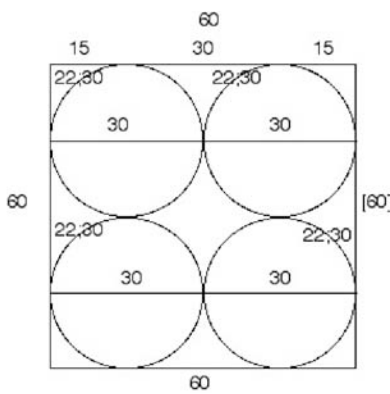
10. As always, I am very grateful to Christopher Walker for his identification and cataloging of the mathematical tablets in the British Museum, which brought this tablet to light. I also warmly thank Duncan Melville, who generously shared a preprint of his article, "The Area and the Side I Added: Some Old Babylonian Geometry," *Revue d'Histoire des Mathématiques* 11 (2005), as well as Eckhart Frahm, Karl-Heinz Kessler, Erica Reiner, Caroline Waerzeggers, and especially Michael Jursa, for invaluable advice on first-millennium orthography and lexicography. BM 47431 is published here with the kind permission of the Trustees of the British Museum.

TRANSLITERATION



1 2 BÁN ŠE.⌜NUMUN ÚŠ⌝.[SA].DU
2 1 BÁN 3 SÌLA ŠE.[NUMUN] 4-*ta* *kip-pat*
3 1 $\frac{1}{2}$ SÌLA ŠE.⌜NUMUN⌝ 4 SAG.DÙ *pa-tar*
4 7 $\frac{1}{2}$ NINDA ŠE.NUMUN ⌜4⌝ SAG.<DÙ> *šal-ḫu*
5 7 $\frac{1}{2}$ NINDA ŠE.NUMUN GANA$_2$.ZÀ.⌜MUK⌝
6 PAB.PAB *2* BÁN ŠE.NUMUN *meš-ḫat* A.ŠÀ [(...)]
7 *a-na a-ma-ri šá* $^{\text{lú}}$*šá-ṭir* [(...)]
8 *ša-ṭi-ir*

TRANSLATION



1 2 BÁN seed measure, surrounding:
2 1 BÁN 3 SÌLA seed measure, 4 circles
3 1 $\frac{1}{2}$ SÌLA seed measure, 4 triangles, daggers
4 7 $\frac{1}{2}$ NINDA seed measure, 4 triangles, outer walls
5 7 $\frac{1}{2}$ NINDA seed measure, concave square
6 Total: *2* BÁN seed measure, the dimension of the area [(...)]
7 In order for the scribe [(...)] to see,
8 it was written.

Notes to the Edition

Obv: The diagram was drawn freehand by the scribe, with little attempt to align the figures within it. In that, it is reminiscent of the diagrams of circles on Old Babylonian rough tablets, intended to be erased or thrown away after use.[11] The analogous diagrams on the OB compilation BM 15285, however, are much more carefully drawn, with rulers and fixed-width compasses (see Appendix B). As in many OB mathematical diagrams (but not BM 15285, as it happens), the lengths of the lines are written, without metrological units, as close as possible to the lines themselves. The configuration of the diagram is identical to BM 15285 §36, minus some verticals and diagonals, and the dimensions are identical (though in BM 47431, as we shall see, the unit of measure is the cubit, not the rod). It is reminiscent too of the diagram of four mutually tangent circles on TSŠ 77 from OB Kisurra, which is often assumed to be a mathematical figure on the basis of its similarity to BM 15285 §36.

Rev. 1: There is room for one or two signs before the final DU. None of the standard first-millennium mathematical uses of that sign seems to fit the context here.[12] The restoration ÚS.SA.DU for *iti*, 'adjoining,' 'surrounding,' or *ṭāḫ*, 'adjacent to' is made on the basis of parallels with NB sale contracts.[13]

Rev. 2: "4" is written with the phonetic complement as 4-*ta* for *erbetta*, 'four (f.)' only in this line, as it describes the feminine *kippat*, 'circle.' Contrary to normal Akkadian usage, NB numbers and nouns agree in gender. *kippatum* is also the standard OB term for circumference and circle (though not with with this CVC spelling of course);[14] it is also written *kip-pat* in W 23291, from late Achæmenid Uruk.[15]

11. For instance HAM 73.2841, published as No. 203 by Marcel Sigrist, *Old Babylonian Account Texts in the Horn Archaeological Museum*, Andrews University Cuneiform Texts, vol. 5; Institute of Archaeology Publications. Assyriological Series, vol. 8 (Berrien Springs, MD: Andrews University Press, 2003), 264; YBC 7302, and YBC 11120, both published by Otto Neugebauer and Abraham Sachs, *Mathematical Cuneiform Texts*, American Oriental Series, vol. 29 (New Haven: American Oriental Society, 1945), 44, all unprovenanced. The second is on a round tablet, the first and third on roughly square ones. All show calculations for finding the area of a circle from its circumference. The first and second have identical calculations, but different layouts; the layouts of the second and third tablets are the same.

12. DU = *alāku*, 'to multiply,' usually written DU-*ik* or DU-*ma* for *alik(ma)*, 'multiply,' or *lu*-DU for *lullik*, 'should I multiply;' DU = GUB in IGI.GUB = *igigubbû*, 'constant, coefficient;' DU = RÁ in A.RÁ = *ana*, 'by,' in multiplication. See Jöran Friberg, " 'Seed and Reeds' Continued: Another Metro-Mathematical Topic Text from Late Babylonian Uruk," *Bagdader Mitteilungen* 28 (1997): 353–55.

13. E.g., Cornelia Wunsch, *Das Egibi-Archiv*, Cuneiform Monographs, vol. 20 (Groningen: STYX, 2000).

14. Robson, *Mesopotamian Mathematics*, 34–35.

15. Friberg, " 'Seed and Reeds' Continued," 353.

Rev. 3–4: *Patru*, 'Dagger, knife,' and *šalḫu*, 'external (city) wall,' are otherwise unattested as geometrical terms.[16] The former term must refer to the thorn (or dagger)-like figures at the centers of the external edges between adjacent circles, the latter to the gaps between the corners of the square and the circles. The sign DÙ is restored in line 4 on the basis of the parallel with rev. 2. The logogram SAG.DÙ = *santakkum* is the standard OB mathematical term for triangle, whose external edges may not necessarily all be straight, as shown by BM 15285 §40. In that instance, SAG.DÙ must refer either to the figures described here as *patru* or to those called *šalḫu*, but not to both (see Appendix B).[17] Compare the writing of GANA$_2$.SAG.DÙ for 'triangle' in the mathematical tablet W 23291 from Late Achæmenid Uruk.[18]

Rev. 5: GANA$_2$.ZÀ.MUK is the first attestation of what must be a Neo-Babylonian logogram for *apsamikku.* This is suggested by the orthographic similarity to the OB writing (GANA$_2$) GEŠTUG$_2$.ZÀ.MÍ (see above) and confirmed by the configuration of the diagram (see the mathematical notes below). GANA$_2$ was commonly used to denote the area of a geometrical figure in both OB and NB mathematics.[19] The replacement of MÍ by MUK may well have had a phonetic rationale, but it also enables a reading ZÀ.MUK = *zagmukku*, 'new year.' Whether the scribe had this allusion in mind, and what he meant by it, is unclear to me.

Rev 6: PAB.PAB X *meš-ḫat* Y (*napḫar* X *mešḫat* Y), 'total: X is the measurement of Y,' is a standard phrase in NB land accounting.[20]

Rev. 7–8: The colophon states the reason why the tablet was made.[21]

16. In rev. 3 the reading SAG.DÙ *àš*(!)*-kut* might also be possible: in NB land sales *aškuttu* refers to little triangular pieces of land. E.g., Karen Nemet-Nejat, *Late Babylonian Field Plans in the British Museum*, Studia Pohl Series Maior, vol. 11 (Rome: Biblical Institute Press), text 47; Wunsch, *Das Egibi-Archiv*, vol. 2, 21, Text 12 line 19. In the late OB compilation of mathematical problems BM 85194 III 1, it is the name of a wedge-shaped volume. See Otto Neugebauer, *Mathematische Keilschrift-Texte*, vol. 1, Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik. Abteilungen A: Quellen, vol. 3 (Berlin: Springer, 1937), 145. If it were the name of a wedge-shaped volume, one would have to explain away: (a) the lack of feminine phonetic complement after the "4," as in rev. 2; (b) the miswriting of *ás* in an otherwise orthographically clear text; and (c) the inclusion of the apparently redundant SAG.DÙ, which is not found with *aškuttu* in the land sale records but apparently used synonymously. See, for instance, Wunsch, *Das Egibi-Archiv*, vol. 2, 21, Text 12, lines 10–11.

17. See for example Robson, "Mesopotamian Mathematics," 40–41.

18. Friberg, " 'Seed and Reeds' Continued," 353.

19. OB: Robson, *Mesopotamian Mathematics*, 38; NB: Friberg, " 'Seed and Reeds' Continued," 353–55 (where it is occasionally also transliterated as AŠA$_5$).

20. *The Assyrian Dictionary*, s.v. *mišiḫtu*, 1.a.2'.

21. For other examples, see Erle Leichty, "The Colophon," in *Studies Presented to A. Leo Oppenheim, June 7, 1964*, eds. R. D. Biggs and J. A. Brinkman (Chicago: The Oriental Institute, 1964), 153; see also Hermann Hunger, *Babylonische und Assyrische Kolophone*, Alter Orient und Altes Testament, vol. 2 (Kevelaer, Butzon und Bercker; Neukirchen-Vluyn: Neukirchener Verlag des Erziehungsvereins, 1968), s.v. *Zweck*.

The word *šāṭiru*, literally 'writer,' appears most often in NB administrative and legal records in the phrase *šāṭir ṭuppi* 'writer of (this) tablet,'[22] although logically that cannot be the referent here. It is not clear whether anything is missing from the end of the line. In the following line, the scribe has written *šaṭir*, the stative form of the verb 'to write,' in a completely different spelling. Such orthographic play is typical of first-millennium colophons.

The tablet is very similar in shape, size (c. 5.5 × 6.5 cm.), and fabric to the Neo-Babylonian field plans and house plans published by Karen Nemet-Nejat; this similarity may suggest a date of Darius I (c. 500 B.C.). There are no other criteria for precise dating. It is also very similar to BM 78822, the Neo-Babylonian mathematical problem and calculation published by Michael Jursa, although that tablet is oriented horizontally, not vertically.[23]

Mathematical Commentary

The diagram on the obverse is annotated in sexagesimal numeration (without metrological units), whereas the prosaic description of the same configuration on the reverse is given in seed measure. Likewise on BM 78822 the prose statements are in seed measure(s), but the calculations on the reverse are in sexagesimal notation.[24] This evidence strongly suggests that, just as in the Old Babylonian period, the sexagesimal place value system was still the preferred notation for calculation, independent of the recording system for numbers, weights, and measures.

From Neo-Babylonian times on, mathematical problems used at least two varieties of seed measure, whereby an area is equated with a fixed capacity of seed, nominally the quantity needed to sow the given area.[25] BM 78822, for instance, implicitly uses both:

'plantation' (*zaq-pa*) seeding rate of 3 00 00 cubits^2 per *pānu*

'cultivation' (*me-re-šú*) seeding rate of 3 20 00 cubits^2 per *pānu*

where 1 square cubit is about 0.25 m^2 and a *pānu* (PI) contains about 36 liters. More often, the first rate was known as the *ašlu* ('cable,' c. 60 m) seed measure, and the second as the *arû* ('multiplication') seed measure. Although nowhere does it state so explicitly, BM 47431 must use the first system, as the area of the figure is stated as 60×60 (cubits) on the obverse and as 2 BÁN (*sūtu*) = $\frac{1}{3}$ *pānu* on the reverse. As in OB times and earlier, we should understand the dimensions of the figure, at about 30 m square, not to be in any sense realistic but rather chosen simply to make calculation as straightforward and elegant as possible.[26]

22. *The Assyrian Dictionary*, s.v. *šaṭāru*, 1.b.

23. Nemet-Nejat, *Late Babylonian Field Plans* and Michael Jursa, "Zweierlei Maß," *Archiv für Orientforschung* 40–41 (1993–4): 71–73.

24. I discuss this tablet in detail in chapter 7 of my forthcoming book, *Mathematics in Ancient Iraq: A Social History* (Princeton: Princeton University Press).

25. See, most comprehensively, Jöran Friberg, Herman Hunger and Farouk Al-Rawi, " 'Seed and Reeds': A Metro-Mathematical Topic Text from Late Babylonian Uruk," *Bagdader Mitteilungen* 21 (1990): 483–557 and Friberg, " 'Seed and Reeds' Continued."

26. OB examples are ubiquitous; for Sargonic examples, see most recently Ben-

For ease of reference, the relevant units of the *ašlu* seed measure system are given below:
1 BÁN (*sūtu*) = 6 SÌLA = 1800 cubits^2, c. 450 m^2
1 SÌLA (*qû*) = 10 NINDA = 300 cubits^2, c. 75 m^2
1 NINDA (*nindanu*) = 30 cubits^2, c. 7.5 m^2.

Although the terminology and metrology of the NB *apsamikku* are radically different than those of the OB *apsamikkum*, the underlying mathematical conception appears to be essentially the same. However, without instructions or calculations given explicitly in the text, this apparent continuity remains unproven.

Rev. 2: Assuming the OB formulæ for the circle, *circumference* = 3 × *diameter* and *Area* = *circumference*^{2}/12,[27] then 4 × *Area* = $\frac{(3\times30)^2}{3}$ = 2700 cubits^2 = 1 BÁN, 3 SÌLA, as given.

Rev. 3–5: If the OB formula *circumference* = 3 × *diameter* is employed, then each quarter-arc *segment* = $\frac{3}{4}$ × *diameter* = $\frac{3}{4}$ × 30 = 22;30 (cubits), as given (but not necessarily used in the calculation). The total square area minus the circles is 3600 − 2700 = 900 cubits^2 or 3 SÌLA. By inspection, this area comprises sixteen corner pieces (*šalḫû*). Together the four central triangles (*patru*) make up half that remaining area: $\frac{3}{2}$ = 1 $\frac{1}{2}$ SÌLA; the four corners (*šalḫû*) make up another quarter = $\frac{3}{4}$ SÌLA, or 7 $\frac{1}{2}$ NINDA; and the central *apsamikku* = $\frac{3}{4}$ SÌLA, or 7 $\frac{1}{2}$ NINDA. All of these figures match those given on the tablet.

The OB method to find the area of the *apsamikku* would be *segment*^{2} × 0;26 40 = 225 cubits^2 = $\frac{3}{4}$ SÌLA, where 0;26 40 is a constant taken from a list (see Appendices A and C).[28] It is thus impossible to determine whether the areas of the central and peripheral figures were calculated or deduced by inspection (although the inscription of the segment lengths on the obverse diagram suggests that calculation was intended). In any case, no fundamentally new conception of the circle (for instance with a different ratio of diameter to circumference) or of the *apsamikku* can be inferred.

Concluding Remarks

This small tablet adds significantly to our knowledge of NB mathematics as well as to the history of the *apsamikku(m)*, which has long been known both as a geometrical term in OB mathematics and as a descriptive figure in other areas of NA and Seleucid intellectual enquiry such as extispicy and celestial description. The text supplies hitherto unattested technical usages for *šalḫû*, ‘outer wall’ and *patru*, ‘knife’ as trilateral figures with respectively one and two curved edges as well as the new writing GANA$_2$.ZÀ.MUK for *apsamikku*. The diagram also confirms that the sexagesimal place value system was still in use for intermediate calculations, even when lengths and areas were expressed in new metrologies.

---

jamin Read Foster and Eleanor Robson, “A New Look at the Sargonic Mathematical Corpus,” *Zeitschrift für Assyriologie* 94 (2004): 1–15.

27. Robson, *Mesopotamian Mathematics*, 36.

28. Ibid., 53.

Most interestingly, however, the last three lines of BM 47431 provide clues as to the context in which this tablet was composed. The colophon clearly demonstrates that the exercise was to be shown to another person, presumably a teacher and apparently a scribe, giving it a clearly educational function. Hitherto, the only demonstrably pedagogical NB mathematical texts have been the metrological lists and tables of squares extracted on multi-subject elementary school tablets.[29] Yet the terms ÚS.SA.DU (rev. 1) and ŠE.NUMUN *meš-ḫat* A.ŠÀ (rev. 6), the former typical of NB land sales and the latter previously known only from NB temple accounts, strongly hint that in mid-first millenium Babylonia mathematical education was not only the preserve of *āšipus* such as the Šangû-Ninurta family of late Achæmenid Uruk, who owned a couple of collections of mathematical problems and several arithmetical and metrological tables,[30] but was—as we might expect—of use to the professionally numerate, too.

29. Petra Gesche, *Schulunterricht in Babylonien im Ersten Jahrtausend v. Chr.*, Alter Orient und Altes Testament, vol. 275 (Münster: Ugarit, 2002) and Eleanor Robson, "Mathematical Cuneiform Tablets in the Ashmolean Museum, Oxford," *SCIAMVS* 5 (2004): 3–65.

30. Egbert von Weiher, *Spätbabylonische Texte aus Planquadrat U 18*, vol. 4, Ausgrabungen der Deutschen Forschungsgemeinschaft in Uruk-Warka, vol. 12 (Mainz am Rhein: von Zabern, 1993), nos. 172–6; Idem, *Spätbabylonische Texte aus Planquadrat U 18*, vol. 5, Ausgrabungen der Deutschen Forschungsgemeinschaft in Uruk-Warka, vol. 13 (Mainz am Rhein: von Zabern, 1998), no. 316; Friberg, Hunger, and Al-Rawi, "Seed and Reeds;" Friberg, "Seed and Reeds Continued;" see also Eckart Frahm, "Zwischen Tradition und Neuerung: Babylonische Priestergelehrte im achæmenidenzeitlichen Uruk," in *Religion und Religionskontakte im Zeitalter der Achämeniden*, ed. Reinhard Kratz, Veröffentlichungen der Wissenschaftlichen Gesellschaft für Theologie, vol. 22 (Gütersloh: Kaiser, Gütersloher Verlagshaus, 2002), 74–108.

Appendix: The apsamikkum *in Old Babylonian Mathematics—Translations*

A: From the Coefficient Lists[31]

A 6 0;26 40, the coefficient of an *apsamikkum* (GEŠTUG$_2$.ZÀ.MÍ).

C 9 An *apsamikkum* (*a-ap-sà-mi-kum*): 0;26 15 is [its coefficient].
C 10 The diameter of an *apsamikkum* (*a-ap-sà-mi-ki*): 0;48 is its coefficient.
C 11 The diagonal of an *apsamikkum* (*a-ap-sà-mi-ki*): 1;20 is its coefficient.

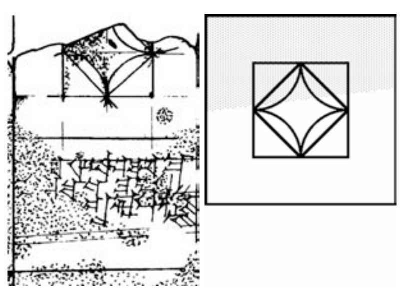
D 22 0;26 40, the coefficient of an *apsamikkum* (*a-pu-sà-am-mi-ki*).
D 23 1;20, the diagonal of an *apsamikkum* (*a-pu-sà-mi-ki*).
D 24 0;33 20, the short transversal of an *apsamikkum* (*a-pu-sà-mi-ki*).

E 4 0;26 40, the area of an *apsamikkum* (GANA$_2$ ZÀ.MÍ).

F 48 0;53 20, of the area of an *apsamikkum* (GANA$_2$ ZÀ.MÍ).
F 50 0;26 40, of an *apsamikkum* (GEŠTUG$_2$.ZÀ.MÍ).

B: From BM 15285[32]

§28

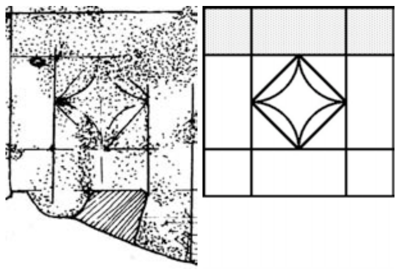


The side of the square is sixty (rods). I made a border each side and I drew a square. Inside the square that I drew is 1 *apsamikkum* (GANA$_2$ GEŠTUG$_2$.ZÀ.MÍ). What is its area?

31. After Robson, *Mesopotamian Mathematics*, 50–54. List A = A 3553 line 6, in Anne Kilmer, "Two New Lists of Key Numbers for Mathematical Operations," *Orientalia* 29 (1960): 275–81. List C = IM 52916 lines 9–11 in Goetze, "Mathematical Compendium." List D = TMS 3, lines 22–4 in Evert Bruins and Marguerite Rutten, *Textes Mathématiques de Suse*, Mémoires de la Mission Archéologique en Iran, vol. 34 (Paris: Librairie Orientaliste Paul Geuthner, 1961), text III; List E = YBC 7243 line 4, in Otto Neugebauer and Abraham Sachs, *Mathematical Cuneiform Texts*, American Oriental Series, vol. 29 (New Haven: American Oriental Society, 1945), 136–39; pl. 23, 49, Text Ue; List F= YBC 5022 lines 48, 50 in Neugebauer and Sachs, *Mathematical Cuneiform Texts*, 132–36; pl. 18, 44, Text Ud.

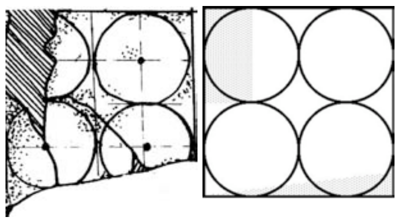
32. Cyril Gadd, "Forms and Colours," *Revue d'Assyriologie Orientale* 19 (1922): 149–59; Neugebauer, *Mathematische Keilschrift-Texte* vol. 1, 137–42; Henry Saggs, "A Babylonian Geometrical Text," *Revue d'Assyriologie Orientale* 54 (1960): 131–46; after Robson, *Mesopotamian Mathematics*, 211–13.

§29



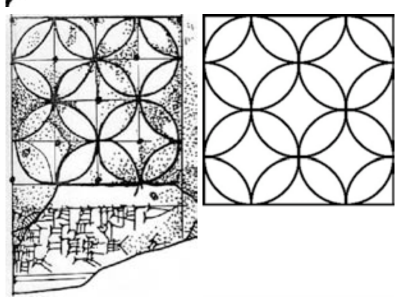
[*Text missing*]

§36



[*Text missing*]

§40



The side of the square is sixty (rods). <Inside it are> 4 triangles, 16 barges, 5 *apsamikkū* (GANA$_2$.GEŠTUG$_2$.ZÀ.MÍ). What are their areas?

C: THE PROBLEMS WITH WORKED SOLUTIONS FROM SUSA

TMS 20[33]

(1) [I added] the area, the side, and the diagonal: [1;16 40.]

You: multiply 0;26 40, the coefficient [of the *apsamikkum*] by 1;16 40: [you will see 0;34 04 26 40]. Turn back.

[Add 1, the length, and 1;20, the diagonal] which you do not know: [you will see 2;20]. Break 2;20 in half: [you will see] 1;[10. Square 1;10: you will see 1:21 40. Add] 1;21 40 to 0;34 04 [26 40: you will see] 1;55 44 [26 40]. What is the side of the square? [The side of the square is] 1;23 20. [Subtract] 1;10 from

33. Bruins and Rutten, *Textes mathématiques*, text XX; after Duncan Melville, "The Area and the Side I Added: Some Old Babylonian Geometry," *Revue d'Histoire des Mathématiques* 11 (2005). As Melville has shown, in these problems, the lengths and diagonals are added to the area of the concave square by treating them as having unit width. The sum is then scaled by the area coefficient and then treated as a standard square-plus-length(s) configuration.

1;23 20: you will see] 0;13 20. Solve the reciprocal of [0;26 40, the coefficient:] you will see 2;15. Multiply 2;15 by 0;13 20: you will see 0;30. The width (*sic,* for length) is 0;30.

(2) An *apsamikkum* (*a-pu-sà-am-mi-*[*ki*]). [I added the area and the side]: 0;36 40.

You: put down 0;36 40. Multiply 0;2[6 40, the coefficient], by 0;36 40: [you will see 0;16 17 46 40]. Turn back.

[Put down] 1, for the [length. Break 1 in half: you will see 0;30.] Square 0;30: you will see 0;15. [Add] 0;15 [to 0;16 17 46 40: you will see] 0;31 17 46 40. [What is the side of the square?] The side of the square is 0;43 20. Subtract 0;30 from 0;43 20: [you will see 0;13 20]. Solve the reciprocal of 0;26 40, the coefficient: you will see 2;15. [Multiply] 2;[15 by 0;13 20]: you will see 0;30. [The length is 0;30.]

TMS 21[34]

(1) [. . .] protrudes by 5 rods each: [. . .] *apsamikkum* (<*a*>*-pu-sà-am-mi-*[*ki*]) [. . .] width from the middle [. . .]. The intermediate area is [35 00]. What is my square side?

[You]: multiply 5, the protrusion (?),[35] by 2: you will see 10. Square [10]: you will see 1 40. Subtract 1 40 from 35 00: you will see 33 20. Put down 1, the *apsamikkum* (*a-pu-sà-mi-ka*). Put down 1;20, the diagonal of the *apsamikkum* (*a-pu-sà-mi-ka*). Multiply 1;20 by 1: you will see 1;20. Put down 1;20 like the area. Turn back.

Square 1;20: you will see 1;46 40. Square 1: you will see 1. Multiply 1 by 0;26 40, the coefficient of the *apsamikkum* (*a-pu-sà-mi-ki*): you will see 0;26 40. Subtract 0;26 40 from 1;46 40: you will see 1;20. Multiply 1;20 by 33 20, the intermediate area: you will see 44 26;40. Multiply 1;[20] by 10, the protrusion(?): you will see [13;20]. Square 13;20: you will see [2 5]7;46 40. Add [2 57;4]6 40 to [44] 26;40. What is the square side? [The square

34. Bruins and Rutten, *Textes mathématiques*, text XXI; after Kazuo Muroi, "Quadratic Equations in the Susa Mathematical Text No. 21," *SCIAMVS* 1 (2000): 3–10. In these two problems, as Muroi has demonstrated, the concave square sits inside a regular square concentric to another figure. In the first problem, the outer figure is also a square, as in BM 15285 §§ 28–29 above; in the second it is a rectangle.

35. Tentatively reading *mi-iṣ-ṣi-ta*—a malformed *piristum* noun from *waṣûm*, 'to go out, emerge, protrude,' —instead of *mi-is-si*$_{20}$*-ta* in Muroi, "Quadratic Equations," 6. In a similar context, see UET 5 864 (facsimile in Hugo Figulla and William Martin, *Letters and Documents of the Old-Babylonian Period*, vol. 5 of *Ur Excavations: Texts* (London: Percy, Lund and Humphries, 1953), pl. CXXXIX. This OB mathematical problem from Ur uses *dikištum*, with the same (correct) morphology, from *dakāšum*, 'to depress, to thrust.' See, most recently, Jens Høyrup, *Lengths, Widths, and Surfaces: A Portrait of Old Babylonian Algebra and Its Kin*, Sources and Studies in the History of Mathematics and Physical Sciences (New York: Springer, 2002), 250–54.

side is 53;20. Subtract 13;20], your holding square,[36] [from 53;20: you will see 40. Solve the reciprocal of 1];20 «which you put down like the area:»[you will see 40]. Multiply [0;45 by 40: you will see 30, the length of the *apsamikkum*. Multiply 30 by 1;20, of the diagonal: you will see 40, the diagonal. Add 10, the protrusion (?) to 40: you will see 50, the square side.]

(2) [*The statement of the problem is missing.*]

[. . .] you will see [. . .] you will see [41 40]. What is the square side? [The square side is 50.] Subtract [10], your [holding square], from 50: [you will see 40]. Solve [the reciprocal of 1;20]: you will see 0;45. Multiply [0;45 by 40]: you will see 30, the length of the *apsamikkum* (*a-pu-sà-mi-ki*): you will see 40.

Multiply [30 by 1;20], the diagonal of the *apsamikkum* (*a-pu-sà-mi-ki*). The diagonal is 40. Add 10, the protrusion (?) of the length, to 40: you will see 50, the length. Add 5, the protrusion of the width, to 40: you will see 45, the width.

36. The noun *takīltum*, literally ‘that which is made to hold itself,’ is used to describe an auxiliary square constructed in the process of “completing the square.” See Robson, “Neither Sherlock Holmes nor Babylon: A Reassessment of Plimpton 322,” *Historia Mathematica* 28 (2001): 190–91 and Høyrup, *Lengths, Widths, Surfaces*, 23.

Bruins, Evert and Marguerite Rutten. *Textes mathématiques de Suse.* Mémoires de la Mission Archéologique en Iran, vol. 34. Paris: Librairie Orientaliste Paul Geuthner, 1961.

Clay, Albert. *Babylonian Records in the Library of J. Pierpont Morgan.* Vol. 4. New Haven: Yale University Press, 1923.

Foster, Benjamin Read and Eleanor Robson. "A New Look at the Sargonic Mathematical Corpus." *Zeitschrift für Assyriologie* 94 (2004): 1–15.

Frahm, Eckart. "Zwischen Tradition und Neuerung: Babylonische Priestergelehrte im achæmenidenzeitlichen Uruk." In *Religion und Religionskontakte im Zeitalter der Achämeniden*, edited by Reinhard Kratz, 74–108. Veröffentlichungen der Wissenschaftlichen Gesellschaft für Theologie, vol. 22. Gütersloh: Kaiser, Gütersloher Verlagshaus, 2002.

Friberg, Jöran. " 'Seed and Reeds' Continued: Another Metro-Mathematical Topic Text from Late Babylonian Uruk." *Bagdader Mitteilungen* 28 (1997): 251–365.

———, Herman Hunger and Farouk Al-Rawi. " 'Seed and Reeds,' A Metro-Mathematical Topic Text from Late Babylonian Uruk." *Bagdader Mitteilungen* 21 (1990): 483–557.

Gadd, Cyril. "Forms and Colours." *Revue d'Assyriologie et d'Archéologie Orientale* 19 (1922): 149–59.

Gesche, Petra. *Schulunterricht in Babylonien im Ersten Jahrtausend v. Chr.* Alter Orient und Altes Testament, vol. 275. Münster: Ugarit, 2002.

Goetze, Albrecht. "A Mathematical Compendium from Tell Harmal." *Sumer* 7 (1951): 126–55.

Høyrup, Jens. *Lengths, Widths, Surfaces: A Portrait of Old Babylonian Algebra and Its Kin.* Sources and Studies in the History of Mathematics and Physical Sciences. New York: Springer, 2002.

Hunger, Hermann. *Babylonische und assyrische Kolophone.* Alter Orient und Altes Testament, vol. 2. Kevelaer, Butzon und Bercker; Neukirchen-Vluyn: Neukirchener Verlag des Erziehungsvereins, 1968.

Jursa, Michael. "Zweierlei Maß." *Archiv für Orientforschung* 40–41 (1993–4): 71–73.

Kilmer, Anne. "Two New Lists of Key Number for Mathematical Operations." *Orientalia* 29 (1960): 273–308.

———. "Sumerian and Akkadian Names for Designs and Geometrical Shapes." In *Investigating Artistic Environments in the Ancient Near East*, edited by Ann Gunter, 83–91. New York: Smithsonian Institution, 1990.

Koch-Westenholz, Ulla. *Babylonian Liver Omens: The Chapters* Manzāzu, Padānu, *and* Pān tākalti *of the Babylonian Extispicy Series, Mainly from Aššurbanipal's Library.* CNI Publications, vol. 25. Copenhagen: The Carsten Niebuhr Institute of Near Eastern Studies, 2000.

Krebernik, Manfred. "Neues zu den Fara-Texten." *Nouvelles assyriologiques brèves et utilitaires* (2006): 15.

Leichty, Erle. "The Colophon." In *Studies Presented to A. Leo Oppenheim, June 7, 1964*, edited by Robert Biggs and J. A. Brinkman, 147–54. Chicago: The Oriental Institute, 1964.

Melville, Duncan. "The Area and the Side I Added: Some Old Babylonian Geometry." *Revue d'Histoire des Mathématiques* 11 (2005): 7–21.

Muroi, Kazuo. "Quadratic Equations in the Susa Mathematical Text No. 21." *SCIAMVS* 1 (2000): 3–10.

Nemet-Nejat, Karen. *Late Babylonian Field Plans in the British Museum.* Studia Pohl Series Maior, vol. 11. Rome: Biblical Institute Press, 1982.

Neugebauer, Otto. *Mathematische Keilschrift-Texte.* 3 vols. Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik. Abteilungen A: Quellen, vol. 3. Berlin: Springer, 1935–37.

——— and Abraham Sachs. *Mathematical Cuneiform Texts.* American Oriental Series, vol. 29. New Haven: American Oriental Society, 1945.

Powell, Marvin. "The Antecedents of Old Babylonian Place Notation and the Early History of Babylonian Mathematics." *Historia Mathematica* 3 (1976): 417–39.

Robson, Eleanor. *Mesopotamian Mathematics, 2100–1600* B.C.*: Technical Constants in Bureaucracy and Education.* Oxford Editions of Cuneiform Texts, vol. 14. Oxford: Clarendon Press, 1999.

———. "Neither Sherlock Holmes nor Babylon: A Reassessment of Plimpton 322." *Historia Mathematica* 28 (2001): 167–206.

———. "Mathematical Cuneiform Tablets in the Ashmolean Museum, Oxford." *SCIAMVS* 5 (2004): 3–65.

Saggs, H. W. F. "A Babylonian Geometrical Text." *Revue d'Assyriologie et d'Archaéologie Orientale* 54 (1960): 131–46.

Sigrist, Marcel. *Old Babylonian Account Texts in the Horn Archaeological Museum.* Andrews University Cuneiform Texts, vol. 5. Institute of Archaeology Publications. Assyriological Series, vol. 8. Berrien Springs, Maryland: Andrews University Press, 2003.

Thompson, Reginald Campbell. *Cuneiform Texts from Babylonian Tablets, etc. in the British Museum.* Vol. 20. London: Longmans, et al., 1904.

Thureau-Dangin, Francçis. *Tablettes d'Uruk à l'usage des prêtres du temple d'Anu au temps des Séleucides.* Textes Cunéiformes du Louvre, vol. 6. Paris: Librarie Orientaliste Paul Geuthner, 1922.

von Weiher, Egbert. *Spätbabylonische Texte aus Planquadrat U 18.* Vol. 4. Ausgrabungen der Deutschen Forschungsgemeinschaft in Uruk-Warka, vol. 12. Mainz am Rhein: von Zabern, 1993.

———. *Spätbabylonische Texte aus Planquadrat U 18.* Vol. 5. Ausgrabungen der Deutschen Forschungsgemeinschaft in Uruk-Warka, vol. 13. Mainz am Rhein: von Zabern, 1998.

Weidner, Ernst. "Eine Beschreibung des Sternhimmels aus Assur." *Archiv für Orientforschung* 4 (1927): 73–85.

Wunsch, Cornelia. *Das Egibi-Archiv.* 2 vols. Cuneiform Monographs, vol. 20. Groningen: STYX, 2000.