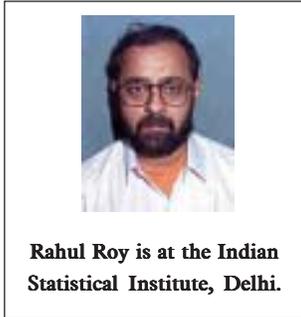

On Ancient Babylonian Algebra and Geometry

Rahul Roy



Introduction

In an earlier article [1] we had discussed some aspects of ancient Babylonian mathematics as deciphered from various clay tablets excavated from modern Iraq, viz. the Pythagoras theorem and also the sexagesimal number system prevalent during the ancient Mesopotamian civilization. In this article, we study the exciting new approach of the last decade in the decipherment and interpretation of the Babylonian mathematical tablets.

In the 1930's, following the discovery of a clay tablet pertaining to mathematics and dating to the Old Babylonian period (2000-1600 BCE), there was considerable activity in Germany and France in the interpretation of the tablets. The work of this period is to be found in the compendiums of Neugebauer [2] and Thureau-Dangin [3] whose analyses of the tablets are understandably influenced deeply by the mathematics current at our times.

However mathematics, like any other subject, is not culture-free; instead it is subject to the socially prevalent mores and conventions. Thus an understanding of the culture, language and history of the Mesopotamian civilization provides a better insight into the thought processes of the ancient Babylonian mathematicians. In this context, consider the following two examples given by Robson [4].

If asked to draw a triangle, most of us would draw a triangle with a horizontal base. However, a typical triangle drawn on the tablets of ancient Babylon has a vertical edge and the other two edges lying to the right of it, and none of them horizontal (see *Figure 1*).

Also, if asked to give the formula for the area of a cir-

Keywords

Babylonian mathematics, sexagesimal system.



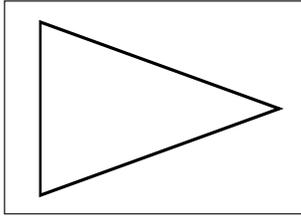


Figure 1. A typical Babylonian triangle.

¹This word will be translated as ‘confrontation’ in the subsequent sections.

In modern mathematics the circle is conceptualised as the area generated by a rotating line, the radius. In ancient Mesopotamia, by contrast, a circle was the shape contained within an equidistant circumference. Even when the diameter of a circle was known, its area was calculated by means of the circumference.

cle, we would immediately say πr^2 . Even in a situation where the radius r and circumference c were given to us, we would not give the formula $\frac{c^2}{4\pi}$. The Babylonians, however, preferred the latter, as has been attested in almost all tablets dealing with areas of circles. Indeed, as Robson [4] writes “in modern mathematics the circle is conceptualised as the area generated by a rotating line, the radius. In ancient Mesopotamia, by contrast, a circle was the shape contained within an equidistant circumference. Even when the diameter of a circle was known, its area was calculated by means of the circumference. We also see this conceptualisation in the language used: the word *kippatum*, literally ‘thing that curves’, means both the two-dimensional disc and the one-dimensional circumference that defines it. The conceptual and linguistic identification of a plane figure and one of its external lines is a key feature of Mesopotamian mathematics. For instance, the word *mithartum*¹ (“thing that is equal and opposite to itself”) means both “square” and “side of square”. We run into big interpretational problems if we ignore these crucial terminological differences between ancient Mesopotamian and our own mathematics”.

Nearly all the problems involving ‘quadratic equations’ are stated in terms of length, width, square, surface, height and volume. The solutions given in the tablets also use these terms. The earlier translations took these terms to be generic for the variables x (length), y (width), x^2 (square), xy (surface), z (height) and xyz (volume). The translations from the tablets were as described by Neugebauer himself as being “substantially accurate” in the sense that the mathematical substance of the text was retained. Jens Høyrup rereads the tablets and presents a “conformal translation”, which he describes as a translation which maps the original structure (i.e., subject, object, verb construction) and retains the etymological meaning of the Mesopotamian words. This



research of Jens Høyrup over the past decade (presented in Høyrup [5]) provides a geometric understanding of the Babylonian methods. The geometric understanding, however, does not preclude the algebraic structure lying behind these solutions.

In Section 2 we consider three algebraic problems of the old Babylonian period. In Section 3 we discuss two geometric problems from this same period, and here we see the difference in the geometry used to study the algebraic and the geometric problems.

The usefulness of the Babylonian study of mathematics is to be seen in the development of astronomy. The study of astronomy in the Babylonian civilization dates from 1800 BCE to 300 BCE – although the most important work was done during the latter half of this period. Indeed, the ‘*Astronomical Diaries*’, which is the record of systematic observations of the heavenly bodies from 800/700 BCE to 100 BCE, is probably the longest uninterrupted stretch of scientific research in the history of any civilization. The Babylonian astronomers were indeed renowned for their work in their times as may be read from the book ‘*Geography*’ by the Greek geographer Strabo of Amasia: “In Babylon a settlement is set apart for the local philosopher, the Chaldaeans, as they are called, who are concerned mostly with astronomy; but some of these who are not approved of by the others profess to be writers of horoscopes. There are also several tribes of the Chaldaean astronomers. For example, some are called Orcheni, others Borsippeni, and several others by different names, as though divided into different sects which hold to various different dogmas about the same subjects. And the mathematicians make mention of some of these men: as, for example, Cidenas, Naburianus and Sudines”.

The influence of this work on Greek astronomical work is apparent. Not only was the Babylonian sexagesimal

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mal number system (*Box 1*) adopted by the Greeks for their astronomical work, the Babylonian development of mathematics for their astronomical studies contributed both the arithmetic and the geometric methods of the Greek studies, in addition to supplying the empirical observations which were used to build these mathematical theories. The Greeks were to further this study by their theories of arcs and chords to determine positions of celestial bodies. It is the latter which found full expression in the trigonometric works of Aryabhata and other Indian mathematicians. The Babylonian work also influenced Indian astronomy, as is shown in Neugebauer [1969], where he shows through various examples how whole sections of Varahamira's *Pancha Siddhantika* may

Box 1. An Aside on the Sexagesimal Number System

To understand the sexagesimal number system, we first look at the familiar decimal number system, which is a positional system with base 10. Here the number 237 is understood to be $(2 \times 10^2) + (3 \times 10^1) + (7 \times 10^0)$, while the number 0.237 is $(2 \times \frac{1}{10}) + (3 \times \frac{1}{10^2}) + (7 \times \frac{1}{10^3})$. We need 10 different digits 0, 1, ..., 9 to express any number in this system. Computers however use a binary number system based on the two digits 0, 1. Thus the decimal number $27 = (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$ will be translated as 11011 by the computer for its calculations, while the binary number 0.1011 is equivalent to the decimal number $(1 \times \frac{1}{2}) + (0 \times \frac{1}{2^2}) + (1 \times \frac{1}{2^3}) + (1 \times \frac{1}{2^4}) = 0.6875$. The sexagesimal system needs 60 digits, which for the present purpose and rather unimaginatively we denote by $\bar{0}, \bar{1}, \dots, \bar{58}, \bar{59}$. Here the sexagesimal number $\bar{249}$ equals $(\mathbf{24} \times 60^1) + (\mathbf{9} \times 60^0) = 1449$ in the decimal system, while the sexagesimal fraction $0;\bar{249}$ equals the decimal number $(\mathbf{24} \times \frac{1}{60}) + (\mathbf{9} \times \frac{1}{60^2}) = 0.4025$. In modern mathematics, whenever a possibility of ambiguity arises, we write $237(\text{mod } 10)$ to express the number 237 in the decimal system, $1011(\text{mod } 2)$ for the binary number 1011 and $0.\bar{249}(\text{mod } 60)$ for the sexagesimal number $\bar{249}$. Note however that when just one number system is in use, one does not need to have a mathematical understanding of decomposition of a number in terms of its base ... $10^2, 10^1, 10^0, 10^{-1}, \dots$ or ... $60^2, 60^1, 60^0, 60^{-1}, \dots$ for day-to-day use.

The representation of the Babylonian number system was rather cumbersome. They had a symbol for 1, a symbol for 10 and a symbol for 60. The digits 1 to 9 were expressed by writing the requisite number of 1's, either consecutively or bunched together in groups of three with one group on top of another. Similarly the 'digits' 10, 20, ..., 50 were expressed by the requisite number of 10's, etc. Thus the number 147 would be represented by two symbols of 60, two of 10 and seven of 1. The digit 0 was not used and presumed to be understood from the context, although in the later Babylonian period it was denoted by a wedge.



be explained by means of the Babylonian planetary texts.

This evidence of a Babylonian influence on Greek and Indian mathematics reveals the exaggeration in the assertions of the ‘greatest contributions’ of India in mathematics (e.g.[6] “the invention of the decimal notation and creation of modern arithmetic; the invention of the sine and cosine functions leading to the creation of modern trigonometry and creation of algebra”) and places the post-Vedic Indian contribution to mathematics and astronomy in the proper historical context. As in modern academics, in the ancient Chinese, Greek and Indian studies we find a continuity which is built on earlier works from other civilizations and other cultures.

Algebra

In this section we discuss the problems from three tablets of the Old Babylonian period.

Consider the problem² from the tablet BM 13901 #1:

I totalled the area and (the side of) my square: it is
0;45.

Clearly the problem may be written as $x^2 + x = 0;45$, where x is the (unknown) length of the side of a square. This equation we wrote involving the symbol x is, of course, a modern transcription of the problem.

To understand the above problem in the Babylonian cultural milieu, we see the ‘conformal’ translation in [5] of the problem together with its solution as given on the tablet³:

The surface and my confrontation I have accumulated: 0;45 is it. $\bar{1}$, the projection, you posit. The moiety of $\bar{1}$ you break, 0;30 and 0;30 you make hold.
0;15 and 0;45 you append: by $\bar{1}$, $\bar{1}$ is the equalside. 0;30 which you have made hold
in the inside of $\bar{1}$ you tear out: 0;30 the confrontation.

² This translation from [2] is by Robson [7] in her review of the book Höystrup [5] which appeared on the MAA Online book review website.

³ The word ‘confrontation’ is used by Höystrup to translate the Babylonian word ‘mithartum’ to convey the sense “a confrontation of equals, viz. the square configuration parametrized by its side”; see also Section 1.



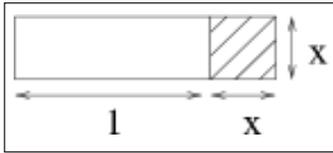


Figure 2. The unknown square is hatched and to it is appended a rectangle of length 1. The area of the resulting rectangle is $x^2 + x$, which is given to be $0;45$.

The first sentence says that the surface of the square and its edge are combined to give an area $0;45$. Since a 1-dimensional object and a two dimensional object cannot be added ‘geometrically’, the second sentence of the problem says that the 1-dimensional line is transformed into a 2-dimensional surface by giving it a thickness 1. Thus geometrically we have *Figure 2*.

The next two sentences ask us to bisect the newly projected surface and let the two pieces hold together. Next it asks us to append the square of area $0;15$ (i.e., of sides $0;30$) to the figure constructed earlier and obtain a square of area $0;15 + 0;45 = 1$. These steps are shown in *Figure 3*.

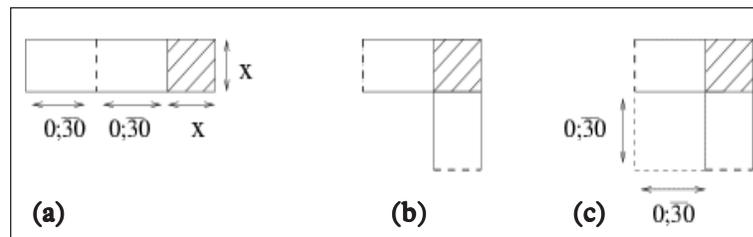
In this larger square of area 1 (*Figure 3c*), the dotted part of the edge has length $0;30$ and thus the length of the edge of the unknown square is $0;30$.

In the language of modern mathematics, the first and the second lines produce the equation $x^2 + x \times 1 = 0;45$ as given in *Figure 2*. In the next few lines, the rectangle $x \times 1$ is broken into $x \times \frac{1}{2} + x \times \frac{1}{2}$ (*Figure 3a* and *3b*) and then we ‘complete the square’ by adding $0;30 \times 0;30$ (*Figure 3c*) to obtain $x^2 + x \times \frac{1}{2} + x \times \frac{1}{2} + 0;30^2 = 0;45 + 0;30^2$, i.e. $(x + 0;30)^2 = 1$; from which we obtain that $x = 0;30$.

⁴ *ibid.* Höyrup [5], pg 71.

Figure 3. In (a) the appended rectangle is bisected, in (b) it is made to ‘hold’ and in (c) the bigger square is formed by appending the smaller square of size $0;30 \times 0;30$. This yields $(x + 0;30)$ as the length of a side of the bigger square of area 1; thereby $x = 0;30$.

Höyrup [5] analyses the texts of many such tablets and arrives at similar geometric solutions. To show that behind all these geometry there is an underlying algebraic structure we consider the bi-quadratic problem from the tablet⁴ BM 13901 # 12:



The surfaces of my two confrontations I have accumulated: $0; \bar{2}1\bar{4}0$.
 My confrontations I have made hold: $0; \bar{1}0$.

This problem asks us to find the lengths of the sides of two squares, given that the sum of their areas is $0; \bar{2}1\bar{4}0$ and the product of the two lengths is $0; \bar{1}0$. The solution as presented on the tablet is:

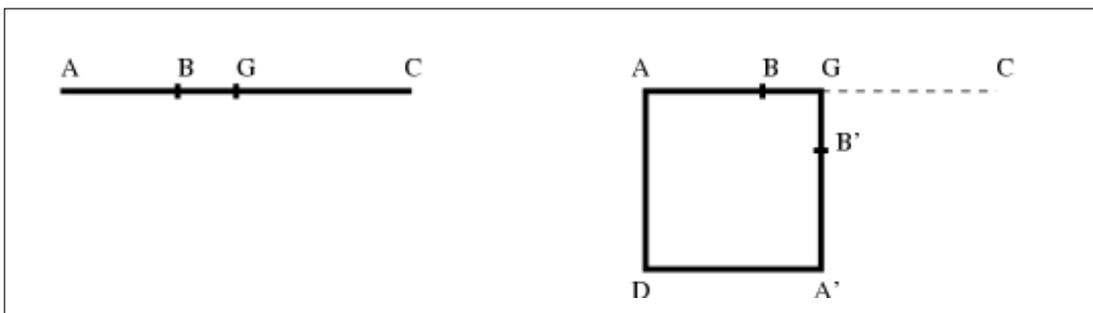
The moiety of $0; \bar{2}1\bar{4}0$ you break: $0; \bar{1}0\bar{5}0$ and $0; \bar{1}0\bar{5}0$ you make hold,
 $0; \bar{1}5\bar{7}2\bar{1}4\bar{0}$ is it. $0; \bar{1}0$ and $0; \bar{1}0$ you make hold, $0; \bar{1}4\bar{0}$ inside $0; \bar{1}5\bar{7}2\bar{1}4\bar{0}$ you tear out: by $0; \bar{0}1\bar{7}2\bar{1}4\bar{0}$, $0; \bar{4}1\bar{0}$ is equalside.
 $0; \bar{4}1\bar{0}$ to one $0; \bar{1}0\bar{5}0$ you append: by $0; \bar{1}5$, $0; \bar{3}0$ is equalside.
 $0; \bar{3}0$ the first confrontation.
 $0; \bar{4}1\bar{0}$ inside the second $0; \bar{1}0\bar{5}0$ you tear out: by $0; \bar{6}4\bar{0}$, $0; \bar{2}0$ is equalside.
 $0; \bar{2}0$ the second confrontation.

In this problem we understand the full import of the algebraic methods used in these geometric solutions.

The first sentence asks us to bisect a line of length $0; \bar{2}1\bar{4}0$ and then make each of the equal parts form a square of area $0; \bar{1}5\bar{7}2\bar{1}4\bar{0}$.

Taking u and v to be the unknown lengths of the two squares we have, from *Figure 4*, $AB = u^2$, $BC = v^2$, G is the mid-point of $AC = AB + BC = 0; \bar{2}1\bar{4}0$ and $AGA'D$ is the square of sides $(u^2 + v^2)/2 = 0; \bar{1}5\bar{7}2\bar{1}4\bar{0}$ each. In *Figure 4*, the point B' is such that $GB' = GB$.

Figure 4. BM 13901 # 12 — holding together.



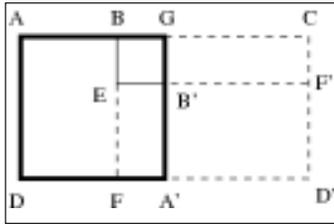


Figure 5. BM 13901 # 12 — tearing out.

The next sentence asks us to tear out an area $0; \bar{1}40 = 0; \bar{1}0 \times 0; \bar{1}0$ from $AGA'D$ to leave an area $0; \bar{0}172\bar{1}40$, which is the area of a square of sides $0; \bar{4}10$ each.

Here although the area $0; \bar{1}40 = 0; \bar{1}0 \times 0; \bar{1}0$ is expressed as that formed by ‘holding’ two sides of length $0; \bar{1}0$ each, experts believe that this area is obtained as the area of a rectangle with sides of length u^2 and v^2 . In Figure 5, this is depicted by the rectangle $EF'D'F$. Now the rectangles $ABFD$ and $A'B'F'D'$ are congruent. Thus the region $ABEB'A'D$ has area $0; \bar{1}40$, and so after ‘tearing’ this region out from the square $AGA'D$ we are left with a square $BGB'E$ of area $0; \bar{0}172\bar{1}40$. Now $BG = (u^2 - v^2)/2 = 0; \bar{4}10$.

The next few sentences obtain u and v from the relation $u^2 = ((u^2 + v^2)/2) + ((u^2 - v^2)/2)$ and $v^2 = ((u^2 + v^2)/2) - ((u^2 - v^2)/2)$.

Finally, we look at a problem from the tablet YBC 6504. This tablet has four problems together with their solutions. In all of these there is a rectangle of size $l \times w$ from which we are asked to ‘tear out’ a square of size $(l - w) \times (l - w)$ to leave a resultant area of $0; \bar{8}20$. In each of the four problems we have to obtain l and w , when (i) $l - w$ or (ii) $l + w$ or (iii) l or (iv) w is given. We discuss the second problem of this tablet, i.e. when $l + w$ is given. As we will see in the next section, a similar construction is made in a geometry problem.

So much as length over width goes beyond, I have made confront itself, from the inside of the surface I have torn it out:

$0; \bar{8}20$. Length and width accumulated: $0; \bar{5}0$. By your proceeding,

$0; \bar{5}0$ you make hold: $0; \bar{4}140$ you posit.

$0; \bar{4}140$ to $0; \bar{8}20$ you append: $0; \bar{5}0$ you posit.

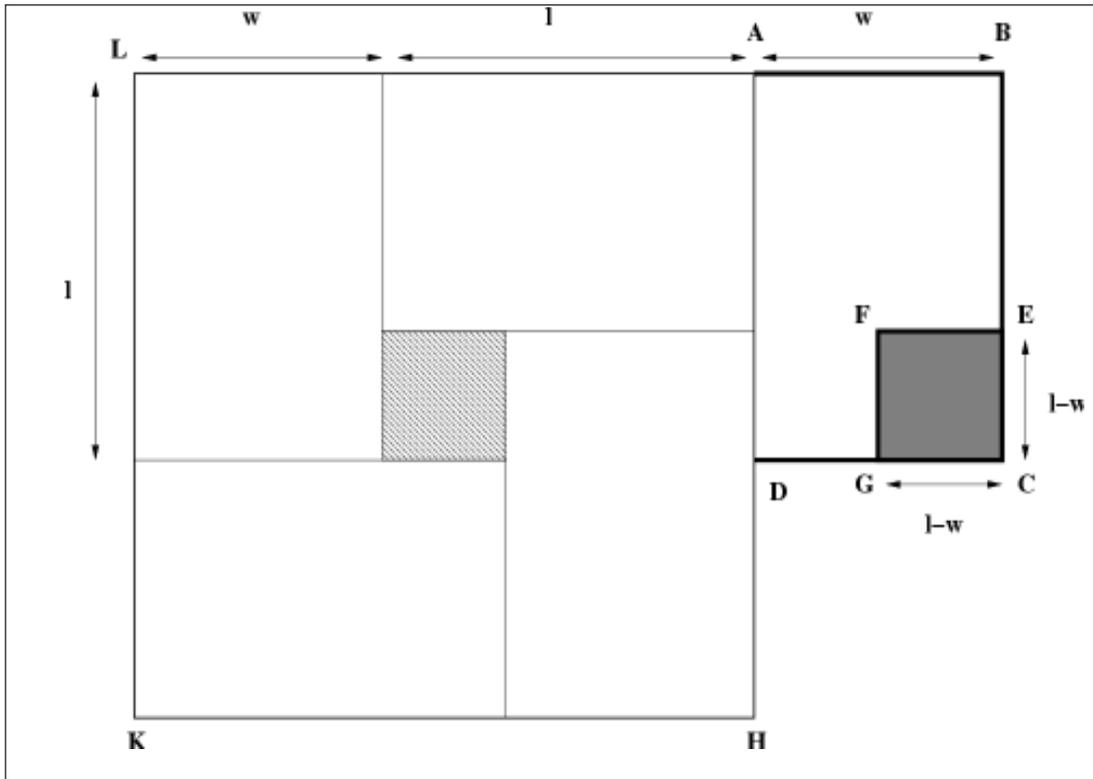
A $\bar{5}$ th part you detach: $0; \bar{1}2$ you posit.

$0; \bar{1}2$ to $0; \bar{5}0$ you raise: $0; \bar{1}0$ you posit

Half of $0; \bar{5}0$ you break: $0; \bar{2}5$ you posit

$0; \bar{2}5$ you make hold: $0; \bar{1}025$ you posit





0; $\bar{1}0$ from 0; $\bar{1}0\bar{2}\bar{5}$ you tear out: 0; $\bar{0}\bar{2}\bar{5}$ you posit.
 By 0; $\bar{0}\bar{2}\bar{5}$, 0; $\bar{5}$ is equalside. 0; $\bar{5}$ to 0; $\bar{2}\bar{5}$ you append:
 0; $\bar{3}0$, the length, you posit.
 0; $\bar{5}$ from 0; $\bar{2}\bar{5}$ you tear out:
 0; $\bar{2}0$, the width, you posit.

Figure 6. The construction for YBC 6504 # 2.

We explain the solution with the help of *Figure 6*, which is the construction suggested by the solution.

In *Figure 6*, as suggested in the first sentence on the tablet, $ABCD$ is the rectangle, from which the square $EFGC$ is ‘torn out’ leaving an area 0; $\bar{8}\bar{2}0$. The second sentence asks us to ‘accumulate’ the length l and width w to form the line AH of length 0; $\bar{5}0$. The third sentence asks us to ‘make hold’, i.e. make the square $AHKL$ from the line AH of area 0; $\bar{5}0 \times 0; \bar{5}0 = 0; \bar{4}1\bar{4}0$. The fourth sentence “0; $\bar{4}1\bar{4}0$ to 0; $\bar{8}\bar{2}0$ you append: 0; $\bar{5}0$ you posit” suggests that we add the area of the square $AHKL$ to



that of the region $ABEFGD$. Since the shaded smaller square inside $AHKL$ is congruent to the ‘torn’ square $EFGC$, we have that $0; \bar{5}0$ is equal to the area of five of the original rectangles $ABCD$. In the fifth sentence we are asked to “a $\bar{5}$ th part you detach: $0; \bar{1}2$ you posit” which translates to $1/5 = 0; \bar{1}2$ and the next sentence says that $0; \bar{1}2 \times 0; \bar{5}0 = 0; \bar{1}0$. Now having obtained $lw = 0; \bar{1}0$ and knowing that $l + w = 0; \bar{5}0$, the next few sentences sets up the equations $(1/2)(l + w) = 0; \bar{2}5$ and $(1/2)(l - w) = 0; \bar{5}$ to obtain $l = 0; \bar{3}0$ and $w = 0; \bar{2}0$.

It should be noted here that regarding this geometric solution *no clay tablet has been found that contains these figures*. Thus evidence as in a ‘smoking gun’ is not present to validate Höyrup’s method based on his conformal translation. Although it may appear that an absence of a confirmation of Höyrup’s analysis, in terms of figures on clay tablets, undermines the validity of his geometric solutions, they are in fact in complete and faithful accord with the texts of the tablets⁵.

⁵ In a personal communication SG Dani points out that “avoiding to draw figures especially in formal communications is to be found even today; the reasons are presumably different but the parallel is intriguing”.

Moreover there is incidental evidence to support Höyrup’s methods. First, as observed from tablets connected with land measurements, there are drawings of plots of land where the lengths, widths and angles are not in accord with those given numerically, i.e. the drawings do not respect scale or angles (e.g., see *Figure 7* from the tablet IM 55357 given in this article). Instead these are⁶ “structure diagrams ... to identify and summarize the role of measured segments”. This suggests that a lot of the mathematics may have been carried out as ‘mental geometry’ (akin to the ‘mental arithmetic’ of our times). Second it has been suggested that cuneiform writing was practised on sand of the school yard; in which case these figures could also be drawn on sand (again similar to calculations on ‘rough paper’ of our times). Indeed the use of sand or dustboard in ancient Greece is also supported by familiar anecdotes of Archimedes drawing figures on sand. The Greek word for the dustboard is ‘abacus’,

⁶ *ibid.* Höyrup [5].



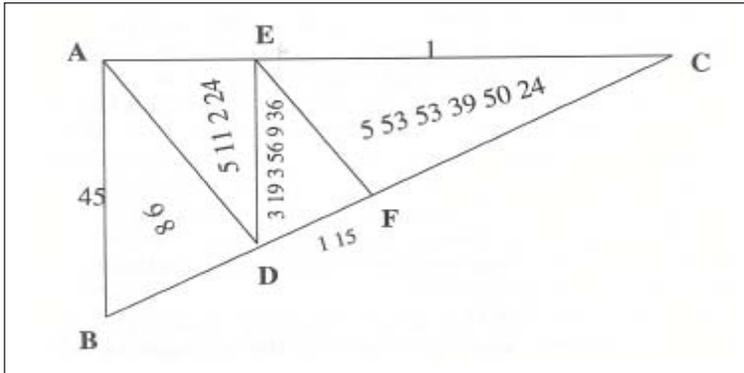


Figure 7. IM 55357.

which is etymologically related to the Semitic word *'abaq* suggesting that the dustboard may have been originally from the Syro-Phoenician area. The close connections in ancient times of Mesopotamia and the Syro-Phoenician region strengthen the contention that the dustboard may have been available to the Babylonians.

To conclude this section we quote from [5] “Old Babylonian ‘algebra’ remained an *art*, not a science, if this is understood as an Aristotelian *episteme* whose aim is *principles*. On this account, however, any supposed algebra before Viète forsakes, however deep its insights. If we accept to speak of (say) Indian, Islamic, or Latin/Italian medieval ‘algebra’ as *algebra*, then we may safely drop the quotation marks and speak of *Old Babylonian algebra* without reserve”.

Geometry

Regarding the geometry of the Babylonians, we had earlier [1] discussed some tablets related to the Pythagoras’ theorem. As far as calculations of areas are concerned, the Babylonians knew methods to calculate the areas of right-angled triangles, rectangles and trapezium. For nearly rectangular quadrilaterals they would use the ‘surveyor’s formula’ which is the product of the average of the lengths and the average of the widths of the quadrilateral. The error inherent in this formula was also realized and so occasionally quadrilaterals were de-



composed into smaller pieces so as to get a good approximation of their areas.

In this section we present problems from two tablets. The first tablet we discuss is IM 55357. *Figure 7* is a reproduction with identifying letters of the figure on the clay tablet of the problem.

A triangle $\bar{1}\bar{0}$ the length, $\bar{1}\bar{1}\bar{5}$ the long length, $\bar{4}\bar{5}$ the upper width.
 $\bar{2}\bar{2}\bar{3}\bar{0}$ the complete surface. In $\bar{2}\bar{2}\bar{3}\bar{0}$ the complete surface, $\bar{8}\bar{6}$ the upper surface.
 $\bar{5}\bar{1}\bar{1}$; $\bar{2}\bar{2}\bar{4}$ the next surface, $\bar{3}\bar{1}\bar{9}$; $\bar{3}\bar{5}\bar{6}\bar{9}\bar{3}\bar{6}$ the 3rd surface.
 $\bar{5}\bar{5}\bar{3}$; $\bar{5}\bar{3}\bar{3}\bar{9}\bar{5}\bar{0}\bar{2}\bar{4}$ the lower surface.
 The upper length, the shoulder length, the lower length and the descendant what?
 You, to know the proceeding, igi $\bar{1}\bar{0}$, the length detach, to $\bar{4}\bar{5}$ raise,
 0 ; $\bar{4}\bar{5}$ you see. 0 ; $\bar{4}\bar{5}$ to $\bar{2}$ raise, $\bar{1}$; $\bar{3}\bar{0}$ you see, to $\bar{8}\bar{6}$ the upper surface
 raise, $\bar{1}\bar{2}\bar{9}$ you see. By $\bar{1}\bar{2}\bar{9}$, what is equalside? $\bar{2}\bar{7}$ is equalside.
 $\bar{2}\bar{7}$ the width, $\bar{2}\bar{7}$ break, $\bar{1}\bar{3}$; $\bar{3}\bar{0}$ you see. Igi $\bar{1}\bar{3}$; $\bar{3}\bar{0}$ detach,
 to $\bar{8}\bar{6}$ the upper surface raise, $\bar{3}\bar{6}$ you see, the length which is the counterpart of the length $\bar{4}\bar{5}$, the width. Turn around. The length $\bar{2}\bar{7}$, of the upper triangle, from $\bar{1}\bar{1}\bar{5}$ tear out,
 $\bar{4}\bar{8}$ leave. Igi $\bar{4}\bar{8}$ detach, 0 ; $\bar{1}\bar{1}\bar{5}$ you see, 0 ; $\bar{1}\bar{1}\bar{5}$ to $\bar{3}\bar{6}$ raise,
 0 ; $\bar{4}\bar{5}$ you see. 0 ; $\bar{4}\bar{5}$ to $\bar{2}$ raise, $\bar{1}$; $\bar{3}\bar{0}$ you see, to $\bar{5}\bar{1}\bar{1}$; $\bar{2}\bar{2}\bar{4}$ raise,
 $\bar{7}\bar{4}\bar{6}$; $\bar{3}\bar{3}\bar{3}\bar{6}$ you see. By $\bar{7}\bar{4}\bar{6}$; $\bar{3}\bar{3}\bar{3}\bar{6}$, what is equalside? $\bar{2}\bar{1}$; $\bar{3}\bar{6}$ is equalside, $\bar{2}\bar{1}$; $\bar{3}\bar{6}$ the width of 2nd triangle. The moiety of $\bar{2}\bar{1}$; $\bar{3}\bar{6}$ break, $\bar{1}\bar{0}$; $\bar{4}\bar{8}$ you see. $\bar{1}\bar{0}$; $\bar{4}\bar{8}$ part detach,
 to ...

⁷ This unconventional use of the word has its root in the history of Babylon. When writing was introduced in the fourth millennium BCE, it was on small tablets and the direction of writing was vertical. Later when writing on larger clay tablets, the scribes rotated the tablets anticlockwise and wrote from left to right; thus what was 'up' earlier became left, although the use of the word 'up' remained.

The text breaks off at this point.

Here, in line 2 the 'upper surface means the surface to the left⁷ and in line 6 'Igi n ' is translated as 'the n -th part'.



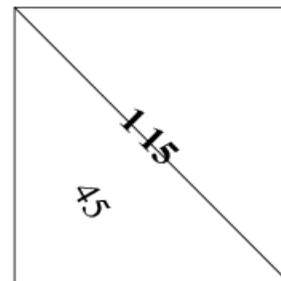
The first five lines of the text sets up the problem and specifies the areas of the various triangles as in *Figure 7*. The sixth line obtains the ratio AB/AC by multiplying $\bar{4}5$ and the reciprocal (*igi*) of $\bar{1}0$. In lines 7 and 8, this ratio, $0;\bar{4}5$ is multiplied with $\bar{2}$ to obtain $\bar{1};\bar{3}0$, which is then multiplied with $\bar{8}\bar{6}$ to obtain $\bar{1}2\bar{9}$, the square of $\bar{2}7$. Here the similarity of the triangle ABD and ABC is used to obtain $AB/AC = BD/AD = 0;\bar{4}5$ and then from the relation $(1/2)BD \cdot AD = \bar{8}\bar{6}$ one obtains $BD^2 = 2(BD/AD)\bar{8}\bar{6} = \bar{1}2\bar{9}$. Line 9 says that $BD = \bar{2}7$ and then using the area of the triangle obtains $AD = 36$. The text proceeds in a similar fashion to obtain the length of the other unknown sides of the inscribed triangles, although the text breaks off before DE and EF are obtained.

One should note here that the text implicitly assumes that the triangle ABC is right-angled – an observation which the text setter probably had in mind because of the proportion $3 : 4 : 5$ of the sides of the triangle ABC . Also, the similarities of the triangles ABD and ABC , ADE and ADC , and EDF and EFC are assumed, though never stated. On the contrary, the lines AD and EF are not drawn perpendicular to BC in the figure on the tablet. As to why this problem was solved by using the similarity properties of the triangles and not by using Pythagoras' theorem, one can only speculate that the solution was illustrative of the use of the 'surveyor's formula' in calculating areas of polygons by decomposing it into smaller triangles/rectangles which were similar, though not congruent.

Our next tablet DB_2-146 from the old Babylonian period exemplifies the use of Pythagoras' theorem. Here the accompanying figure from the text is reproduced in *Figure 8*.

If, about a rectangle with diagonal, somebody asks
you

Figure 8. Db_2-146 .



thus, $\bar{1}; \bar{15}$ the diagonal, $0; \bar{45}$ the surface;
length and width corresponding to what? You, by
your proceeding,
 $\bar{1}; \bar{15}$, your diagonal, its counterpart lay down:
make them hold: $\bar{1}; \bar{3345}$ come up,
 $\bar{1}; \bar{3345}$ may your hand hold
 $0; \bar{45}$ your surface to two bring: $\bar{1}; \bar{30}$ comes up.
From $\bar{1}; \bar{3345}$ cut off: ... $0; \bar{3345}$ the remainder
The equalside of $0; \bar{345}$ take: $0; \bar{15}$ comes up. Its
half-part,
 $0; \bar{730}$ comes up, to $0; \bar{730}$ raise: $0; \bar{05615}$ comes up
 $0; \bar{05615}$ your hand. $0; \bar{45}$ your surface over your
hand,
 $0; \bar{455615}$ comes up. The equalside of $0; \bar{455615}$ take:
 $0; \bar{5230}$ comes up, $0; \bar{5230}$ its counterpart lay down,
 $0; \bar{730}$ which you have made hold to one
append: from one
cut off.
 $\bar{1}$ your length, $0; \bar{45}$ the width. If $\bar{1}$ the length,
 $0; \bar{45}$ the width, the surface and the diagonal corre-
sponding to what?
You by your making, the length make hold:
 $\bar{1}$ comes up ... may your head hold.
...: $0; \bar{45}$, the width make hold:
 $0; \bar{3345}$ comes up. To your length append:
 $\bar{1}; \bar{3345}$ comes up. The equalside of $\bar{1}; \bar{3345}$ take:
 $\bar{1}; \bar{15}$ comes up. $\bar{1}; \bar{15}$ your diagonal. Your length
to the width raise, $0; \bar{45}$ your surface.
Thus the procedure.

Here the first three lines sets out the problem: given a rectangle with diagonal $\bar{1}; \bar{15}$ and area $0; \bar{45}$ what are its length and width? The fourth and fifth lines require that the diagonal of the rectangle be made to hold a square, i.e. a square with sides of length $\bar{1}; \bar{15}$. This square has area $\bar{1}; \bar{3345}$, which we are asked to keep at hand in line six. In lines seven to nine, two of the original rectangles are now ‘cut off’ from the square to yield a square of area $0; \bar{345}$ ⁸ and sides of length $0; \bar{15}$. Comparing with the texts of other tablets, as well as that of the tablet YBC 6504, Høyrup concludes that bringing the surface ‘to two’ and then to ‘cut off’ from the square results in the diagram given in *Figure 9*. Once the middle square

⁸ Note the ‘typo’ $0; \bar{33} \bar{45}$ in the text of the tablet.



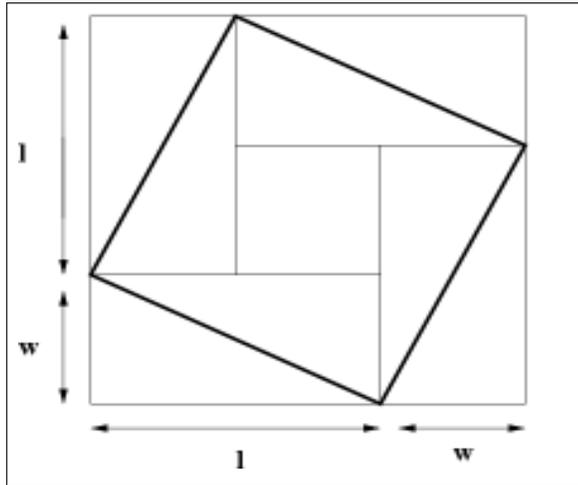


Figure 9. The diagram which follows from the text of *Db₂-146*.

is known i.e. $(1/2)(l - w) = 0; \bar{7}30$, where l and w are the unknown length and width of the rectangle, lines 10 to 15 establish $(1/2)(l + w) = \sqrt{[(1/2)(l - w)]^2 + lw} = \sqrt{0; \bar{0}5615 + 0; \bar{4}5} = \sqrt{0; \bar{4}55615} = 0; \bar{5}230$. Thus, line 16 concludes that $l = \bar{1}$ and $w = 0; \bar{4}5$. The remainder of the text verifies that the solution is correct, by doing a ‘back calculation’.

As an aside compare the figure constructed for the previous problem with the Hsuan-thu⁹ diagram from the ancient Chinese text *Chou Pei Suan Ching* (The arith-

⁹ See Joseph [10], Smith [11] for more details.

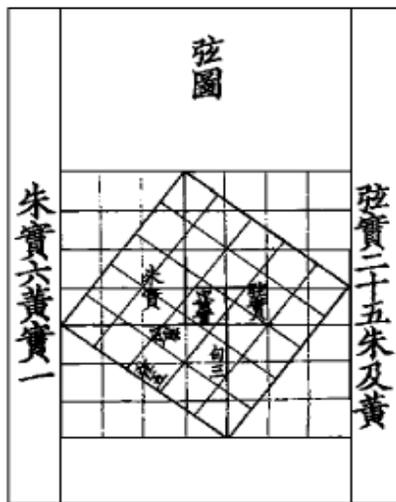


Figure 10. The Hsuan-thu diagram.

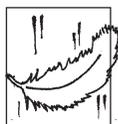
¹⁰ The ancient Chinese nomenclature for the Pythagoras' theorem.

metic classic of the Gnomon and the circular paths of the heavens) from around 1000 BCE. This diagram was used in the book to illustrate the *kou-ku*¹⁰ theorem.

Suggested Reading

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- [5] J Hoyrup, *Lengths, widths, surfaces: a portrait of old Babylonian algebra and its kin*, Springer-Verlag, New York, 2002.
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- [9] O Neugebauer, *The exact sciences in antiquity*, 2nd edition, Dover, New York, 1969.
- [10] G G Joseph, *The crest of the peacock: non-European roots of mathematics*, Penguin, London, 2000.
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“We were on the steamer from America to Japan, and I liked to take part in the social life on the steamer and so, for instance, I took part in the dances in the evening. Paul, somehow, didn't like that too much but he would sit in a chair and look at the dances. Once I came back from a dance and took the chair beside him and he asked me, 'Heisenberg, why do you dance?' 'I said, 'Well, when there are nice girls it is a pleasure to dance'. He thought for a long time about it, and after about five minutes he said, 'Heisenberg, how do you know **before hand** that the girls are nice?' ”

– Werner Heisenberg on Dirac

