Book Review

From Numerate Apprenticeship to Divine Quantification

Reviewed by Peter Damerow

Mathematics in Ancient Iraq: A Social History
Eleanor Robson
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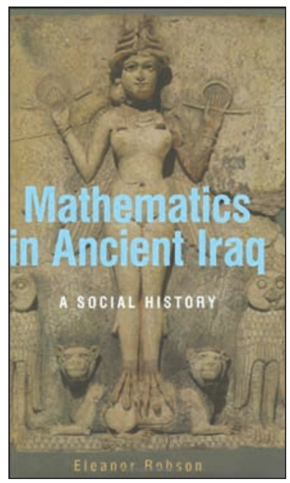

To make it clear from the very beginning, Eleanor Robson's book *Mathematics in Ancient Iraq* is presently unique and will surely become a classic in the history of early mathematics. Despite the meticulous and detailed presentation of a representative selection of available sources, the book is very readable and captures the attention of the interested reader from the first to the last page. I recommend it to anyone who would like to learn something about the fascinating story of the development of mathematical activities in Mesopotamia, from its roots in bookkeeping practices at the beginning of the third millennium BCE to the divine quantification practiced by the educated scholarly priests of the declining Babylonian culture about 3,000 years later in the Hellenistic period founded by Alexander the Great.

This story deserves closer attention, not only because it sheds light on the origins of the long tradition of scholarly activities concerned with arithmetical problems, but also because it challenges the familiar understanding of mathematics in general. It is commonly assumed that mathematics deals with provable and therefore absolutely true knowledge, in contrast to empirical sciences, which provide knowledge that can always be disproved by new discoveries. The book makes evident that this understanding of mathematics is heavily biased by our access to the historical sources that contribute to our knowledge about its early history. The term "mathematics" is a Greek term, usually confounded with the notion of mathematical proof as introduced by Greek mathematicians. Up to the 1920s, any mathematical activity predating classical Antiquity was conceived as more or less based on empirically discovered patterns that were transformed into rules without justification by proof. When, around seventy years after the basic decipherment of cuneiform writing, it turned out that some of the thousands of cuneiform tablets unearthed at that time contained problems and their correct solutions, which could be interpreted as solutions of second-degree equations, the pioneers of the decipherment of these tablets coined the term "mathematical cuneiform texts". They maintained that such tablets represent a kind of "Babylonian algebra" from which Greek mathematics adopted a range of content and methods and transformed them into a geometrical conceptual framework.

While it is true that none of the mathematical cuneiform tablets discovered contained anything comparable with or similar to the Greek way of representing mathematical material by "theorem" and "proof", these tablets made evident that it was no longer reasonable to restrict the term "mathematics" to the specific Greek form of representation. For several decades, conflicting opinions of classicists and orientalists about the relation of Babylonian and Greek mathematics were vehemently expressed without any substantial progress. One of the reasons for the

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many fruitless debates was that what was indiscriminately designated by the term "Babylonian mathematics" incorporated neither deductive theories nor algebraic transformations in the modern sense. It is only due to recently pursued, careful philological studies and investigations of specific cultural contexts represented by archaeological evidence that we are able to achieve a better understanding of the intellectual background of the challenging mathematical cuneiform sources. This intellectual background is based partly on mental representations of geometrical relationships and partly on reflections about ingeniously designed arithmetical procedures.

This situation is the starting point of Eleanor Robson's eminent study. It is the first to combine the analysis of mathematical content with the results of recent philological and archaeological investigations in order to gain an extended reinterpretation of the conflicting traditional interpretation of available sources. Her profound knowledge of these investigations in combination with her mathematical competence make her statements truly reliable. Nearly all of her claims are justified by references to the publications on which they are based. The extensive bibliography documents this diligent attempt to produce a verifiable integration of what can be derived from the sources into a coherent picture of the origins and nature of mathematics in the ancient Near East. If the available evidence does not allow for a definite answer to a question, the alternative interpretations of the sources are usually made explicit, sometimes indicating an inclination to accept one or the other alternative.

In spite of the accuracy in presenting details of the development of early mathematics, the presentation is never boring. It always follows a clear line of argument, frequently interrupted only by a close reading of some challenging mathematical cuneiform tablets to reveal their genuine mathematical content. This way of discussing sources in order to substantiate general claims is applied in the introduction (called "Scopes, Methods, Sources"), which specifies the scope of the study, the methods applied, and the range of sources on which the study is built. In this way the author's program is made understandable, even to readers who never have heard about mathematics documented by cuneiform tablets. A simple mathematical exercise concerning the squaring of a number is used to explain basic characteristics of the writing system, the system of numerical notations, the method of interpreting the written text as well as the use of archaeological information that may contribute to the reconstruction of the function of mathematical cuneiform tablets in the ancient social context of learning and applying mathematical techniques. Even this introduction can be considered to be a masterpiece of presenting complex information in an understandable and concise manner without losing its full meaning.

The succeeding chapters follow the course of historical development. The presentation begins with the earliest sources documenting enumeration and abstraction, dating back to the period before the mid-third millennium, and ends with mathematical and astronomical texts written in the later first millennium BCE, which represent the last blossoming of cuneiform culture before the end of local rule in Mesopotamia. This long history of approximately 3,000 years is divided roughly by the chapters of the book into periods of about 500 years each.

The second chapter, "Before the Mid-Third Millennium", starts with the incipient forms of any explicit mathematical activity beginning with the emergence of sedentary settlements around 10,000 BCE and ending around the middle of the third millennium, when early forms of writing and calculating were fully developed. The author identifies the origins of these administrative tools in techniques that were developed as an outcome of the emergence of sedentary settlements. The early settlers constructed dwellings and storerooms, indicating a long-term administration of domesticated plants and animals. It is likely that this social change from hunting and gathering to farming and animal husbandry was the condition for the invention of counters in the form of clay tokens with different shapes and incisions used as administrative tools. Beginning around the fifth millennium BCE, influenced by population growth and the emergence of increasingly large cities, the use of counters became complemented by further administrative tools such as cylinder seals, sealed tablets with impressions of styluses representing quantities, and sealed envelopes containing counters. Around 3000 BCE, the arsenal of tools at the disposal of the administrators of cities and city states was further enlarged by the invention of signs incised on clay tablets. These signs complemented the documentation of quantities by stylus impressions with means to indicate objects, agents, and institutions. Thus they initiated the development of a writing system representing the Sumerian language leading finally to the abstraction of arithmetical and geometrical notions.

In order to adequately conceptualize this development from incipient forms of mathematical activities and record-keeping to a numerate and literate urban culture, the author redefines the traditional concept of mathematics, which was based on the study of classical Greek mathematics. For her, scholarly mathematics is "an intellectual, supra-utilitarian end in itself, written for the purpose of communicating or recording a mathematical technique or aiding a mathematical procedure to be carried out in the course of scribal training" to be distinguished from "numeracy as the routine application of mathematical skills by professional scribes" (pp. 28f.). This definition implies that numerical activities performed in the context of scribal training, which do not directly serve administrative purposes but instead relate to the study and elaboration of their inherent potential in administrative contexts, have to be considered as mathematics. Accordingly, a specific focus of the chapter is the identification and analysis of "pedagogical exercises", in contrast to sources documenting actual accounting practices.

The third chapter, "The Later Third Millennium", focuses on the origins of what was probably the most influential innovation in southern Mesopotamia to foster the development of Babylonian mathematics, i.e., the invention of the sexagesimal place value system. Before this invention, all mathematical activities in Mesopotamia were based on commodity-specific metrological notations and context-dependent symbolic operations. Robson documents in this chapter how the administrative needs of developing empires led to the expansion, standardization, and integration of metrological systems and the development of ever more sophisticated methods of predicting and managing the storage and distribution of commodities, the allocation of labor, and the distribution of arable land. This development eventually resulted in the invention of an abstract numerical notation system, the sexagesimal place value system, which brought about a radical unification and simplification of all kinds of calculation as applied by the scribes of the state bureaucracy. Almost nothing is known about the training of the enormous number of scribes, who are known from the tens of thousands of preserved administrative documents dating to the period of the third dynasty of Ur when, during the last one hundred years of the third millennium, the rulers of this dynasty temporarily integrated all city states of Mesopotamia into one huge empire. Robson suggests that this missing evidence indicates that scribal education in this period was still realized by a kind of apprenticeship, a kind of learning through participation in administrative activities, as is attested for the earlier Sargonic period.

Chapter Four, "The Early Second Millennium", deals with the outstanding product of the Old Babylonian scribal culture that followed, conventionally designated as Babylonian mathematics. This kind of mathematics is represented by some 700 cuneiform tablets containing—besides lists, tables, diagrams, and calculations—some 150 tablets containing hundreds of mathematical problems and their numerical solutions, which differ from all earlier cuneiform tablets documenting mathematical activities. Among them are tablets with sophisticated mathematical exercises, for example finding a number that exceeds its reciprocal by 7 (pp. 113–115), which implicitly require the resolution of second-degree equations. The way such mathematical knowledge was represented and how the solutions to such problems were achieved differs from the kind of mathematics for which Euclidean geometry became a prominent paradigm, as well as from the methods of modern symbolic algebra. In the present case, for instance, the number 5 is calculated as the solution. This shows that the product of a number and its reciprocal is assumed not to be 1 but 60—i.e., the base of the sexagesimal positional system. This example indicates that sexagesimal notation represents entities that were not considered to be absolute numbers in the modern sense.

Following the work of Jens Høyrup, the author argues that the terminology used in the calculation of the result suggests that the solution is based on the mental image of a field with the area 60, and that the calculation procedure reflects some mentally performed cut-and-paste operations for which the calculation was a generic outcome.

While such problems were obviously somehow derived from the mathematical methods developed in the context of administrative practices of the state bureaucracies, it goes without saying that problems such as the given example no longer had any practical value within this context. Robson focuses in Chapter Four on the question of what might explain the development from the mathematical activities of practitioners to mathematics as a kind of esoteric art. She writes:

"While the invention of the sexagesimal place value system was a necessary condition for the creation of mathematics as an intellectual activity, divorced from the mundane necessities of central administration, the sexagesimal place value system alone is not sufficient to explain that extraordinary development." (p. 123)

In order to figure out what might explain this development, the author goes beyond the mathematical analysis of the texts. She also pays close attention to their linguistic, material, and social context in an educational environment that brought about specific numeracy and literacy in the Old Babylonian period. She investigates the archaeological context of the findings, the social context of learning that then took place in schools, the social function of the competence achieved in these schools, the material that defined the curriculum of the education system, the way in which the contents of the curriculum were perceived, and the consequences of an education establishing a "royal ideology" (p. 124) among the administrators of the first empires in the region of ancient Iraq. In her conclusion she writes:

"Thus, while modern scholars have chosen to portray Old Babylonian literature and mathematics as amongst the world's first truly creative and non-utilitarian writings (…) for their producers and consumers they represented above all idealised abstractions of the ordered urban state, with god, king, and scribe at its centre." (p. 124)

The short Chapter Five, "Assyria", sidesteps to a certain extent the chronological sequence of the main chapters. In this chapter Robson discusses the often neglected development of mathematical activities in the region commonly designated as Assyria in the northern parts of the rivers Euphrates and Tigris. Highly influenced by cultural traditions of the south, this development is an example of the diffusion of knowledge in the wake of the spreading of writing. In a spirit of "fascination with Babylonian intellectual culture" (pp. 149f), Assyria adopted techniques of administrative control together with related mathematical activities, as well as ideological notions such as the ideal of metrological justice. However, the transmission of knowledge from the south to the north involved adaptations to local conditions that in particular resulted in "a rather different flavour" as she puts it (p. 125). These differences resulted, for instance, from the fact that in the northern region merchants played a more important role than administrators of a centralized bureaucracy. As a consequence, no institutions comparable to the Old Babylonian scribal schools developed at this time in this region. Moreover, in addition to the imported sexagesimal notations, a decimal numerical notation system was used in this region, reflecting the decimal counting system of Semitic languages. Therefore, calculations had to be performed taking into account a hybridization of both systems.

The sixth chapter, "The Later Second Millennium", returns from the geographical digression of the preceding chapter to the chronological presentation of the development of mathematical activities. Archaeological evidence of the survival of the scribal culture of the Old Babylonian kingdom after its decline is scarce. On the other hand, the spreading of cuneiform writing and its adaptation to other languages such as Hittite, accompanied by the diffusion of knowledge transmitted by means of writing such as sexagesimal numeracy, reached a climax, especially in this period. The expansion of the influence of knowledge based on Babylonian origins can be traced by archaeological findings to places as far as the eastern Mediterranean coast, Anatolia, Syria, the southwest of Iran, and Egypt.

This chapter assembles the poor evidence of the scribal culture and in particular of mathematical activities in the heartland of Babylonia, showing that there was a continuity of reproducing, transmitting, and disseminating mathematical techniques, even in the so-called dark age of Babylonian culture and the ensuing occupation of the southern plain by people of Kassite ethnicity invading from an Iranian region east of the Zagros mountains. It turns out that substantial parts of Old Babylonian numeracy survived, although the political ideology that supported it was tending to disappear. The author discusses in particular the extensive innovations developed by scribes of southern Babylonia in the preceding Old Babylonian period based probably on some poorly attested earlier attempts—that is to say, the use of a tabular accounting technique. She furthermore discusses the work of northern Babylonian land surveyors who adopted the surveying methods of the south. This work is known as well from cuneiform tablets documenting the results of the surveying practice as also from a considerable number of extant so-called Kudurrus—i.e., stelae that contain monumental copies of legal documents concerning royal grants of land to courtiers or other high-ranking officials.

The seventh chapter, "The Early First Millennium", deals with a period in which Babylonia came under Assyrian control, followed by a period of Babylonian control over Assyria, and ending with the conquest by the Achaemenid Persians. Again, for this period documents related to mathematical activities are rare. Robson, however, assumes that the available evidence may at least partly be enriched in the future when the numerous excavated but still unpublished cuneiform tablets from the first millennium, now located in museums and archives, are made accessible. But even the scanty evidence available so far shows that scribal education was gradually revived, ending with a formalized curriculum in spite of the political turmoil at this time. Basic metrology and numerical knowledge related to its use in practical contexts were part of this education, which otherwise lacked the sophisticated characteristic of Babylonian mathematics as created in the Old Babylonian period. This is demonstrated in the chapter using two examples that are documented by documents from this period. The first example is the use of numeracy in urban household archives; the second is the calculation methods of land surveyors. Moreover, a few surviving examples of school exercises documenting the revival of a formalized education system are presented, and their relation to administrative practices is extensively discussed.

Chapter Eight deals with the final time period covered by the book, that is "The Later First Millennium", which according to Robson can be characterized by a growing erosion of the indigenous Babylonian culture due to Greek and Iranian influences on the one hand, and on the other by a last blossoming of cuneiform culture, in which scholarly mathematics and a newly created mathematical astronomy were central components. The surviving tablets from this period show the same intellectual complexity as their Old Babylonian ancestors but are complemented with an unprecedented application of arithmetical knowledge to systematically collected empirical data about astronomical events. This Babylonian astronomy constitutes probably the oldest empirical science based on systematic observation and measurement, integrated with inductively achieved rules. By contrast, the available sources seem to indicate that at the same time numeracy as a professional skill of scribes lost its importance. As a hypothesis, the author suggests that Babylonian mathematics underwent a major conceptual shift in the period between the fifth century BCE and the mature Hellenistic period, indicated by a "dramatic re-thinking of the status of number". According to this assumption numeracy lost its close ties to administrative practices and became considered "as an entity in its own right (to) be accounted for" (p. 261). Correspondingly, the locus of intellectual mathematical activities moved from the practitioners in institutions of a central economical administration to the age-old temples, which became widely independent of royal patronage. Babylonian mathematics, according to this assumption, no longer served predominantly practical purposes but religious ones instead and diminished with the decline of this intellectual tradition in the Hellenistic period.

The book ends with an extensive "Epilogue", drawing a "big picture" (p. 263) of three millennia of the development of Babylonian mathematics. Robson derives as a conclusion from her meticulous investigation of this development that three phases can be distinguished, characterized by numerate apprenticeship, metrological justice, and divine quantification. The actors practising Babylonian mathematics, the problems they were dealing with, and the methods they applied changed accordingly.

In the first phase of numerate apprenticeship "a small cadre of bureaucrats managed the domestic economics of big institutions" (p. 263). They developed for this purpose complex standardized metrologies and used them to control the flow of commodities by symbolic means. For all we know, the professional knowledge of these bureaucrats was transmitted from one generation to the next by apprenticeship and participation in administrative practices.

The situation changed with the occurrence of the first empires, in particular with the rise of the empire of the Third Dynasty of Ur at the end of the third millennium BCE. The standardization and unification of metrology supported the development of an ideology of metrological justice as the divine goal of bureaucracy. The author writes: "It is probably at this time that pedagogical curricula began to be formalised […]. Increased numbers of bureaucrats were needed, who had to be trained in consistent transterritorial accounting methods and in writing Sumerian, the language of bureaucracy and state" (p. 265). Thus, systematic training in school-like institutions replaced the small-scale education by apprenticeship of the earlier period of bookkeeping.

The situation changed again with the Assyrian, Neo-Babylonian, Persian, and Seleucid empires of the first millennium BCE. Empirically based astronomy was created in accordance with a growing need by the rulers to foresee the future by determining the will of the gods and acting in accordance with their wishes. Mathematics was conceived of in terms of divine quantification. Astronomical observations played an increasing role in omens foretelling the future, and arithmetical techniques became a major instrument for predicting astronomical events from observed data. Accordingly, in this last phase of cuneiform literacy, mathematics developed into a matter of priestly concern. Robson writes: "The secret knowledge of mathematics, astronomy, and ritual was communicated through apprenticeship within a tightly restricted social circle, either down the bloodline or between male members of a tiny number of families […]. These men resisted the allure of newfangled Persian and Hellenistic culture, clinging to the old ways of belief while constantly renewing and improving their mathematical methodologies" (p. 268).

This "big picture" of three millennia of the development of Babylonian mathematics is not the only content of the epilogue. It is followed by an outline of the rediscovery and reappraisal of ancient mathematics in the modern world. Robson maintains that misinterpretations of Babylonian mathematics in the standard literature on the history of mathematics result from the chronology of decipherment: Babylonian mathematics similar to mathematical activities of other ancient non-Greek cultures was studied only when ancient Greek mathematics had long achieved the status of representing the origins of mathematics per se. Therefore, mathematical activities of other ancient cultures were widely neglected or interpreted as being deficient if compared with the Greek style. For readers who are unaware of the dramatic changes of our understanding of non-Greek ancient mathematics in recent decades, Robson ends the epilogue with a glimpse of the fascinating story of the development of our modern understanding of Babylonian mathematics, from the enthusiasm of the pioneers of its decipherment to the integration of our current knowledge, leading to the composition of her book.

Admittedly, the book does not provide "a definitive history of mathematics and numeracy in and around ancient Iraq" (p. 263). It should rather be considered as a first attempt to meet "the need to break down the monolithic twentieth-century construction of Babylonian (or Mesopotamian or cuneiform) mathematics into more historically manageable pieces" (p. 288). Numerous tables listing specific groups of sources and providing tools to study them invite scholars to join the program she has sketched with her publication.