Mesopotamian mathematics, seen “from the inside” (by Assyriologists) and “from the outside” (by historians of mathematics)

Jens Høyrup
jensh@ruc.dk
http://ruc.dk/~jensh/

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"Through a glass, darkly": historians of mathematics before Assyriology

Until 1850, historians of mathematics had no other way to know about pre-classical Near Eastern mathematics[1] than use of the information they could draw from classical authors, at best submitted to historical and epistemological common sense – whence the quotations from 1 Corinthians 13:12 in the above headline (which entails no promise like that of St. Paul that in the end we see "face to face"). This, for instance, is what Montucla [1758: I, 46f] writes about "what is told" about the birth of arithmetic – the Enlightenment spirit is unmistakeable:

> Les Phéniciens, ont dit quelques-uns, furent les premiers & les plus habiles commerçans de l'univers; mais l'Arithmétique n'est nulle part plus utile & plus néccessaire que dans le commerce: ainsi ces peuples ont dû être aussi les premiers Arithméticiens. Strabon[2] nous donne cette opinion comme accréditée de son tems; & même si nous en croyons un historien,[3] Phœnix ls d' *Agenor* écrivit le premier une arithmétique en langue Phénicienne. D'un autre côté l'Égypte se faisoit gloire d'avoir été le berceau de cet art;[4] & comme une intelligence humaine parut à peine suffire pour une invention si utile, on imagina cette pieuse fable qu'une Divinité en étoit l'auteur, & qu'elle en avoit fait part aux hommes, [5] C'étoit du moins l'opinion générale, suivant Socrate ou Platon[6] que *Theut* étoit l'inventeur des nombres, du calcul et de la Géométrie; & il est fort probable que c'est de là que les Grecs ont pris l'idée, de donner à leur *Mercure*, avec qui le *Theut*, ou l'*Hermes* Egyptien a un rapport marqué, l'intendance du commerce & de l'Arithmétique. [7]

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[1] A conceptual clari cation: The "Near East" encompasses Egypt, the Palestino-Syrian area, Arabia and Mesopotamia – sometimes other neighbouring areas are included as well. Mesopotamia largely coincides with present-day Iraq. Its northern third is Assyria, and the remainder is Babylonia. Chaldea strictly speaking is the southern third (in the third millennium BCE Sumer), but often in the quotations (as in the present one) it stands for the whole of Babylonia, from where the astrology spoken of in Greek and Latin sources as "Chaldean" claimed to come.

[2] *Géograph.* lib. XVII.

[3] Cedrenus [an 11th-century Byzantine historian].

[4] Diog. Laer. in proemio [ed. Hicks 1925: I, 12].

[5] In *Phædro* p. 1240 ed. 1602. [274c].

[6] Ibid.

[7] [At this point, the astronomer Joseph-Jérôme Lalande adds the following in the second edition – the rst reasoned reference to *Babylonian* mathematics [Montucla 1799: I, 43f]:

> Il est même bien difficile de ne pas les associer les Chaldéens. Car puisque ces peuples nous présentent les premières traces des connaissances astronomiques, il falloit bien qu'ils eussent une arithmétique, et même fort avancée. Comment, sans ce secours auroient-ils pu parvenir à la découverte de plusieurs périodes astronomiques, dont la connoissance est venue jusqu'à nous!

> Mais je n’insisterai pas davantage sur ces traits fabuleux ou hazardés; quand on voudra discuter un peu philosophiquement l’origine de nos connoissances, on verra que l’Arithmétique a dû précéder toutes les autres. Les premières sociétés policées ne purent s’en passer; car il suffit de posséder quelque chose pour être obligé de faire usage des nombres, & même les premiers hommes n’eussent-ils que compté les jours, les années, leur âge, leurs troupeaux, en voilà allez pour dire qu’ils étoient en possession de l’Arithmétique. Il est vrai que les sociétés les plus riches ou les plus commerçantes ont pû étendre les limites de cette Arithmétique naturelle, en inventant peut-être des lignes ou des procédés abrégés pour soulager l’esprit dans les supputations un peu compliquées: & en ce sens Strabon n’a rien dit que de conforme à la raison. Quant au récit de Josephe[8] qui nous donne *Abraham* comme le plus ancien Arithméticien, & qui lui fait enseigner aux Egyptiens les premiers élémens de l’Arithmétique, il est aisé de voir que cet historien a voulu parer le premier pere de sa nation de quelques-unes des connoissances qu’il voyoit en estime chez les étrangers. C’est un de ces traits qui ne peuvent trouver de l’accueil qu’auprès de quelque compilateur dénué de critique & de raisonnement.

The last sentence could be directed at Petrus Ramus, in whose *Scholae mathematicae* [1569: 2] this story is taken for a fact (but with a correct reference to chapter 8).[9]

Abraham Gotthelf Kästner has no more sources and is even more cautious in his *Geschichte der Mathematik* [1796: I, 2]:

> Für uns sind die ältesten Lehrer der Mathematik, die Griechen: Was sie selbst von den Morgenländern gelernt haben, wissen wir nur aus ihren eignen Geständnissen, und wir weit ihre Lehrer für sich fortgegangen sind, das aufzuzeichnen, war ihnen nicht nöthig: vielmehr, hinderten sie wenigstens den Gedanken nicht, daß die Lehrer gegenseitig von ihnen könnten gelernt haben.

These two excerpts, with the addition quoted in note 7, illustrates how much could be known about the mathematics of Mesopotamia and neighbouring areas until the birth of Assyriology.

Apart from that, the passage is unchanged.]

[8] Ant. Jud. liv. I c. 9.

[9] In any case, Abraham is absent from Giuseppe Biancani’s *Clarorum mathematicorum chronologia* [1615: 39], and also from Vossius’s *De universae mathesios natura et constitutione liber* and *Chronologia mathematicorum* [1650] – but other direct readers of Josephus can be imagined, and so can compilers who had drawn on Ramus. Since Montucla does not abstain from chiding Ramus for following “le penchant du vulgaire vers tout ce qui porte le charactere de merveilleux” (p. 450), the present reference is most likely at least not to be to Ramus alone.

The beginnings of Assyriology

The earliest dead languages and writing systems to be deciphered were Aramaic dialects – rst Palmyrene in 1754, then in 1764 and 1768 Phoenician and Egyptian Aramaic [Daniels 1988: 431]; all three scripts were alphabetic, and the basis was provided by bilingual texts containing proper names, which were skilfully exploited by Jean-Jacques Barthélemy.

Much more famous is Jean-François Champollion's use of the Rosetta Stone in the decipherment of the hieroglyphs and the Demotic script [1824], proving the mixed alphabetic-ideographic character of the former as well as the existence of homophones in the alphabet.

The decipherment of the cuneiform scriptures was a more involved affair – a short description will illustrate how much more. In the beginning, everything was based on the trilingual inscriptions from Persepolis, which Pietro della Valle had seen in 1621 to be written from left to right. [10] The development until around 1800 is described by Fossey as on the whole a "période de tâtonnements, des hypothèses hasardés et contradictoires" (p. 90). Noteworthy positive contributions were, rstly, Carsten Niebuhr's new and more precise copies of the Persepolis inscriptions – his discovery that three different scripts were involved – and his con rmation of the writing direction [1774: II, 138 f, pl. XXIII, XXIV, XXXI]; and secondly, at the very close of the period, Friederich Münter's dating of the inscriptions to the Achaemenid era (1798, published in Danish in 1800); his con rmation that three scripts are involved; and his arguments that the rst of these is alphabetic, the second apparently mixed alphabetic-syllabic and the third perhaps mixed alphabetic-logographic [Münter 1802: 83–86]; his identi cation of a few signs from the alphabetic script as vowels (ibid., pp. 104–109); and his identi cation of its language as Old Iranian (more precisely he suggests Zend). Also of importance was Münter's veri cation that the Persepolis scripts had been used in Babylon too, and that it had probably originated in Mesopotamia (ibid, pp. 129–144).

In [1802], Georg Friedrich Grotefend presented a memoir to the Göttingen Academy[11] which is habitually taken as the beginning of decipherment proper. He came to the same conclusions as Münter (whose work only appeared in German during the same year, and which Grotefend may not have known). He went further on two decisive points, showing that all inscriptions were linked

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[10] What follows about work done before 1860 is drawn, when no original sources are referred to, from Charles Fossey's very detailed exposition of (good and bad) arguments and results [1904: 85–220].

[11] Published only in full in [Meyer 1893], for which reason I build on Fossey's account [1904: 102–111] of which of the arguments circulated.

to Darius and Xerxes; nding the royal names mentioned in the inscriptions as well as the word for king; and using this to identify a number of letters (he claimed identi cation of 29 letters of the alphabetic script, 12 of which were later con rmed).

Over the next four decades or so, a number of scholars extended and corrected Grotefend's work, removing false values and adding new ones (not always correctly at rst), and identifying the language as an Old Persian dialect distinct from Zend (adding also new inscriptions to the corpus) [Fossey 1904: 112–146]. However, all of this concerned the alphabetic script, which was certainly derived from the cuneiform script of Mesopotamia but had a totally different character (and which moreover concerned matters without the slightest relation to mathematics).

Decipherment of the second script (Elamite), using about one hundred signs and being in a language with no known kin, made some but little progress during the same period, and is anyhow irrelevant for the present purpose. Grotefend made some attempts at the third script, which is in Akkadian. His rm belief that the language had to be an Iranian dialect was one of the reasons he had no success – but until the second half of the 1840s nobody else did much better. In the meantime, excavations had begun, and a much larger, geographically wider and chronologically broader text corpus was now available.

From 1845 onward, a large number of workers took up discussion and competition about the third script, from which some 300 signs were known: Isidore Löwenstern (1845 and onward); Henry Rawlinson (1846 and onward); Paul-Émile Botta (1847 and onward); Edward Hincks (1846 and onward); Félicien de Saulcy (1847 and onward); Henry de Longpérier (1847); Charles William Wall (1848); and M. A. Stern (1850) – of whom Rawlinson, Botta and Hincks were by far the most important. Before 1855 it was known that the language of the third script was that of Babylonia and Assyria; that this language (Akkadian) was a Semitic language, and thus a cognate of Arabic and Hebrew; that the same sign might have (mostly several) phonetic and (often several) logographic values, and even function as determinatives (an unexpected function which Champollion had discovered in Hieroglyphics); that the original shape of the signs had been pictographic. Moreover, Hincks had shown that the inventors of the script must have spoken a non-Semitic language. This is all summarized in a letter written by the young Jules Oppert in 1855 (published as [Oppert 1856]), together with observations and hypotheses of his own. So, from now on large-scale reading of documents could begin – and we may speak of the birth of Assyriology. IN [1859], Oppert himself was to stabilize the eld – in his obituary of Oppert, Léon Heuzey [1906: 7] was eventually to write as follows:

> Après quelques travaux sur l'ancien perse, Oppert porta son principal effort sur les inscriptions assyriennes. Chargé avec Fresnel d'une mission en pays babylonien, il publia à son retour, en 1859, un volume, le tome second (en réalité premier en date) de son Expédition en Méslopotamie [sic], où, s'aidant des recueils de signes ou syllabaires récemment découverts, il fixa les principales règles du déchiffrement. Ce volume, le chef-d'oeuvre d'Oppert, marque une date ; il mit fin aux tâtonnements et fonda définitivement l'Assyriologie.

Assyriologists' history of mathematics, 1847–1930

On one account Oppert says nothing in his letter from 1855, even though this was to be one of the things that occupied him during his later brilliant career: mathematics.

However, already in a paper read in 1847 (published as [Hincks 1848]), Hincks had described the "non-scholarly" number system correctly.[12] In comparison, of the 76 syllabic values identified in this early paper only 18 turned out eventually to be correct or almost correct, while 46 had the right consonant but erred in the vowel, and 12 were wholly wrong [Fossey 1904: 185] – which however was already a significant step forward. The discovery of the place-value system followed soon. It was also due to Hincks [1854a: 232], who discovered it in a tabulated "estimate of the magnitude of the illuminated portion of the lunar disk on each of the thirty days of the month".[13] A slightly later publication dealing with the numbers associated with the gods [Hincks 1854b: 406f] refers to the "use of the different numbers to express sixty times what they would most naturally do" on the tablet just mentioned; there, 240 is indeed written as iv (Hincks uses Roman numerals for the cuneiform numbers), while "iii.xxviii, iii.xii, ii.lvi, ii.xl, etc." stand for "208, 192, 176, 160, etc.".

Rawlinson also contributed to the topic in [1855] (already communicated to Hincks when the second paper of the latter was in print, in December 1854). A five-page footnote (pp. 217–221) within an article on "The Early History of Babylonia" points out that the values ascribed by Berossos [ed. Cory 1832: 32] to σάρος (šār), νῆρος (nēru) and σῶσσος (šūš), respectively 3600, 600 and 60 years, are "abundantly proved by the monuments" (p. 217). As further confirmation Rawlinson presents an extract of "a table of squares, which extends in due order from 1 to 60" (pp. 218–219), in which the place-value character of the

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[12] This system is sexagesimal but not positional until 100, after which it is combined with word-signs for 100 and 1000.

[13] Archibald Henry Sayce, when returning to the text in [1875: 490; cf. Sayce 1887: 337–340], reinterprets the topic as a table of lunar longitudes. Geometrically, the two interpretations are equivalent, but the final verb of the lines (DU, "to go") suggested this new understanding.

notation is obvious but only claimed indirectly by Rawlinson. The note goes on as follows:

> while I am now discussing the notation of the Babylonians, I may as well give the phonetic reading of the numbers, as they are found in the Assyrian vocabularies.

All three of “cuneiform’s ‘holy’ triad”, as Rawlinson, Hincks and Oppert were called by Samuel Noah Kramer [1963: 15], were indeed quite aware that numbers and what had to do with them was important for understanding Mesopotamian history and culture. [14]

The reason that this was so is reflected further on in Heuzey’s obituary:

> L’activité scientifique d’Oppert se porta dans des directions très diverses : textes historiques et textes religieux, textes bilingues (suméro-assyriens) et textes purement sumériens, textes juridiques et textes divinatoires, textes perses et textes néo-susiens, il n’est presque pas une branche de la vaste littérature des inscriptions cunéiformes qu’il n’ait explorée. Les questions les plus spéciales, juridiques, métrologiques, chronologiques, attiraient sa curiosité [...].

Evidently, administrative, economical and historiographic documents could – and can – only be understood if numeration and metrology were/are understood. Reversely, such documents – in particular administrative and economical records – are and were important sources for understanding numeration and metrology. Oppert’s observations on “La notation des mesures de capacité dans les documents juridiques cunéiformes” from [1886] offers an example.

For a long time, however, they were far from the only sources for knowledge and assumptions. Already the decipherment of the scripts had drawn much on sources from classical Antiquity (how else could the names of the Achaemenid kings have been known?) and on comparison with Zend, Hebrew and Arabic. Similarly, known or supposedly known metrologies and numerical writings from classical Antiquity were drawn upon – sometimes with success (Rawlinson’s use of Berossos is an example), sometimes with exaggerated confidence in the stability and uniformity of metrologies. Didactical material such as bilingual lexical lists and tables also played their role (as also in the decipherment); so did astronomical texts (as they had already done for Hincks and Rawlinson in 1854–55).

Three illustrative examples are [Norris 1856], [Smith 1872] and [Oppert 1872]. Edwin Norris not only draws much on Biblical material in his dubious article but also reads the cuneiform signs on Mesopotamian weight standards as Hebrew characters (“I thought the first word looked like מבה” – p. 215). George Smith

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[14] In contrast, the just published Blackwell *Encyclopedia of Ancient History* planned the same number of pages for Mesopotamian mathematics and Mesopotamian hairstyles. It should be added that those who planned the volume had little idea about Mesopotamia.

makes use of metrological lists in order to establish the sequence of length units and their mutual relations, and of "lion" and "duck weights" (that is, stone sculptures of these animals on which their weight is inscribed) and of written documents in order to reach a similar understanding of the weight system (which turns out to be contradictory).

Oppert makes use of similar material. But he also believes in a shared stable "ancient" metrology, [15] and draws in particular on Hebrew parallels (and on Hebrew measures which he assumes must have a parallel[16]). Jöran Friberg [1982: 2] justly characterizes the outcome as "somewhat premature", even in comparison with other publications from the period.

The use of the place-value principle not only for integers but also for fractions was established in Josef Strassmaier's, Josef Epping's and Franz Xaver Kugler's analysis of the Late Babylonian astronomical texts, beginning with [Strassmaier & Epping 1881], but so far it was understood only in analogy with the use of sexagesimal fractions in Ptolemaic and modern astronomy.[17]

In a way, Hermann Hilprecht's *Mathematical, Metrological and Chronological Tablets from the Temple Library of Nippur* from [1906] constitutes a decisive step. As we have seen, tables of squares and metrological lists had already been used in the early period by Rawlinson [1855] and Smith [1872]. Hilprecht, however, put at the disposal of Assyriologists a large number of arithmetical and metrological tables. Unfortunately, his failing understanding of the floating-point character of the place-value system; the still strong conviction that the classical authors could provide interpretations of Mesopotamian texts; and a belief that everything Babylonian had to be read in a mystico-religious key [18] caused him

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[15] Five years earlier, however, Oppert [1886: 90] had pointed out that there were "en Assyrie et en Chaldée, comme partout ailleurs, des variations continuelles dans les mesures", which should have warned against the dangers inherent in the comparative method.

[16] See for example p. 427 on the postulated unit "hair", which leads him to rather far-fetched hypotheses (presented "sous toute réserve", it is true).

[17] Basing himself on indirect evidence and on Greek writings, Johannes Brandis [1866: 18] had already claimed that the unending sexagesimal fraction system of the Greek astronomers had to be of Chaldean origin "selbst wenn es uns nicht durch mittelbare und unmittelbare Zeugnisse als ihnen eigenthümlich dargestellt würde".

[18] Hilprecht quotes this passage from Carl Bezold's *Kurzgefasster Überblick über die babylonisch-assyrische Literatur* [1886: 225]:

> Die Mathematik stand bei den Babyloniern-Assyrern, so viel wir bis jetzt wissen, vornehmlich im Dienste der Astronomie und letztere widerum in dem einer Pseudowissenschaft, der Astrologie, die wahrscheinlich in Mesopotamien entstand, sich von dort aus verbreitete und bis hinein in die gnostischen Schriften und auf's

not only to read very large numbers into the texts but also to understand a division of 1;10 (or 70) by 1 as 195,955,200,000,000÷216,000 (p. 27), where the denominator was then explained from a (dubious) interpretation of the passage about the "nuptial number" in Plato's *Republic* VIII, 546B–D (pp. 29–34) and coupled to postulated cosmological speculations:

> How can this number influence or determine the birth and future of a child? The correct solution of the problem is closely connected with the Babylonian conception of the world, which stands in the centre of the Babylonian religion. The Universe and everything within, whether great or small, are created and sustained by the same fundamental laws. The same powers and principles, therefore, which rule in the world at large, the macrocosm, are valid in the life of man, the microcosm.

So, while materially a step forward, the approach remained that of the nineteenth century.

Franz Heinrich Weißbach's "Über die babylonischen, assyrischen und altpersischen Gewichte" from [1907], on the other hand, inaugurated a new trend. As formulated by Marvin Powell [1971: 188], "the study of Mesopotamian weight norms can be divided into two eras: the pre-Weissbach and the post-Weissbach eras". Weißbach discarded the comparative method and (like George Smith) concentrated on what could be derived from Mesopotamian sources and artefacts. He did not convince those who were committed to the "comparativist paradigm"; instead, the process confirms the observation made by Max Planck [1950: 33] (concerning Boltzmann) and famously quoted by Thomas Kuhn [1970: 151], namely that

> a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it.

The following generations of Assyriologists, indeed, less trained in Hebrew and classical scholarship but familiar with the results of a mature discipline, followed the model set by Weißbach and by the immensely influential François Thureau-Dangin. The earliest work on metrology and mathematical techniques of the latter had been published in [1897] (a sophisticated interpretation of the calculations on a field plan from the outgoing third millennium BCE); he was going to publish on metrological questions for decades to come.[19]

> Mittelalter vererbte, ohne dass wir aber bis jetzt im Stande sind, die Kette dieser ganzen Ueberlieferung, deren Glieder vielfach zerstückt sind, widerherzustellen.

[19] Outside Assyriology, in particular among natural scientists taking interest in Antiquity and its mysteries, the trend is still alive – see [Berriman 1953], [Rottländer 2006] and [Lelgemann 2004].

So far, Assyriologists had been concerned with mathematics in use, namely in use in non-mathematical documents (including astronomy and texts serving in elementary mathematical training). The first to come to grips with what became known among historians of mathematics as “Babylonian mathematics” from the 1930s onward – namely mathematics which was complicated enough to be counted as mathematics by those same historians in the twentieth century – was the 25-years old Ernst Weidner in [1916] (he had already published a volume on Babylonian astronomy in 1911 [Jaritz 1993: 15]). Weidner’s article begins with the observation that

> Ueber die Kenntnisse der Akkader auf mathematischem Gebiete sind wir heute noch recht schlecht orientiert.[20] Ausser einigen Tafeln mit Quadrat- und Kubikzahlen und verhältnismässig zahlreichen Multiplikationstabellen kommen eigentlich nur die Bau- und Felderpläne in Betracht, die uns schon für recht frühe Zeit bei den Akkadern die Fähigkeit auch schwierigere Rechnungen auszuführen, voraussetzen lassen

which summarizes the situation perfectly. Two sophisticated texts had been published in 1900, Weidner says[21] (but only in cuneiform, with neither transliteration nor translation), but these texts are then characterized as “wohl das schwierigste in Keilschrift überlieferte”, which explains that nothing had been done on them. In the Berlin Museum he had now seen other texts of the same type, and he analyses two problems from one of them (VAT 6598), two different approximate calculations of the diagonal of a rectangle (a first and a corrupt second approximation, see [Høyrup 2002: 268–272]).

Compared to the analysis of the same text offered by Neugebauer in 1935, Weidner’s interpretation contains some important errors, for which reason it can certainly be characterized as premature. However, Weidner’s short paper, together with the commentaries of Heinrich Zimmern [1916] and Arthur Ungnad [1916; 1918] provided the first understanding of Babylonian mathematical terminology.[22]

In [1922], Cyril John Gadd published a text with calculations concerning subdivided squares, and added some further important terms (not least those for square, triangle and circle). Since the text in question contains no calculations,

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[20] [A footnote refers to Moritz Cantor’s *Vorlesungen* I, on which below.]

[21] Now known as BM 85194 and BM 85210.

[22] Only the terms for (what can approximately be translated as) square and cube roots were known since Moritz Cantor’s use of Hilprecht’s material in [1908].

Quite a few of Weidner’s readings later turned out to be philologically wrong while their technical interpretation was adequate. ; What was correct, however, was important later on, and some of the philological errors were also taken over in Neugebauer’s early interpretations without great damage.

only terms for mathematical objects occur, none for operations. In [1928], finally, Carl Frank published the collection of Straßburger Keilschrifttexte – a spelling that adequately reflects his working situation: he had made the copies before the War, when Strasbourg was Strassburg, and only received his own material in 1925, with no possibility of collating. None the less, Frank's book added another batch of terms. Because Frank's texts are even more difficult that the short ones dealt with by Weidner, Zimmern and Ungnad, and because Frank translated all sexagesimal place value numbers into modern numbers (repeatedly choosing a wrong order of magnitude), his understanding of the texts was rather deficient.

This is how far Assyriologists went in the exploration of cuneiform mathematics until 1930 – the year where Assyriology had half of its present age.

Historians of mathematics until c. 1930

On the whole, historians of mathematics depended during the same period not only on the material put a their disposal by Assyriologist but also on their interpretations.

In [1874], Hermann Hankel dealt with "die Babylonier" (once, on p. 65, accompanied by the Assyrians) on scattered pages of his discussion of the "vorwissenschaftliche Periode". Given the difficulties of Assyriologist with not only absolute but also relative chronologies until [Hommel 1885], it is no wonder that his observations are messy on this account. Substantially, he speaks about the sexagesimal divisions of metrologies (pp. 48f; not mentioning that not all subdivisions are sexagesimal, which was known at least since [Smith 1872]); a hunch of sexagesimal fractions (pp. 63, 65; but only to one place, and considered written with a denominator which is "usually omitted"); the existence of tables of squares and astronomical tables (the two texts used by Hincks and Rawlinson in 1854–55), from which the hypothesis is derived that the Babylonians were interested in arithmetical series (p. 67); and a low level of geometry, concluded on the basis of the "styllose Baukunst" (p. 73). Iamblichos's claim that Pythagoras had his knowledge of the harmonic proportion from the Babylonians is mentioned but explicitly not endorsed (p. 105).

In the first edition of volume I of his Vorlesungen from [1880], Moritz Cantor dedicates separate chapters to the Egyptians and the Babylonians – the latter on pp. 67–94. He is much better informed that Hankel – in part, it must be admitted, from publications that had appeared too late to be taken into account by Hankel, such as [Oppert 1872].[23] He offers an orderly exposition of the

[23] Already Cantor's Mathematische Beiträge zum Kulturleben der Völker had contained a chapter on the Babylonians [Cantor 1863: 22–38]. At the time, however, he had only been able to speak about the decipherment; about "Oriental" culture in general; and about

numerals and the “natural fractions” $^1/_6$, $^1/_3$, $^1/_2$, $^2/_3$ and $^5/_6$. Further, he describes the tables of squares and cubes (which in [1908] he was going to see as tables of the corresponding roots), and he discusses the sexagesimal principle in connection with astronomy. Geometry is dealt with on the basis of geometric decorations, Herodotos and other Greek authors, and the Old Testament. Babylonian numerology is also discussed, in particular the ascription of numbers to the gods.

In the third edition from [1907], Cantor deals with the Babylonians before the Egyptians (pp. 19–51). The ve extra pages allow him to tell the historiography of the eld, but apart from a suggestion of that reinterpretation of Rawlinson’s tables of squares as tables of square roots which he was to publish in full in [1908], nothing substantial is changed in the account of Babylonian mathematics. There is, however, some remarks about the material published by Hilprecht in [1906], with faithful adoption of his immense numbers (pp. 28 f).

Hans Georg Zeuthen’s *Geschichte der Mathematik im Altertum und im Mittelalter* [1896] dedicates a chapter (pp. 8–13) to what the Egyptians and the Babylonians knew in mathematics at the moment they came into touch with the Greeks, and which the Greeks might possibly have taken over from them (thus pp. 8 f). Of the six pages, 26 lines deal with the Babylonians. 21 of these lines refer to astronomy and the division of the circle into 360°, and 5 to the possibility that Greek numerology was in debt to Babylonians and Chaldeans.

Johannes Tropfke follows Hankel’s pattern in the rst volume of his *Geschichte der Elementarmathematik* [1902], mentioning the Babylonians now and then but not treating Babylonian mathematics per se – he has to, since the full title is *Geschichte der Elementar-Mathematik in systematischer Darstellung*. But he only speaks about the sexagesimal system (mentioning Rawlinson’s “square table” but without describing it). Only on two (quite dubious) points does he go beyond Hankel: he considers Iamblichos a certain source, and he claims (p. 304) that the Babylonians knew the solution 3–4–5 to the “Pythagorean equation”; he gives no source, and would have been unable to, since no pertinent texts were known at the time. Most likely, he misremembers Cantor’s idea [1880: 56] (“allerdings noch ohne jede Begründung”, thus Cantor) that *the Egyptians* might have used 3–4–5 triangles on ropes to construct right angles.

In general, historians of mathematics were not interested in Babylonian matters during the period. Inspection of 21 of the rst 26 volumes of the series *Abhandlungen zur Geschichte der Mathematik*[24] (1877–1907) reveals no single article

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the writing system, integer numerals and the possible use of some kind of abacus (a hypothesis which he repeats in the *Vorlesungen*).

[24] In the moment of writing I had no access to vols 2, 16, 19–20 and 25.

on the subject. For good reasons, as revealed by what Hankel and Cantor are able to say about it – what Assyriologists had been able to nd out was still so tentative and so incoherent that it invited more to speculation than to solid work. The other possible explanation – that they should have been interested only in the higher level of mathematics – can be safely disregarded for the period before 1914, witness the many articles on elementary topics published in the same series.

The long 1930s – Neugebauer, Struve, Thureau-Dangin, and others

Beginning in 1929, the distinction between Assyriologists and historians of mathematics becomes irrelevant (for a while). This is the period when the advanced level of Old Babylonian and Seleucid mathematics was deciphered for good, after the modest but decisive beginnings made by Weidner, Zimmern, Ungnad, Gadd and Frank.

Admittedly, Otto Neugebauer is normally counted as a historian of mathematics. If anything, historian of astronomy would be the correct denomination – as we shall see, mathematics only occupied a rather short stretch of his life. But he had also been trained in Assyriology by nobody less than Anton Deimel, as he tells with gratitude in [1927: 5]. Vasilij Vasil'evic Struve was an Egyptologist but had also been trained in Assyriology, which was soon to become his main eld. Thureau-Dangin was one of the most eminent Assyriologists of his times (of all times, indeed), but the contrast between his works from the 1920s (and before) and those from the 1930s demonstrate how much the discussions (and competition) with Neugebauer and the perspective of the history of mathematics had changed his approach. Hans Siegfried Schuster, who made an important contribution in c. 1929, was an Assyriologist but participated in Neugebauer's seminar in Göttingen (Kurt Vogel, personal information; [Neugebauer 1929: 80]); Heinz Waschow studied not only Oriental philology (including Assyriology) from 1930 until 1934 but also applied mathematics [Waschow 1936, unpaginated CV]. Albert Schott, the last of Neugebauer's contacts, had a strong interest in astronomy (but the numerous references to his assistance in [MKT] all refer to strictly philological matters). Kurt Vogel, who sometimes took part in the discussion, was a mathematician and historian of mathematics but also trained in Egyptian (and Greek, and later also in medieval Italian and German) philology.

Since Neugebauer's person was all-important for what happened in the 1930s, some words about his background may be tting. His Doktorarbeitfrom [1926] had dealt with the Egyptian fraction system, but already while working at it he had become interested in the mathematics of the Sumerian cultural orbit as a parallel that might throw light on Egyptian thought, and been convinced (with due reference to Thureau-Dangin) that metrology was all-important for the development of early mathematics [Neugebauer 1927: 5].

In 1929, he launched *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik* together with Julius Stenzel and Otto Toeplitz. Since the co-editors were 17 respectively 19 years older than Neugebauer, there can be little doubt that the initiative was his. He may not have written the *Geleitwort*, but if not he must at least have agreed with it [Neugebauer, Stenzel & Toeplitz 1929: 1–2]:

> [...]
>
> Durch den Titel "*Quellen und Studien*" wollen wir zum Ausdruck bringen, daß wir in der steten Bezugnahme auf die Originalquellen die notwendige Bedingung aller ernst zu nehmenden historischen Forschung erblicken. Es wird daher unser erstes Ziel sein, Quellen zu erschließen, d. h. sie nach Möglichkeit in einer Form darzubieten, die sowohl den Anforderungen der modernen Philologie genügen kann, als auch durch Übersetzung und Kommentar den Nichtphilologen in den Stand setzt, sich selbst in jedem Augenblick von dem Wortlaut des Originales zu überzeugen. Den berechtigten Ansprüchen *beider* Gruppen, Philologen und Mathematikern, nach wirklicher Sachkenntnis Genüge zu leisten, wird nur möglich sein, wenn es gelingt, eine enge *Zusammenarbeit* zwischen ihnen herzustellen. Diese anzubahnen soll eine der wichtigsten Aufgaben unseres Unternehmens sein.
>
> Die technische Durchführung dieses Programmes denken wir uns so, daß in zwangloser Folge zwei Publikationsreihen erscheinen. Die eine, A, "*Quellen*", soll die eigentlichen Editionen größeren Umfanges umfassen, enthaltend den Text in der Sprache des Originales, philologischen Apparat und Kommentar und eine möglichst getreue Übersetzung, die auch dem Nichtphilologen den Inhalt des Textes so bequem als irgend tunlich zugänglich macht. Jedes Heft dieser "Quellen" wird ein für sich geschlossenes Ganzes bilden – Die Hefte der Abteilung B, "*Studien*", sollen jeweils eine Reihe von Abhandlungen zusammenfassen, die in engerem oder weiterem Zusammenhang mit dem aus den Quellen gewonnenen Material stehen können.
>
> Die "Quellen und Studien" sollen Beiträge zur *Geschichte* der Mathematik liefern. Sie wenden sich aber nicht ausschließlich an Spezialisten der Wissenschaftsgeschichte. Sie wollen zwar ihr Material in einer Form darbieten, die *auch* dem Spezialisten nützen kann. Sie wenden sich aber weiter an alle jene, die fühlen, daß Mathematik und mathematisches Denken nicht nur Sache einer Spezialwissenschaft, sondern aufs tiefste mit unserer Gesamtkultur und ihrer geschichtlichen Entwicklung verbunden sind, daß in der Betrachtung des geschichtlichen Werdens mathematischen Denkens eine Brücke zwischen den sogenannten "Geisteswissenschaften" und den scheinbar so ahistorischen "exakten Wissenschaften" gefunden werden kann. Unser letztes Ziel ist, an einer solchen Brücke mit bauen zu können.
>
> [...]

So, a common endeavour between philologists and historians of mathematics was aimed at, for the benefit of both groups as well as a broader educated public. That those publications about Babylonian mathematics that appeared in the journal did not cast much light on the role of mathematics in general culture was not a result of failing will; as Neugebauer had to point out in [1934: 204], one should “nicht vergessen, daß wir über die ganze Stellung der babylonischen Mathematik im Rahmen der Gesamtkultur praktisch noch gar nichts wissen”.

In the first issue, Neugebauer and Struve [1929] published an article “Über die Geometrie des Kreises in Babylonien” (but actually also about other geometric objects). Among the results is the identification of a technical term for the height of geometric plane or solid figures. The explanation is philologically mistaken, but as in the case of Weidner’s similar errors this is not decisive, as pointed out by Thureau-Dangin [1932a: 80] in the note where he gives the correction.

The preceding article in the same issue is by Neugebauer alone [1929]. It offers a new analysis of some of Frank’s texts, and manages to elucidate much which had remained in the dark for Frank. Neugebauer’s main tool is of astonishing simplicity: he retains the sexagesimal shape of numbers, while Frank, in order to get something more familiar to a modern mathematical eye, had translated them into decimal numbers (and often translated them into a wrong order of magnitude, as observed above). Beyond that, Neugebauer offers a number of improved readings.[25] Some of the problems, it turns out, contain problems of the second degree. Neugebauer concludes (pp. 79f) in these words:

> Man darf wohl sagen, daß in den vorliegenden Texten. ein gutes Stück babylonischer Mathematik zutage liegt, das geeignet ist, unsere nur allzu dürftigen Kenntnisse dieses Gebietes um wesentliche Züge zu bereichern. Ganz abgesehen von der Verwendung von Dreiecks- und Trapezformel sehen wir, daß komplizierte lineare Gleichungs-systeme aufgestellt und gelöst werden, daß man ganz systematisch Aufgaben quadratischen charakters stellt und zweifellos auch zu lösen verstand – und all dies mit einer Rechentechnik, die der Unseren völlig äquivalent ist. Bei einer solchen Lage der Dinge bereits in altbabyloniseher Zeit wird man in Hinkunft auch die spätere Entwicklung mit anderen Augen anzusehen lernen müssen.

A note added after the proofs had been finished then reveals that a text has been found which *solves* mixed second-degree problems, referring to the essential role played Schuster for understanding this, while an article written by Schuster [1930] and appearing in the second issue analyses the solution of four such problems in a Seleucid text.

The conclusion just quoted announces the approach which was to dominate the 1930s. Since the meaning of terms for mathematical operations were derived from the numbers that resulted from their use, they were by necessity understood as arithmetical operations; as a rather natural consequence, problems were understood as (arithmetical) equations and equation systems. And of course Neugebauer, as everybody else, expressed amazement that complicated matters such as second-degree equations were dealt with correctly.

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[25] “Schließlich lassen sich, nachdem einmal der sachliche Inhalt klargestellt ist, auch die Lesungen selbst nicht unerheblich verbessern” (p. 67).

Neugebauer knew very well that Old Babylonian (1800–1600 BCE, according to the "middle chronology") and Seleucid (third-second century BCE) mathematics were formulated in different terminologies. But he believed that the difference was one of terminology and implicitly supposed, as we see, that there must have been steady progress of knowledge from the early to the late period.

A number of publications from Neugebauer's hand (and three from Waschow's [1932a; 1932b; 1932c]) followed in *Quellen und Studien* B until 1936 (in vol. 4 from 1937–38, Neugebauer has turned completely to astronomy). In 1935–37, Neugebauer also published the monumental *Mathematische Keilschrifttexte* [MKT I–III]. They can be said to bring to completion the interpretation of his [1929]-paper; but they also make clear that Neugebauer had not left behind his interest in metrology and other simple matters – he was not looking merely after matters that might be seen as analogous to modern equation algebra. The conclusion of volume III [MKT III, 79 f] gives two warnings to the reader. Firstly, that MKT is a *source edition* – "Es gehört nicht zu den Aufgaben, die ich mir in dieser Edition gestellt habe, die Konsequenzen zu entwickeln, die sich nun aus diesem Textmaterial ziehen lassen". Secondly,

> Da unsere Kenntnis von diesen Dingen relativ neu ist, und übliche Datierungen erheblich verschoben werden mußten, liegt die Gefahr nahe, die babylonische Mathematik zu überschätzen. Um die leere der quellenmäßigen Grundlagen einigermaßen zu überdecken, sind in vielen geläu gen Büchern oft die elementaren mathematischen Dinge zu "Sätzen" und "Entdeckungen" gemacht worden, die großen Männern zugeordnet werden mußten. Mir scheint, man muß jetzt nicht die Babylonier zu solchen Entdeckern stempeln. Was man oft übersieht und nicht genug hervorheben kann, ist die ungeheure Schwierigkeit und Langsamkeit der Entwicklung der *allereinfachsten* mathematischen Grundbegriffe, vor allem einer wirklichen Rechen technik. Dies ist aber nicht die Leistung Einzelner, sondern nur aus historischen Prozessen verständlich, die mit der Entstehung einer Kultur überhaupt unlöslich verknüpt sind. Ist dieses Stadium erst einmal erreicht, so bedeutet die babylonische Mathematik an keiner Stelle etwas, was als unerwartete Glanzleistung angesehen werden müßte.

The last sentence refers to Neugebauer's hypothesis (which he considers an established fact) [MKT III, 79],

> daß die babylonische Mathematik zunächst aus den numerischen Methoden des sexagesimalen Zahlenrechnens erwachsen ist, dessen praktische Vorteile man voll erkannt hat und dann rasch, entscheidend gefördert von der ideographischen Ausdrucksmöglichkeit, zu einer stark "algebraisch"[26] gerichteten Behandlung rein

[26] [The quotes around the word *algebraisch* indicate that Neugebauer refuses to make hypothesis about which kind of algebraic thought is involved in the texts. The many algebraic formulas in his commentary are not meant to map the thinking of the authors of the texts; they show why the calculations are pertinent (or, rarely, why they are not).]

mathematischer Aufgaben linearen und quadratischen (bzw. darauf reduzierbaren) Charakters gelangt ist.

Thureau-Dangin, as we have seen, had been interested in metrology and mathematical techniques since [1897]. He started dialogue with Neugebauer in [1931] (making a philological correction that also concerns Frank, whom he does not mention). His weighty *Esquisse d'une histoire du système sexagésimal* [1932c], however, is rather a crown on his work from the 1920s, describing both the sexagesimal place-value system and the non-positional system and non-sexagesimal fractions, together with their uses.[27] But very soon, Thureau-Dangin moved from purely philological emendations and addenda to the publication of new mathematical texts and to considerations of their mathematical substance – for example in [1932b], [1934] and [1936] – and to a synthesis about "La méthode de fausse position et l'origine de l'algèbre" [1938] along with the source edition *Textes mathématiques babyloniens* [TMB] from the same year.[28] In several of these works Thureau-Dangin can be seen to be much less wary than Neugebauer when speaking of the algebraic thinking of the Babylonians. He also shows himself

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[27] This booklet had no strong impact – it drowned in the fury surrounding the new discoveries of the time. However, a revised English translation (including much about the Babylonian "algebra") appeared in *Osiris* in [1939] on George Sarton's initiative (p. 99).

[28] This is what von Soden [1939: 144] tells about the purpose of this parallel edition:

> Dieses neue Werk hat nicht die Aufgabe, Neugebauer's MKT zu ersetzen; werden doch weder die Lichtdrucke und Autographien der Texte wiederholt noch auch alle Texte neubearbeitet. Th.-D. 's Ziel war es vielmehr, unter vollständiger Beiseitelassung der Rechentabellen (nur die Einleitung geht kurz auf sie ein) diejenigen Aufgabentexte, deren Erhaltungszustand ein wenigstens im großen und ganzen befriedigendes Verständnis ermöglicht, in einer wohlfeileren Ausgabe möglichst vielen Forschern zugänglich zu machen, da der leider so hohe Preis der MKT ihrer weiteren Verbreitung im Wege steht.

But further:

> Wird also der Fachforscher nach wie vor auf Neugebauer's MKT als das, abgesehen von der eben erwähnten Ausnahme [two small texts from Susa with area calculations published by Vincengt Scheil in 1938], immer noch vollständige Quellenwerk nicht verzichten können, so kann gerade er aber auch nicht an Th.-D.'s neuer Ausgabe vorbeitgehen, da niemand die große Zahl der berichtigten Lesungen und die vielen, bei aller meisterhaften Knappheit ungeheuer inhaltreichen lexikalischen, grammatischen und sachlichen Anmerkungen auf einmal verarbeiten kann.

In Thureau-Dangin's own words [TMB, p. xl]:

> Le présent volume ne comprend aucun texte qui n'ait été édité ailleurs dans sa forme originale [that is, without a tranlation of ideograms into syllabic Akkadian]. Le principal objet que je me suis proposé en le rédigeant a été de mettre des documents à la disposition des historiens de la pensée mathématique.

familiar with a very wide range of later mathematical sources, from Diophantos, Ptolemy and al-Khwārizmī to Stevin and Wallis.

Then, in 1937–38, this “heroic period” ended abruptly. In 1945, it is true, Neugebauer and Abraham Sachs published *Mathematical Cuneiform Texts* [MCT],[29] an edition of texts from American collections that had not been included in MKT, and Neugebauer’s popularization *The Exact Sciences in Antiquity* from [1951] (revised in 1957) contains a chapter on the topic; but apart from that Neugebauer only published two or three small items on Babylonian mathematics after 1937, dedicating instead himself wholly to the history of astronomy (and to the launching of the *Mathematical Reviews*, after the National Socialists had taken over power over his earlier creation *Zentralblatt für Mathematik*). Schuster published nothing in the area after 1930 (he is better known as a Hittitologist), while Waschow entered the army in 1934, writing at the same time a dissertation on Kassite letters [1936].[30] In 1938 he published a book (*4000 Jahre Kampf um die Mauer*) about siege techniques since Old Babylonian times, after which I have been unable to find information about his fate (I would guess that he fell during the war). Albert Schott concentrated on astronomy, while Kurt Vogel’s *Habilitationsschrift* [1936] dealt with Greek logistics. Thureau-Dangin returned to other Assyriological questions.

In 1961, Evert Bruins and Marguerite Rutten published a volume with mathematical texts from Susa. They had started work around 1938, and Bruins was very proud of having been trained by Thureau-Dangin.[31] No wonder that the volume is wholly in the style of the 1930s – yet on a much lower philological level than what had been published during this epoch, and full of groundless speculations and misreadings (with interspersed good ideas, it should be added).

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[29] Curiously enough, MCT is much less afraid of ascribing modern mathematical concepts to the Babylonians than Neugebauer had been in the 1930s – such as logarithms, p. 35, cf. [MKT I, 363–365]. Whether this is due to Sachs’s influence or Neugebauer himself had been convinced of what others had read into [MKT] I am not able to say.

[30] The edition of one long Seleucid text (BM 34568) in [MKT III: 14–22] is also, according to Neugebauer, “bis auf einige Kleinigkeiten Herrn Dr. Waschow zu danken”. This work must be dated between 1935 and 1937.

[31] He returns to this link time and again in the numerous angry letters I have from his hand. I suppose he can be believed on this account, in spite of his general unreliability.

According to the preface [TMS, xi], Rutten made the hand copies and collaborated with Bruins on the translation. However, already the translation of word signs into Akkadian contains so many blunders of a kind no competent Assyriologist would commit that Bruins can be clearly seen to have had the upper hand concerning everything apart from the hand copies.

Assyriologists, 1940–1980

After 1940, Assyriologists would usually put aside any tablet containing too many numbers in place-value notation as "at matter for Neugebauer" (thus Hans Nissen, at one of the Berlin workshops on "Concept Formation in Mesopotamian Mathematics" in the 1980s). In consequence, very few new texts (apart from the batch from Susa) were published during the following four decades.

There is one important exception to this generalization (and a few other less important). Between 1950 and 1962 the Iraqi Assyriologist Taha Baqir published four papers in the journal Sumerwith new texts excavated between 1945 and 1962 [Baqir 1950a; 1950b; 1951; 1962]. These were highly important for several reasons: They came from a region from which until then no mathematical texts were known; like the Susa texts their provenience was known, since they were regularly excavated; but unlike the Susa texts the excavations were carefully made, for which reason the texts can also be dated.[32] Wolfram von Soden [1952] suggested a number of improved readings with implications for the interpretation,[33] and Bruins [1953] tried (as usually) to show that everything von Soden had said was absurd; but the impact of Baqir's papers on historians of mathematics was almost imperceptible – one joint article by the mathematician Karl-Bernhard Gundlach and Wolfram von Soden [1963] deals with one of Baqir's texts and a text from Susa.

Already in 1945, Goetze had contributed a chapter "The Akkadian Dialects of the Old-Babylonian Mathematical Texts" to [MCT, 146–151]. In contrast to the volume as a whole, this chapter falls outside what had been done in the 1930s.[34] In these pages, Goetze makes a careful classi cation of all Old Babylonian mathematical texts known by then that contained enough syllabic writing to allow orthographic analysis.

Occasionally, some Assyriologist publication would touch at numero-metrological questions, but not very often.[35] we have to wait until the early

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[32] A further text covering three tablets was found on the ground, left behind by illegal diggers as too damaged. It was published by Albrecht Goetze in [1951].

[33] Until then, von Soden had never worked directly on mathematical questions himself; but he had always been interested in the topic, as can be seen from his careful and extensive reviews of MKT [1937] and TMB [1939]. He also made a review of TMS in [1964], an indispensable companion piece to the edition itself.

[34] The outcome can be seen as an extension of a division of the corpus into a "Northern" and a "southern" group which Neugebauer had suggested in [1932: 6 f], but Neugebauer's arguments had been of a wholly different nature.

[35] I disregard publications in Russian, most noteworthy of which is [Vajman 1961] – my reading of Russian, which reached the level of "rudimentary" 25 years ago, has vanished

1970s before an Assyriologist took up systematically the kind of work which Thureau-Dangin and others had pursued in the 1920s. In [1971], Marvin Powell submitted his doctoral dissertation on Sumerian Numeration and Metrology soon followed by a major paper on “Sumerian Area Measures and the Alleged Decimal Substratum” [1972a]. Also in 1972 a short paper from his hand [1972b] on “The Origin of the Sexagesimal System: The Interaction of Language and Writing”, followed, and in [1976] a longer one on “The Antecedents of Old Babylonian Place Notation and the Early History of Babylonian Mathematics”, published in Historia Mathematica. The latter two articles took up topics which both Thureau-Dangin and Neugebauer had tried their teeth on around 1930, yet for lack of adequate sources from the third millennium without reaching solid results. In [1978] and [1979], Jöran Friberg, paradoxically a mathematician of merit and no Assyriologist but using approaches and methods that had been characteristic of the Assyriological tradition, made a break-through on the numerical and metrological notation of the fourth millennium; with minor corrections, his results were later confirmed by the Berlin Uruk project [Damerow & Englund 1987].

Historians of mathematics

During the same decades, little original work on Mesopotamian mathematics was made by scholars who would primarily be classified as historians of mathematics. They can be seen to have regarded the analysis in MKT and MCT as exhaustive – as it actually was on most accounts, as long as Neugebauer’s and Sachs’s approach as understood by historians of mathematics was taken for granted.

There are a few exceptions. The most substantial of these is a sequence of proposed interpretations of the famous text Plimpton 322, originally published in [MCT, 38–41] and considered there as an early instance of number theory. Most noteworthy during the early period is [Bruins 1957], where a derivation of the Pythagorean triples from pairs of reciprocals is proposed (an interpretation which has been confirmed with modifications and extra arguments in recent times by Friberg [1981] and Eleanor Robson [2001]). It may be considered a manifestation of the new “modernizing” orientation of MCT that this possibility had been overlooked, given that Neugebauer had believed in the 1930s that the whole second-degree “algebra” came from the place-value system (above, text around note 26).

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completely since then for lack of practice.

An exhaustive survey, often with discussion, of all at least minimally pertinent publications (also those in Russian) for the period 1945–1980 will be found in [Friberg 1982: 67–130]

Other exceptions are a publication of some merit by Solomon Gandz [1948], which however had been delivered to the journal around 1938 but delayed by the war; and a republication of one of Gandz's results in [1955] by Peter Huber, who had not noticed Gandz's work.

However, while little new research was done on Mesopotamian mathematics by historians of mathematics, "Babylonian mathematics" was close to becoming the standard introduction to histories of mathematics.

The way it was dealt with is well illustrated by Asger Aaboe's *Episodes from the Early History of mathematics* [1964]. Aaboe starts by observing that a modern schoolboy transposed to Babylonia or ancient Greece would find the "physics" of classical Antiquity utterly unfamiliar (p. 1). Mathematics, however, would

> look familiar to our schoolboy: he could solve quadratic equations with his Babylonian fellows and perform geometrical constructions with the Greeks. This is not to say that he would see no differences, but they would be in form only, and not in content; the Babylonian number system was not the same as ours, but the Babylonian formula for solving quadratic equations is still in use.

That is, firstly: mathematics is a topic outside history, changing "in form" only. Secondly, the "contents" of mathematics consists in "formulae". Aaboe himself may have believed to continue Neugebauer's approach, but in reality the programme of *Quellen und Studien* has been betrayed. The "scheinbar so ahistorischen 'exakten Wissenschaften´" have become, exactly, *ahistorisch*. The lack of information about social context is no longer a deplorable fact, as for Neugebauer in 1934 – the absence of information about its creators is just taken note of, institutional setting etc. constitute non-questions. [36]

Coming to the contents the reader learns that the sexagesimal place-value system is *the* Babylonian number system. Aaboe ignores that it was used only for intermediate calculations; in school; and in (late Babylonian) mathematical astronomy, and that a different system was used in "real-life" juridical and economical documents[37] – he only knows about inconsistency and failing rationality. [38]

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[36] "Of the creators of Babylonian mathematics we know nothing whatsoever except the result of their work" (p. 6). That the texts are school texts is intimated by photos of presumed schoolrooms from Mari (which are actually store-rooms) and occasional references to a "schoolboy" – but schooling seems to be just as timeless as mathematics. In 1964, it should be noted, more was known about the Old Babylonian scribe school than in 1934, cf. [Kramer 1949], [Falkenstein 1953] and [Gadd 1956].

[37] However, all of this is described in [Thureau-Dangin 1939], who distinguishes the "abstract" (namely place-value) system "intended only to serve as an instrument of calculation" (p. 117) from the ordinary sexagesimal but non-positional system.

[38] "It should be added that an entirely consistent use of the sexagesimal system is to be

When going beyond computation with the place-value system Aaboe deals with three more advanced topics. The first is treated through two "algebraic" problem about square areas and appurtenant sides from BM 13901, quoted in Neugebauer's translation but then immediately transformed into modern algebraic symbols; the second is YBC 7289, the tablet showing a square with diagonals and three inscribed numbers corresponding to the side, the diagonal and their approximate ratio, which allows immediate discussion in terms of $\sqrt{2}$; the third the calculation of a height in an isosceles trapezium. It is mentioned (p. 23) that the first two are Old Babylonian and the third Seleucid, but it is claimed (as did not correspond to the information that could be extracted from MKT) that all three could have been written in any period. The conclusion discusses "algebras" once again, and states that

> Quadratic equations are often given in the equivalent form of two equations with two unknowns, such as
> $$x + y = a\,,\quad xy = b\,,$$
> whence one finds immediately that x and y are the solutions of
> $$z^2 - az + b = 0$$

without mentioning that such problems deal with rectangular areas and sides, nor that the "one" who "finds immediately" is Aaboe himself or some other modern calculator, and that no corresponding step can be found in the original texts.

Aaboe's book was intended as supplementary high-school reading, and can thus be understood according to Toeplitz' "genetic method" [1927], the introduction of modern concepts through pedagogically motivating idealized quasi- (or pseudo-)history.[39] However, the typical general histories of mathematics published during the period share the basic character of Aaboe's presentation – see my anatomies of [Hofmann 1953], [Boyer 1968] and [Kline 1972] in [Høyrup 2010]. Only another book written for the high-school level (but here the German *Gymnasium*), Vogel's *Die Mathematik der Babylonier* from [1959]

found only in the mathematical and astronomical texts, and even in astronomical texts one can find year numbers written as, e.g., 1-*me* 15 (meaning 1 hundred 15) instead of 1,55. In practical life the Babylonians showed the same profound disregard for rationality in their use of units for weight and measure as does the modern English-speaking world" (p. 20). The year number in question is written in precisely that number system which Hincks had deciphered in 1847, cf. note 12 – the very first contribution to the study of Assyro-Babylonian mathematics!

[39] "Nichts liegt mir ferner als eine Geschichte der Infinitesimalrechnung zu lesen; ich selbst bin als Student aus einer ähnlichen Vorlesung weggelaufen. Nicht um die Geschichte handelt es sich, sondern um die Genesis der Probleme, der Tatsachen und Beweise, um die entscheidenden Wendepunkte in dieser Genesis" [Toeplitz 1927: 94].

stands out – with its awareness that the place-value system was a scholarly system; because of its interest in metrologies and in computations dealing with everyday life; and with its discussions of ways of thought.[40] Vogel, indeed, had worked on the material himself already in the 1930s and always been interested in ways of thought and in the mathematics of practical life, while Aaboe had only worked on Seleucid astronomical texts, and Hofmann, Boyer and Kline at best on Neugebauer's translations – but apparently more often on his popularizations and his explanatory commentaries without distinguishing the latter from what was done in the sources.

After 1980

After c. 1945, the historiography of Mesopotamian mathematics had thus been an almost dead topic, little considered by Assyriologists and treated under the point of view of "historical mathematics" by those who otherwise wrote about the history of mathematics.[41]

Beginning with the works of Powell and Friberg that were mentioned, this situation was going to change once again. But this is where my own work in the field started, first on the connection between mathematics, general socio-cultural context and educational situation, from 1982 onward on the concepts and operations of Old Babylonian mathematics, so here I shall stop – adding only that in recent years a number of younger scholars trained in mathematics as well as Assyriology have entered the field, adding new approaches and returning to the Assyriological questions of the earlier twentieth century with the luggage of a century of extra textual and archaeological discoveries, thus being able to integrate the mathematical dimension with studies of social, political and economic history. So, the field remains alive – but mathematicians may not find it very interesting for their purpose.

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[40] Dirk Struik's *Concise History of Mathematics* from [1948] deals with Mesopotamian mathematics too briefly to allow description in depth as Vogel (pp. 23–32). Struik's layout, however, is similar: The analysis is embedded in general social history, non-positional as well as place-value system is described, but like Vogel's, Struik's analysis has no possibility to go beyond Neugebauer's.

[41] Boyer had written about *The Concepts of the Calculus* [1949], and Kline's title refers to "mathematical thought". Hofmann had written among other things about Ramon Lull's squaring of the circle in [1942], and had tried there to penetrate the thinking and motives of Lull (without which he would not have been able to conclude anything of interest).

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