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Algebra in the scribal school – Schools in old Babylonia algebra?*

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Dedicated to Hubert L. L. Busard and Menso Folkerts in gratitude for so many important editions

Zusammenfassung

Eine Reihe von mittelalterlichen Schriften zur Landmessung (vom 9. islamischen Jahrhundert bis zu Fibonacci und Pacioli) enthält eine besondere Art von „algebraischen" Aufgaben. Darin werden z.B. die Summe der Fläche und einer oder alle vier Seiten eines Quadrates beschrieben und nach der Seite gefragt. Es zeigt sich erstens, daß dieser Aufgabentyp mindestens seit dem frühesten 2. vorchristlichen Jahrtausend von geometrischen Praktikern tradiert wurde, und zweitens, daß er die Entwicklung einer „Algebra" in der altbabylonischen Schreiberschule inspirierte.

Der Aufsatz untersucht, in welcher Weise die Überführung der „sub-wissenschaftlichen" Praktikertradition in einen systematischen Schulunterricht den mathematischen Inhalt und den Denkstil künftiger Praktiker prägte. Im letzten Kapitel wird diskutiert, inwieweit es sinnvoll ist, innerhalb dieser Schulalgebra von „Schulen" zu sprechen.

I. From tradition to schooling

In the second, geometrical part of his *Summa de Arithmetica*, Luca Pacioli [1] states that

> "Benche nela parte de arithmetica dicessimo de la regola dalghebra assai copiosamente: Niente dimeno e necessario alcuna cosa qui dirne."

The main interest of this passage lies in the assertion that it is *necessary* to say something about algebra when presenting practical geometry. As to the content, Luca follows the corresponding section of Leonardo Fibonacci's *Pratica geometrie* (ed. [Boncompagni 1862: 56ff]) in a characteristic interest in problems dealing (e.g.) with a square area and *the four* sides, and has obviously copied

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directly (so much so that misprints in the diagrams of the 1523 edition are easily corrected with recourse to Leonardo's text).

Leonardo's problems refer to genuine geometrical configurations, the linear extensions of which are given in a real unit (*pertica*). Often he asks, not for "the side" of a square but for "each of its sides". In many cases he has recourse to standard algebra, but in others he argues on geometrical diagrams (very often a problem is solved in both ways). The arguments from diagrams are tainted by familiarity with *Elements* II[1], but basically most of them are elaborations of the same kind of naive-geometric considerations which constitute the fundamental method of Abū Bakr's *Liber mensurationum* (ed. [Busard 1968]), as I have argued elsewhere (e.g., [Høyrup 1986])[2]. A number of other affinities, some regarding mathematical content or style, some the terminology and the formulations, and some the organization of the subject-matter) also demonstrate that Leonardo builds on a source which is very close to Abū Bakr's work – maybe simply on another version than the one we know, and which is partially corrupt, maybe on a work equally used by Abū Bakr[3].

Savasorda's *Liber embadorum* contains a similar though shorter "algebraic" section, also interested in square area and four sides and making use of similar diagrams (this time supported by explicit references to *Elements* II).

Many aspects of Abū Bakr's work, on its part, point back to the style and content of certain parts of Old Babylonian mathematical texts (cf. [Høyrup 1986] and [1992a]), in particular to problems themselves pointing towards an origin in a surveyors' environment (see [Høyrup 1990: 271–275, 309–314], and [Høyrup 1990a: 79f]; to be explicated below), from which the Old Babylonian scribe school will probably have borrowed its interest in second-degree "algebraic" problems[4].

Abū Bakr is separated from the Old Babylonian scribe school teachers by c. 2500 years, and Luca Pacioli from the earliest traces of "mensuration algebra" by almost 4000. Continuity and (at least concerning the central 2500 years) immutability of essential characteristics testify to the existence of a *tradition*.[5] It also suggests how we may distinguish *a tradition* from *a school* in the pre-modern context, in particular when we compare the characteristics of the surveying tradition with its scribe-school offspring.

If Q signifies the area and s the side of the square, s_s "its sides" (i.e., the sum of the four sides), s_4 "its four sides" and s_u "each of its sides" ([unum]quodque latus), the section on squares in Abū Bakr's work contains this sequence of problems:

1. $s = 10, Q$?
2. $s = 10, d$?
3. $s+Q = 110, s$?
4. $s_4+Q = 140, s_u$?
5. $Q-s = 90, s$?

6. $Q-s_s = 60$, s_u?
7. $s_s = 2/5 \cdot Q$, s_u?
8. $s_s = Q$, s_u?
9. $s_s-Q = 3$, s_u?
10. $d = \sqrt{200}$; s?
11. $d = \sqrt{200}$; Q?
12. $s_s+Q = 60$, s?[6]
13. $Q-3s = 18$, s?
14. $s_s = 3/8 \cdot Q$, s_u?[7]
15. $Q/d = 7\frac{1}{2}$, s_u?
16. $d-s = 4$, s?
17. $d-s = 5$, no question, refers to the previous case.
18. $d = s_u+4$, no reference to N° 16.
19. $Q/d = 7\frac{1}{14}$, s?, d?

We observe that everything remains very close to the observable geometric configurations. With one exception, what occurs is either *the side* or *the sides*, and the single area. Similarly, Leonardo's text only brings one problem that in modern terms would be non-normalized (p. 60). He does not tell, however, that thrice the area added to the four sides yields 279, but that the sum of the square on the diagonal, the area and the sides equals this number.

The picture offered by both authors in the sections on "quadrates one side of which is longer"[8] and on rhombi is analogous. Again, we encounter the length, the width, the two sides (i.e., the two different sides), the four sides, the diagonal, the diagonals, etc.

Certain Old Babylonian algebra problems are strictly similar – thus BM 13901 N° 23, AO 8862 N° 1, and the single problem on Db_2–146. The first is of the type $s_4+Qb = A$, while the second tells the sum of the length and width of a rectangle and the sum of the area and the excess of length over width, and the third coincides with N° 27 of the *Liber mensurationum* (given area and diagonal of a rectangle; also in Leonardo, p. 64).

But in spite of the strong similarities between Abū Bakr's "mensuration algebra" and the Old Babylonian texts as concerns certain techniques (not least nor however solely the naive-geometric fundament) and the grammatical and rhetorical structure of the text, Old Babylonian algebra differs strongly from what we encounter in our Medieval treatises – so strongly that a direct descent from Old Babylonian scribe-school mathematics is highly improbable. This becomes visible when we ascend from the single problems to the larger structures in which they are organized.

One such larger structure is the tablet BM 13901, which contained the problem $s_4+Q = A$ as its N° 23. Using the same symbols as before (and subscript numbers when several squares are involved), it contains the following problems (the length of the side is always asked for; "□" between lines refers to the construction of a rectangular area; numbers are transcribed according to Thureau-Dangin's convention, ′, ″ etc. indicating descending and ‵, ‶ etc. ascending sexagesimal orders of magnitude):

1. $Q+s$ = 45′
2. $Q-s$ = 14‵30
3. $(⅔)Q+(⅓)s$ = 20′
4. $(⅔)Q+s$ = 4‵46°40′
5. $Q+s+(⅓s$ = 55′
6. $Q+(⅔)s$ = 35′
7. $11Q+7s$ = 6°15′
8. Q_1+Q_2 = 21′40″, s_1+s_2 = 50′ (reconstructed)
9. Q_1+Q_2 = 21′40″, $s_2 = s_1$+10′
10. Q_1+Q_2 = 21°15′, $s_2 = s_1-(1/7)s_1$
11. Q_1+Q_2 = 28°15′, $s_2 = s_1+(1/7)s_1$
12. Q_1+Q_2 = 21′40″, s_1-s_2 = 10′
13. Q_1+Q_2 = 28′20″, $s_2 = (¼)s_1$
14. Q_1+Q_2 = 25‵25, $s_2 = (⅔)s_1+5$
15. $Q_1+Q_2+Q_3+Q_4$ = 27‵5, $(s_2,s_3,s_4) = (⅔, ½, ⅓)s_1$
16. $Q-(⅓)s$ = 5′
17. $Q_1+Q_2+Q_3$ = 10‵12°45′, $s_2 = (1/7)s_1$, $s_3 = (1/7)s_2$
18. $Q_1+Q_2+Q_3$ = 23‵20, $s_2 = s_1+10$, $s_3 = s_2+10$
19. $Q_1+Q_2+(s_1-s_2)\square(s_1-s_2)$ = 23‵20, s_1+s_2 = 50
20. [missing]
21. [missing]
22. [missing]
23. s_s+Q = 41′40″
24. $Q_1+Q_2+Q_3$ = 29‵10, $s_2 = (⅔)s_1+5$, $s_3 = (½)s_2$+2°30′

What most of all characterizes this list (and other Old Babylonian texts) in contrast to the one from the *Liber mensurationum* is a rather systematic variation of coefficients quite unfettered by the actual geometric configuration dealt with. Another difference, less conspicuous but none the less present, is the introduction of *representation:* Even though the entities appearing in the equations are measurable lines and surfaces, they can be used to represent entities of other kinds which are involved in structurally similar relationships; in precisely the same way, the x's and y's of modern elementary algebra, though conceptualized as pure numbers, may represent prices, weights, etc. N° 12, indeed, is solved in a way which lets *the areas* of the two squares be represented by *the length* and *the width* of a rectangle, whose area is determined as $(s_1\square s_2)^2$.

In principle, the difference between the two mathematical enterprises could be explained in two ways. Surveyors borrowing and continuing the algebraic tradition of the Old Babylonian scribe school might change its character, leaving out what had little appeal within their professional environment. Alternatively, the scribe school might have been inspired by a pre-existing surveyors' "sub-scientific tradition"[9] and have developed a limited array of "algebraic riddles" dealing with real geometrical configurations into a mathematical discipline *sui generis*.

The presence of isolated pieces of characteristic mensuration algebra in what appears to be the earliest Old Babylonian mathematical tablets (and hence the earliest algebraic texts at all), occasionally in a language which even inside this context seems to contain certain deliberate archaisms, speaks against a theory of *gesunkenes Kulturgut*.[10] We are left with the conclusion that algebra originated as a set of surveyors' puzzles, and was only expanded, systematized and recast in the scribal school.[11] What we find in the *Liber mensurationum* will be a descendant of the original surveyors' tradition, most probably of course with significant admixture, first from the Babylonian and later from the Alexandrian school tradition.

The organization of Old Babylonian algebraic texts can thus illustrate the process that takes place when a branch of sub-scientific mathematics is adopted into and transformed by an intellectually strict school environment. Further investigation may also reveal whether the process resulted in anything which can reasonably be regarded as a formation of *a mathematical school* – maybe even of different schools.

II. The impact of schooling

A first striking difference in character between the sub-scientific tradition and its scribe-school offspring is the contrast between the stability of the former and the relatively rapid change of the latter. The Old Babylonian era, in total, lasted from c. 2000 B.C. to 1600 B.C. Within this time span, at most one sixth of the distance that separates its termination from the epoch of Abū Bakr, took place the transformation of the stock of surveyors' puzzles into the organized discipline reflected in BM 13901, and the further reorganization of this in the series texts, which is the theme of the following paragraphs.

The contrast between the systematic variation of data in BM 13901 and the structure of the corresponding section of the *Liber mensurationum*[12] was already dealt with. Other texts, according to internal evidence of a somewhat later but still Old Babylonian date, bring this spirit of systematization to a culmination (and certainly a peak of boredom for the students, if they ever had to solve the problems in sequence!).

These texts are the "series texts", thus called because the single tablets are elements of larger series. Series organization of tablets belonging together is known from other domains of Old Babylonian culture: Omina, incantations, lexical series. Like omina, incantations, and lexical series, the mathematical series contain long lists of single cases, ordered according to some principle or principles. At times only the statements of problems are given, at times also the solutions.

YBC 4714 – a texts which informs about the solutions – is told to be Tablet 4 of its series, and contains problems which are somewhat more intricate than those of BM 13901 but of the same character; we may imagine that the subject-matter of the latter tablet was dealt with in Tablet 3 of the series (only Tablet 4 is extant). The problems of Tablet 4 are ordered as follows (some of them are damaged and reconstructed from context; {L} stands for a set of linear equations involving the sides of the squares, always precisely as many equations as needed and often tediously complex; the "25 nindan" in N^{os} 30–39 is presented as a "second width", the nindan being the basic unit of horizontal distance):

1. Q_1+Q_2 = 21`40, s_1+s_2 = 50
2. $Q_1+Q_2+Q_3+Q_4$ = 1``30`, $s_1+s_2+s_3+s_4$ = 2`20
3. $Q_1+Q_2+Q_3+Q_4+Q_5+Q_6$ = 1``52`55, $s_1+s_2+s_3+s_4+s_5+s_6$ = 3`15
4. $Q_1+Q_2+Q_3$ = 30`50, $s_2 = (1/7)s_1+15$, $s_3 = (1/2)s_2+5$
5. $Q_1+Q_2+Q_3$ = 1``8`5 (or $Q_1+Q_2+Q_3+s_1+s_2+s_3$ = 1``9`46), {L}
6. $Q_1+Q_2+Q_3+s_1+s_2+s_3$ = 27`50, {L}
7. $Q_1+Q_2+Q_3$ = 1``17`30, {L}
8. $Q_1+Q_2+Q_3+Q_4$ = 2``23`20, {L}
9. $Q_1+Q_2+Q_3+Q_4+s_1+s_2+s_3+s_4$ = [?], {L}
10. $Q_1+Q_2+Q_3+Q_4$ = 1``15`50, {L}
11. $Q_1+Q_2+Q_3+Q_4$ ¿[$+s_1+s_2+s_3+s_4$]? = [?], {L}
12. $Q_1+Q_2+Q_3+Q_4$ = 1``36`15, {L}
13. $Q_1+Q_2+Q_3$ = 2``47`5, $s_1$ = 1`20, {L}
14. $Q_1+Q_2+Q_3$ = 2``47`5, s_2 = 45, {L}
15. $Q_1+Q_2+Q_3$ = 2``47`5, s_3 = 40, {L}
16. $Q_1+Q_2+Q_3$ = 2``47`5, {L}
17. $Q_1+Q_2+Q_3$ = 2``47`5, {L}
18. $Q_1+Q_2+Q_3$ = 2``47`5, {L}
19. $Q_1+Q_2+Q_3$ = 2``47`5, {L}
20. [too damaged for reconstruction]
21. $Q_1+Q_2+Q_3+Q_4$ = 52`30, $s_{i+1} = s_i+(1/7)s_1$
22. $Q_1+Q_2+Q_3+Q_4+s_1+s_2+s_3+s_4$ = 54`20, $s_{i+1} = s_i+(1/7)s_1$
23. $Q_1+Q_2+Q_3+Q_4$ = 52`30, $s_{i+1} = s_i+(1/4)s_4$
24. $Q_1+Q_2+Q_3+Q_4+s_1+s_2+s_3+s_4$ = 54`20, $s_{i+1} = s_i+(1/4)s_4$
25. $Q_1+Q_2+Q_3+Q_4$ = 52`30, $s_{i+1} = s_i+(1/5)s_3$
26. $Q_1+Q_2+Q_3+Q_4+s_1+s_2+s_3+s_4$ = 54`20, $s_{i+1} = s_i+(1/5)s_3$
27. $Q_1+Q_2+Q_3+Q_4$ = 52`30, $s_{i+1} = s_i+(1/2)183(1/3)s_2$
28. $Q_1+Q_2+Q_3+Q_4+s_1+s_2+s_3+s_4$ = 54`20, $s_{i+1} = s_i+(1/2)\cdot(1/3)s_2$
29. Q_1+Q_2 = 48`45, $s_1\square s_2$ = 22`30
30. Q_1-Q_2 = (25 nindan)$\square s_2$, {L}

31. $Q_1 - Q_2$ = (25 nindan)□s_2, {L}
32. $Q_1 - Q_2$ = (25 nindan)□s_2, {L}
33. $Q_1 - Q_2$ = (25 nindan)□s_2, {L}
34. $Q_1 - Q_2$ = (25 nindan)□s_2, {L}
35. $Q_1 - Q_2$ = (25 nindan)□s_2, {L}
36. $Q_1 - Q_2$ = (25 nindan)□s_2, {L}
37. $Q_1 - Q_2$ = (25 nindan)□s_2, {L}
38. $Q_1 - Q_2$ = (25 nindan)□s_2, {L}
39. $Q_1 - Q_2$ = (25 nindan)□s_2, {L}

The affinity between the mathematical substance of this text and that of BM 13901 is obvious, as is the higher level of systematization and complexity (achieved by trivial means). Most interesting is probably the use of a "two-dimensional" system of variation, e.g. in N^{os} 21–28.

Other series texts go further in this direction. A section of the very long text YBC 4668 deals with rectangles (x□y) of area 1 eše (= 600 nindan^2), where the other condition regards the entities $X = x\cdot(x/y)$ and $Y = y\cdot(y/x)$. The subsection made up by N^{os} C 38 to C 53 (MKT I, 431f) varies the condition $X+(1/19)(X-Y) = A$ systematically along four dimensions: Replacing + with –, replacing the numerator with 2, replacing the denominator with 7, replacing the first X with Y – all in all, this gives a Cartesian product of $2^4 = 16$ problems.[13]

The series texts do not differ from procedure texts like BM 13901 solely by systematic and pluridimensional variation of data. Their language is also specific:

BM 13901, like the two other early algebra texts mentioned above, is written in predominantly syllabic Akkadian, using Sumerograms only for a few specific terms. Other procedure texts use logograms less sparingly, but it is always clear that we are confronted with logographic writing of standardized but still full Akkadian phrases.

The series texts are quite different, so much so that Neugebauer ([1932: 222]; MKT I, viii) regarded their Sumerograms not only as non-logographic ideograms but even as a *mathematical symbolism*. In any case, only a few prepositions and the relative pronoun *ša* are written at times syllabically (but the relative pronoun is often omitted, and many prepositions are replaced by Sumerian case suffixes), and the use of Sumerograms is highly condensed. Like mathematical symbols they function like ideograms rather than logograms.

Speaking of a "mathematical symbolism" is, none the less, misguided. A mathematical symbolism is meant to be *unambiguous*, which is the very reason that it allows operation *at the level of symbols*, without continuous thought for the interpretation of these. A text like YBC 4714, on the other hand, abbreviates "the [sum of the] 2 sides" and "the 2nd side" in exactly the same way, to mention but one glaring example. It is thus *stenographic*, and not symbolic, and the interpretation of the single passages depends on the user's understanding of the context.

Mathematical symbolism has a strong impact on the mode of mathematical thought, precisely because it allows easy operation at what would be a high level of abstraction if the symbolism was not available. Stenographic writing also has an impact, but of a different kind. While the flexibility and accuracy of symbolic expression allow the formulation of *new, unexpected* problems, stenographic writing *increases the rigidity* of the discipline and decreases inventiveness: if interpretation depends on context, then only problems of an already familiar type can be formulated.

One reason that the scribe-school texts become tediously systematic is of course their drilling function. The algebraic problems of the surveying tradition are *recreational problems* – hence their interest in the striking *[four] sides* of a square, resembling the appetite of the camel which will devour *precisely* all the grain it has to carry unless a clever trick is applied[14]. They serve professional identity and pride (carrying an implicit message "Tell me, if you are a clever calculator, ..."). This is also one function of Babylonian second-degree algebra, whose problems have no practical purpose whatsoever. But it has another function, which is best fulfilled precisely by the boringly repetitious variation of basic patterns: The training of computational skills, in particular computation with the sexagesimal place value system. From this point of view, the important part of N^{os} 30 to 39 of YBC 4714 is not the recurrent second-degree equation but the reduction of the more or less weird first-degree equations (e.g., in N^{o} 36, $s_2-(1/3)(s_1-s_2) = 16°40$, which can be reduced to $s_1 = 4s_2-50$ by means of techniques carefully explained in the tablet TMS XVI–cf. [Høyrup 1990: 299–305]), and the transformation of the second-degree equations by means of the information thus obtained.

The didactical function, however, cannot be the sole and immediate explanation, as it may be illustrated by a cursory comparison with Egyptian scribe school teaching. The training of computational skills was a no less important task for the Egyptian than for the Babylonian school, yet neither the Rhind nor the Moscow Mathematical Papyrus (the only sources extensive enough to allow comparison) show any trace of trivially complex[15], not to speak of pluridimensional variation.

In the Mesopotamian school, on the other hand, the drive toward pluridimensional hierarchical order and exhaustive coverage of possibilities was old and pervasive. Its first traces are found already in the proto-literate "profession list", which means that the priestly rulers of the late fourth millennium will have imposed, or at least tried to impose, a similar order on the actual social fabric of the city-states, just as they imposed a higher level of mathematical regularity on metrological and numerical systems than required by practical considerations.[16] In fully developed shape it characterizes the Old Babylonian grammatical texts, which go so far as to invent non-existing grammatical forms for the sake of completeness [Reiner 1990: 98f]; omen texts from the period try to list all possible liver shapes (including those which we would consider impossible) and has a series of contrasts (up/down, large/small, short/long, etc.) as one of their organizing principles [Larsen 1987: 213f].

The quest for exhaustive variation and pluridimensionality is thus to be explained from the interplay of two factors: on one hand, the task of the scribal school and the function of algebra problems in relation to this task; on the other, the general intellectual framework of the Sumero-Babylonian school.

Mensuration algebra appears to have had only two "favourite numbers" in the domain of coefficients, namely 4, the number of sides of a square or a rectangle, and 2, the number of *different* sides of a rectangle and the number of diagonals in a rhombus (the latter number being of course no real coefficient in the modern sense)[17]. These are the only contexts in which these numbers stand out as remarkable. Square sides are preferably 10 – again, this is the only function in which 10 possesses a special status[18].

Old Babylonian school algebra had several sets of favourite numbers, each set bound up once again with a specific role[19]. Sides of squares and rectangles are almost invariably chosen to be multiples of 5 (or 5′), mostly between 15 (15′) and 35 (35′); the absolute favourites are 20 (20′) and 30 (30′)[20]. "Simple" variations of basic equations are created by means of coefficients ⅓, ½ and ⅔ (cf. the initial problems of BM 13901). "Complex variations" involve the numbers 4, 7, 11, 13, 17 and 19 as factors or denominators of coefficients or relative differences between unknowns.

The fractions ⅓, ½ and ⅔ are precisely those which possess their own specific cuneiform sign, which may suffice to explain their role. The choice of round, but not too round, values for the unknowns is age-old in Mesopotamian mathematics teaching – indeed one of the features which allow us to single out certain proto-literate texts as didactical and not administrative – but it is equally familiar from other teaching traditions (the same principle makes Abū Bakr and his predecessors choose 6 and 8 as the sides of the standard rectangle). Sides of 20 or 30 (more rarely 15, 35, or 35) may thus have been felt as "natural" choices.

Because the specific role of 7 belongs to our own Biblical (whence ultimately Mesopotamian) legacy, we may find the group of "complex" multiplicative-partitive numbers equally natural, albeit with some amazement concerning the appearance of 4. Yet if these numbers were *particular per se* and thus independently of function we should not expect them to occur only in multiplicative and partitive roles – why only be interested in pairs of squares whose sides differ by ⅐ of either the larger or the smaller, and never in squares whose sides differ by 7 nindan?

7, 11, 13, 17, and 19 are immediately recognized as *prime* and *sexagesimally irregular*. It *is* thus precisely in the multiplicative-partitive domain that they have been discovered to be particular: they are absent from the normal table of reciprocals; with the exception of 7 they do not possess their own multiplication tables (while they occur as multipliers in other tables, which is not the case for primes above 20); and they cannot be reduced to simpler cases through factorization (which distinguishes them from 14).

7 is singled out already in mid-third millennium Sumerian school texts, and 11 (more precisely: $33 = 3 \cdot 11$) not much later in Ebla[21]. The heavily computing Ur-III period (21st c. B.C.) will have stumbled upon the rest if it had not happened before.

By being irreducible through factorization, 7 etc. function as representatives of *number in general*, and their only partial absence from standard tables makes it *difficult, but not too difficult*, to deal with them. Reduction, for instance, of the equation $X+(1/19)(X-Y) = 46°40$ presupposes that $n = 19$ be looked up in (or remembered from) the tables $n \cdot 40$ and $n \cdot 6°40'$, and that the results be added; if the denominator had been 18, a simple factorization strategies could have been used ($3 \cdot 46°40' = 2`20$, $6 \cdot 2`20 = 14`$, two pieces of elementary mental arithmetic for a trained calculator), while a denominator 23 would involve the addition of four partial products of two sexagesimal places (exceeding certainly the capacity of most calculators for mental arithmetic). In many cases, of course, the choice of (e.g.) 7 instead of 6 or 8 produces no particular difficulty – thus in BM 13901 N° 10, which is solved by means of a false position $s_1 = 6$, $s_2 = 7$; in such cases, the special role of 7 has already been hypostatized and its concrete role as a complexifier been left behind.

In a similarly vein, 4 can be regarded as the first "non-small" number (cf. that 1/4 is the first fraction which has to be expressed in igi-formalism). As already the production of complexity through coefficients, the rise of 4, 7, 11, 13, 17 and 19 to prominence as "remarkable numbers" in the partitive-multiplicative domain of Old Babylonian algebra is thus a consequence, it appears, in part of the obligation of the school to train computational skill, in part of a "psychology of numbers" imprinted by the use of the sexagesimal place-value system with appurtenant tables of multiplication and reciprocals.

A third point where Old Babylonian algebra differs from its sub-scientific precursor is in the creation of a distinction which has been understood as a differentiation between "positive" and "negative" number (wrongly so, as I shall argue).

In many of the problems dealing with rectangles it is told by Abū Bakr and Leonardo that one side exceeds the other by a certain amount (this is the only part of the statement which is then told in the third person singular, present tense). Precisely the same expression (and the same characteristic grammatical form) is used time and again in the Old Babylonian algebra texts, not least in corresponding passages of the statements. This is what was rendered in the above symbolic transcriptions by $B = A+d$ (in Sumerographic writing: *B* ugu *A d* dirig).

The coincidence of function as well as grammatical form leaves no reasonable doubt that this expression is part of the sub-scientific legacy. At times, however, the Old Babylonian texts use a different phrase with no counterpart in material closer to the sub-scientific tradition, "*B* falls short of *A* by *d*", transcribed above as $B = A-d$ (*B A d* ba-lal). This made Neugebauer speak of an "Unterschied von positivem Überschuß und negativem 'Abgezogenen'" (MKT III, 13). The distinction must be counted as an indubitable creation of the Old Babylonian school, not only because it appears to be absent from the background tradition as far as we know it (after all, our evidence is late and certainly incomplete) but also because of its particular function.

Neugebauer's description when taken to the letter is indeed irrefutable. The widespread extrapolation speaking about an introduction or discovery of negative numbers is much less so (Neugebauer did not subscribe to it, cf. the absence of the claim from his [1969]). All occurrences of the "reverse comparison" turn out to belong to one of two categories[22]. One type is illustrated by N^{os} 10–11 of BM 13901. N^{o} 11 tells s_2 to exceed s_1 by one seventh, which thus introduces a "favourite" coefficient. N^{o} 10 could have told instead that one square side exceeds the other by one sixth, but this would have forced the author to make use of a less favoured coefficient ⅙. In such cases (there are more of them in other texts), comparison of the smaller with the larger is thus introduced simply in order to allow the use of a favourite coefficient (correspondingly, this reverse construction is never used when the difference is told in absolute terms and thus the same both ways, cf. YBC 4714 N^{os} 21–28 and BM 13901, N^{o} 18).

The other type is only found in the series texts. Here, at times, an expression A is compared to another expression B which turns out to be larger. This can happen if A is a complex and B a simple expression, or if one or both expressions are submitted to systematic variation and A comes out sometimes larger and sometimes smaller than B. Since complex expressions are regularly mentioned first (it would indeed be forbiddingly clumsy to subordinate them to a preposition), and since the compact writing of the series texts presupposes that as much as possible – including order – be kept constant from one case to the next, reversal of the standard expression is necessary in such cases.

Both types thus make use of reversed comparison, not because it investigates "falling short" as a particular *mathematical* category (which, at a pinch, might allow us to speak of "negative numbers") but for *stylistic reasons*. The innovation introduced by the school in this domain was not conceptual, and no text contains the slightest hint of a shift from style to concept.

The transfer of algebra to the school will certainly have had other consequences. "Rapid change" of algebra did not only entail systematic variation and trivial sophistication of already familiar patterns but also attempts to see how far existing methods could be used to solve *new* kinds of problems – at times with relative success, as in the attack on third-degree problems in BM 85200+VAT 6599[23], at times less so, as in the division of a trapezium into three bisectable trapezia in AO 17264[24].

Even the idea of "representation" will have emerged as a consequence of the transfer of algebraic techniques to the scribal school. In the surveying tradition, algebraic techniques were applied only within an orbit defined by professional practice, i.e., used to solve artificial problems dealing with real geometric configurations. The same *principle* holds when we look at scribe-school algebra – but the orbit of professional practice had become wider. While second- and higher-degree algebra was still not used to solve problems of practical relevance, the scribe-school teachers found out that it could be used to solve artificial problems concerning numbers belonging together in the table of reciprocals, buying and selling rates for goods, etc. Thus, *if* real problems had presented themselves which could be dealt with by means of higher algebra, the transformation brought about by the insertion of the discipline into the school curriculum would have made it more fit for the job than its sub-scientific ancestor.

III. School formation?

A "mathematical school" is something different from a school teaching mathematics, not least when this school is not meant to train mathematicians but practical people[25]. Thus, even though algebra was transformed by becoming a school subject.we may still ask whether we can reasonably speak of the formation of a *mathematical school* – and, since this concept is only vaguely defined, whether our indefinite feelings about what seems "reasonable" can help us to delimit the concept more clearly.

The introduction of the idea of representation, together with the systematic search for problem types which the algebraic techniques would solve and the attempt to see how far they could carry – all this may be said to characterize Babylonian algebra as *a discipline*. We will probably not find that it allows us to speak of an *algebraic school* – and if not, then because we presuppose that a school is characterized by a *choice* (conscious or not) *between several actually possible ways to work within a discipline*[26].

A choice can be said to be involved in the heavy use of favourite numbers. Equations *might* just as well have been constructed with solutions 3, 7, 12, 16, 19, etc., and coefficients and relative differences *might* have been constructed from regular numbers or primes beyond 20. None the less, we will probably hesitate before applying the school concept to this choice, since it appears to have affected neither *thinking and working within the discipline*, nor (to the modest degree we can trace it) *the conceptualization of the discipline*.

The same applies to the differentiation between "excess" and "deficiency". *If* any trace had been present of an investigation of the mathematical possibilities inherent in the differentiation, the notion of a school might have been applicable. However, since only questions of style seem to be involved, it is not. Idiosyncratic and stylistic manners with negligible impact on thinking, techniques and aims do not invite for description as a "school".

If a school is characterized, firstly by being a choice of one particular among several possible ways to work within a discipline; secondly, by this choice having appreciable impact on thinking and techniques: then "a school" is an ephemeral or transitory entity (in absolute contrast to the quasi-perennial character of a "tradition"). Either it is resorbed, its characteristic way of thought and its distinctive techniques finding no echo; or, if ways of thought and the application of specific techniques find echo and thus become consequential, the "school" either conquers the whole discipline or acquires the characteristics of a discipline of its own.

One phenomenon among those discussed in the previous chapter may warrant the application of the notion of a school: The pluridimensional systematization of series texts, together with their stenographic language and compact style. The pluridimensional organization, it is true, is superposed upon single problems which, even though tangled, do not differ in substance from what is known from earlier texts like BM 13901, and offer no challenge which cannot be confronted successfully on this basis by a careful and tenacious student. But concentration of attention on the higher-level organization of the subject instead of its substance in itself constitutes a change of character; it also entails that attention is diverted, for instance, from the search for new applications of familiar techniques and ideas. Both in itself and through its consequences, the search for pluridimensional structure thus implies a change of the discipline-algebraic techniques, we might say, being no longer the subject in focus but a mere material for organization.

Stenographic language will have supported this change of character. Stenographic language, it is true, can only be formulated and used by those who can translate it into a fuller idiom; it thus presupposes the existence of a non-stenographic technical language. But if the creation of compact and highly structured texts becomes the central aim, only problems which *allow* formulation within such a framework are interesting; freely imaginative creativity will have no appeal.

Both the higher-level organization of the series texts and their language thus point to a specific molding of the algebraic discipline at that intermediate level which invites us to characterize it as a "school formation". It remains to be asked whether this tendency characterizes late Old Babylonian algebra at large, or it is a separate style. In the former case, it could be argued to be a ripening of tendencies inherent in the discipline itself in its interaction with general cultural patterns, and thus not the outcome of a choice between several *actually* possible ways to work within the discipline. In the latter, characterization as "school formation" is not only tempting but fully justified.

"Catalogue texts" containing long lists of mathematical problems organized in groups were produced in many places in Babylonia. That much can be said with confidence in spite of the fact that most mathematical texts have been found in the market and not *in situ*. But the particular genre of mathematical series texts may have been a specialty of Kiš (see MKT I, 387f). That it was at least *some kind* of local or otherwise separate development (and thus the product of a genuine *mathematical school*) is at least rather evident from the character of other late

Old Babylonian mathematical texts. Most important is the corpus of Susa texts, whose catalogue texts are different in character, and which shows no evidence of that kind of cognitive *Gleichschaltung* toward which the series texts point; but the late Tell Harmal texts as well as a number of "anthology texts" (thus BM 85194 and 85210) point in the same direction.

The Kiš(?)-school may not have been the only distinct mathematical algebraic school. A number of (mostly terminological) indicators *might* point to the existence of separate styles, perhaps so cognitively discrete that one should speak of schools. To some extent these indicators can be correlated with the separation of the text material into linguistic groups that was undertaken by Goetze (in MCT, 146–151) and with each other. The correlation is imperfect, however; moreover, I have not been able to associate any of the distinctive characteristics with noteworthy differences in mathematical substance, technique or orientation. So far the series texts thus represent our only evidence that the Old Babylonian epoch produced not only *school mathematics* but also at least one current which with some justification can be termed a *mathematical school.*

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Notes

1 Luca, when copying, adds a reference "per la .6ª. del secondo". Leonardo, it is true, tells nothing similar, even though he starts the treatise from Euclidean definitions; but an argument "quoniam recta .ba. divisa est in duo equalia et totidem inequalia super punctis .gc." (p. 63) obviously presupposes *Elements* II.5, as do a number of other formulations.

2 They are thus also related to the diagrams used by al-Khwārizmī in his *Algebra* to prove solutions of mixed second-degree equations. But they are clearly not derived from al-Khwārizmī.

3 That Leonardo "builds on" such a source does not mean that he translates it, which is clearly not the case. Leonardo presents the subject-matter found in the source in his own phrases, respecting mostly its ordering and borrowing often single formulations, but probably also abbreviating in some places and expanding elsewhere with supplementary illustrations and

explanations (borrowed from other sources or of his own making). His careful explanations of diagrams, proving for instance that diagonals coincide if one square is embedded in the corner of another, are clear instances of expansion, inspired as they are by the expository style of the *Elements*. I leave my discussion of this for another occasion.

It is quite clear that Leonardo does *not* build exclusively on Gherardo of Cremona's translation – for one thing because he brings the diagrams which are presupposed by Gherardo's text but actually absent. But in some places he uses it.

4 As to the question whether or in which sense Old Babylonian "algebra" can legitimately be described as *algebra tout court*, I shall only refer to the discussion in [Høyrup 1989]. It is immaterial to my present purpose, and I shall henceforth speak of *algebra*, *equations*, *coefficients* etc. without quotes, with the implication that such terms are to be understood in the sense which suits the Babylonian texts.

5 So much so that a sensible first reaction is disbelief. Recipe-books for cooking, however, offer a parallel case of continuity from Old Babylonian to Medieval times in Iraq (Simo Parpola, personal communication). The mathematical problem of precisely 30 consecutive doublings is also found for the first time in an Old Babylonian text, and then again in Roman Egypt and in the Islamic (and Latin) Middle Ages (see [Høyrup 1992a: 104f]).

6 But the solution speaks of s_u.

7 The text is corrupt (or possibly intentionally enigmatic, as is indeed N° 50). More or less at the corresponding place in his exposition, Leonardo (ed. [Boncompagni 1862: 61]) discusses the problem $s_s+\frac{3}{8}Q = 77\frac{1}{2}$.

8 *Quadratum altera parte longius* – thus Abū Bakr/Gherardo. Leonardo speaks of "quadrilaterals" and not of "quadrates", but shows that his source uses *murabba'* in both places (and thus both senses) precisely as Abū Bakr must have done ("Explicit de quadrilatero [= quadrate]. Incipit de [quadrilateri = quadrangles] parte altera longiori" (p. 63).

9 For further discussion of this notion I refer to the presentation in [Høyrup 1990a].

10 Language and writing make Neugebauer (MKT III, 10) and Thureau-Dangin [1936: 27] ascribe an early date to BM 13901. Comparison with the structure of the similar but clearly later series-text tablet YBC 4714 (see below) also places BM 13901 in a comparatively early phase of the development.

Language and writing are also the reasons that Neugebauer and Thureau-Dangin consider AO 8862 as particularly early. The Northern Db_2-146, finally, is found in a stratum dated Ibalpiel II, year 8 or 9 (c. 1775 B.C.).

One Tell Harmal text (IM 55357, in [Baqir 1950]) is still earlier (c. 1840 B.C.). It is not algebraic but concerned with subdivisions of a right triangle, and thus not foreign to mensuration and surveyors' riddles.

11 It is possible to make a reasoned conjecture as to where the transfer from the surveying environment to the school took place. Firstly, the method of quadratic completion appears to have carried the name "the Akkadian [method/trick]" (see [Høyrup 1990: 326]). Secondly, Db_2-146, like a number of Tell Harmal texts, carries the introductory formula "If somebody asks you thus", followed by a statement in the first person singular, past tense, and a subsequent procedure prescription in the second person singular, present tense or the imperative. Precisely this form recurs in the *Liber mensurationum*, while the texts from Southern Babylonia have expunged the "Somebody", making the problem deal with a situation which the speaking "I" (the teacher) has prepared.

Both the name of the method and the distribution of introductory formulas are most easily explained if the surveying tradition had its focus in the Northern (Akkadian) region, where surveying is also more likely than in the Sumerian South to have been a non-scribal occupation. (But cf. [Krecher 1973: 173–176] for third-millennium Sumerian juridical documents which distinguish the surveyor and the scribe, both of whom are involved in the transaction).

It may be noticed that both Tell Dhibaï (where Db_2-146 was found) and Tell Harmal belonged to the Kingdom Ešnunna, which produced the earliest law collection written in Akkadian, apparently in the nineteenth century B.C., to which also IM 55357 (cf. note 10) belongs.

The hypothesis that the surveyors' tradition had a Northern focus is supported by the particular role played by the number 10 (cf. below, note 20). 10 will have been a much more outstanding number in an oral Semitic-speaking environment than if the basis was Sumerian (or the Sumerian-based sexagesimal place-value system which made 20 and 30 the favourite sides of rectangles and squares in the scribal school, cf. below).

12 And even this structure, we should observe, is not identical with that of the surveyors' tradition but its reflection in a book, written in an attempt to impose some scientific system and order on the rules and favourite problems of the tradition.

13 Formally, the system is of the sixth degree, but once again the sophistication is trivial, at least when the solution of a first problem has shown the trick: since $X\square Y = x\square y = 600$, X and Y can be found by familiar second-degree methods to be 45 and 13°20′; then x/y will be $\sqrt[3]{45/13°20'} = 3/2$, whence $x^2 = (x\square y)\cdot(x/y) = 600\cdot 3/2 = 900$.

14 *Propositiones ad acuendos iuvenes*, *N°* 52/ii, ed. [Folkerts 1978: 74].

15 Evidently, some variation on a shared pattern is found here and there in the Rhind Papyrus. Thus N^{os} 1–6 ("Dividing *n* loaves among 10 men", ed. [Peet 1923: 51–53]); N^{os} 24–27 (simple 'h'-problems; ibid., 61–63); etc. But variation is never obtained by means of sham intricacy beyond the mathematical level really involved, as in the sixth-degree problems of YBC 4668.

16 This question I deal with, e.g. in [Høyrup 1991: 36–38].

17 "Favourite" numbers are numbers which turn up time and again by deliberate choice. Numbers which come up during calculations do so by necessity and not as the result of any choice, and they are thus irrelevant for the definition of the category. Choice, however, is involved in the value of solutions (since problems are constructed backwards from known solutions both in the surveyors' riddles and in Old Babylonian algebra), and in the selection of coefficients and other parameters of the equations which are formulated.

At times, the possible values of solutions are submitted to constraints – thus when problems deal with rectangles with expressible diagonal or with bisectable trapezia. In such cases, even the values of solutions do not count too much – thus, e.g., when the length, width and diagonal of Abū Bakr's standard rectangle is the second-most simple Pythagorean triple.

18 That 10 is Abū Bakr's preferred value for the side of the square is obvious from the above listing of his problems. It also plays a preponderant role for Leonardo, and N^{os} 16 and 19 of the *Liber mensurationum* demonstrate that the habit is considerably older: Both lead to irrational solutions, but both are obviously derived from approximate values (14 and 14 1/7, respectively) for $\sqrt{200}$, i.e., for the diagonal of the 10 by 10 square (the latter approximation is in fact given by Leonardo [ed. Boncompagni 1862: 61]).

One may also surmise that al-Khwārizmī's choice of the particular irrational root $\sqrt{200}$ in his discussion of the addition and subtraction of binomials *(Algebra*, ed. [Hughes 1986: 243–246]) is a consequence of the fact that this is the length of the diagonal of the favourite square.

19 This distinction between categories builds upon [Høyrup, forthcoming/a/].

20 This rule does not hold if the "rectangles" represent entities belonging to other categories, as for instance pairs of numbers belonging together in the table of reciprocals, buying and selling rates for oil, or the length and width of a real rectangle multiplied by their respective ratios to the other (as in YBC 4668, cf. above).

An exception of a different kind is found in BM 13901. All problems dealing with only one square give its side as 30 (in varying orders of magnitude), with a single exception: in N° 23, the problem apparently borrowed from the surveyors' tradition, it is 10, precisely as in later sources for this tradition. Its order of magnitude, however, is that of minutes, which implies that an originally "natural" choice has lost this characteristic through interaction with the place value system.

21 See, respectively, texts discussed in [Høyrup 1982] and [Friberg 1986].

22 The material on which this is built is presented in [Høyrup, forthcoming/b].

23 "While it remains true that the Babylonians were unable to treat problems of the third degree in general [...], the techniques displayed here must be recognized as not merely ingenious artifices but the very best that could be done by means of the mathematical techniques at hand" [Høyrup 1992: 351].

24 "[...] the text actually gives a correct solution of this problem, but this correct solution has been found by the wrong formulas (2) and (3)" [Brack-Bernsen & Schmidt 1990: 8].

25 Admittedly, this statement presupposes that "a mathematical school" is defined from cognitive and not from sociological criteria. If "a school" is a social network held together by teacher-student connections, then even the sequence of mathematics teachers of the scribal school must be presumed to constitute a "mathematical school". Such a definition, however, reduces the concept to an undifferentiated expression for a phenomenon which other terminologies describe more clearly and with more shades, and leads to discussions whether "a school" needs to have one, or possibly a group, of strong personalities as its centre, whether the sequence of professors at University X since 1400 or 1850 can legitimately be spoken of as "the X-school", etc.
A sociologically defined "school"-concept might gain some independent interest if the concrete constitution of the network is understood as having cognitive implications. At least in pre-Modern contexts, however, where we are not likely to possess much explicit information about the biographical links between persons, it seems more rewarding to go directly for the cognitive characteristics of one or the other mathematical endeavour: not forgetting, of course, that such characteristics will have had their background in specific social interactions and structures, since the correlation between sociology and cognition remains the justification for the choice of the double-edged metaphor – a real school being, expressly, a structure or place where people are brought together in order to make (some of) them learn or understand.

26 Or, possibly, parallel choices for work within several disciplines. I omit this possibility from the following discussion, since my textual evidence is taken from a single discipline.

Anschrift des Verfassers

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