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Recursive Knowledge Procedures Informing the Design of the Parthenon: One Instance of Continuity between Greek and Near Eastern Mathematical Practices

Abstract: This paper argues for reconsideration of two generally held views in the historiography of Greek mathematics, a narrow one about fourth and fifth-century Greek mathematics and a more general one about the continuity between Greek and earlier near Eastern mathematics. The narrower view called into question, owed to Knorr 1975, holds that two mathematical notions crucial for the development of Greek mathematics—(1) continuous proportion (and its non-reducible-to-a-unit-or-multiples-of-a-unit correlate, *anthyphairesis*) and (2) the reflective awareness of incommensurability owing to a misfit between magnitudes and multitudes—were understood for the first time in the mathematics of the first decades of the fourth century. Against this view, we propose to show that the Parthenon's construction displays at least a *foundational* comprehensive understanding of both of these mathematical ideas. Building on this narrower reconsideration, we further claim that the specific manner, which can be called 'algorithmic-problematic', of the Parthenon's engagement with these themes gives us reason to believe that the relationship of fifth-century Greek mathematics to its near Eastern precedents shows more continuity than is generally held.

Keywords: Greek Miracle, Incommensurability, Knorr, Szabó, Anthyphairesis.

I Introduction

In what follows, we shall re-examine two views in the historiography of Greek mathematics that have enjoyed relative stability for about a generation. First, that the development of a specifically theoretical understanding about two mathematical notions of great importance not only for Greek mathematics but for the history of mathematics as such—(1) continuous proportion (and its non-reducible-to-a-unit-or-multiples-of-a-unit correlate, *anthyphairesis*), and (2) the relationship of the geometric and harmonic mean (crucial for the theorization of incommensurability)—can be traced to the first decades of the fourth century. We hope to show that this dating might unnecessarily rule out genuinely theoretical knowledge at least one or two generations older in provenance, arguing that the Parthenon's construction (447-432 BC) entails a properly theoretical (not 'merely' practical) understanding of both notions. On this basis, we shall also call into question a more general historiographical standpoint which holds

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that one can find much more discontinuity than continuity between Greek mathematics and its near Eastern predecessors. For, we shall see, the specific manner in which the mathematical principles of the Parthenon are put into practice (we shall call it 'algorithmic-problematic', as opposed to 'proof-theoretical') gives us reason to believe that the relationship of fifth century Greek mathematics to its near Eastern precedents shows more continuity than is generally held.

Already 50 years ago, Arpad Szabó and Wilbur Knorr debated on when Greek mathematics transitioned from a 'merely' practical to a properly theoretical understanding of the key mathematical notions later enshrined in Euclid's *Elements* as the basis of mathematical knowledge. If the position of Knorr in that debate is right—and there is a relative consensus that he was—then the kind of theoretical understanding we believe rests beneath the Parthenon's construction would not have been possible at that time. Yet, notwithstanding the scholarly consensus adduced, we will suggest that they were known in this reflective way at that time.

It is crucial to speak tentatively here given the fact that, as Waschkies (2004, 16) notes, any "idea concerning the origins of Greek mathematics" is effectively a guess—hopefully an educated one—given the lack of original sources prior to the time around the turn of the fifth to the fourth century. Yet, as Netz (2014, 167–71) recounts, even if definitive statements about the pre-Euclidian period are impossible, we can be relatively sure that the path toward the consolidation offered by *Elements* relies on knowledge networks that can be read out of Proclus' summary of the early mathematicians. The reconstruction of the earlier historiographical debate in section II hopes on this basis to suggest that it is at least not impossible that one such network was active in Athens in the mid-fifth century though largely unrecorded, Proclus' mention of Oenopides of Chios (fl. in Athens around 450 BC) notwithstanding.[1]

Building on this first part, the balance of the present work will suggest reading the Parthenon itself as a source that demonstrates the work of this network and how it fits into what Netz (2014, 181) calls the "gears" by which the knowledge networks generate and transmit the practical mathematical knowledge that came to be canonized only a century or more later. First, in section III, we identify one potential source of the insufficiency of choosing between the views of Szabó and Knorr: namely, that both authors understand the relationship between practical mathematical knowledge and theoretical reflection on it in an overly narrow way. We examine two potential correctives to this failing: (1) following Fowler 1999 in paying more attention to the role *logistike*, *anthyphairesis* in particular, plays as part of the theoretical mathematics of the early Academy; (2) re-thinking the possible 'legacies' of near Eastern mathematics in the reflective comprehension (*theory*) as well as the *practice* of mathematical procedures and their results.

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1 For an exhaustive discussion on Oenopides, about whom quite little is known definitively, see Bodnar 2007.

Finally, the concluding third part (section IV) offers a new reading of the mathematical principles of the Parthenon's design that emerge from, but cannot be answered by, the *practical* application of procedural mathematical knowledge. If we begin with the view that such features could not have been consciously implemented in the Parthenon's construction absent a rich reflective awareness of these principles, then we will find that the designers involved could not but have had a *theoretical* understanding of 'cutting edge' mathematics of the kind being developed in the networks that Netz 2014 describes, to one of which those responsible for the design of the Parthenon would have belonged. Crucially, we will discover, such mathematical knowledge was sufficiently advanced to have accomplished something that Knorr believes to have only been possible only 30–50 years after the period of the Parthenon's construction, after the work of Theaetetus, Archytas and Eudoxus.

II The Szabó/Knorr Debate

A return to the Szabó/Knorr debate is crucial for telling a story about the historical development of the theory of incommensurability, which itself seems central to pre-Euclidian mathematical knowledge. There are two central components of Szabó's analysis that are relevant for us. First, is his thesis that there was much more of a theoretical development in Greek mathematics during the fifth century than is acknowledged by the conventional view, which holds that theoretically advanced mathematics began only in the early fourth century—that is, during Plato's intellectual career. It is important to note, though, that views do differ as to how central Plato and the Academy were in this development, and also about just how far theoretical advances went at what time in the relevant period, i.e., 410-370 BC.[2] Second, is his view of the priority of music theory, both in terms of time of circulation and in terms of intellectual formation. This priority finds primary expression in Szabó's chief dissent from the consensus that developed after his debate with Knorr. Namely: if he dates significant advances further back chronologically in the development of Greek mathematics than is commonly done in the literature, this is not because of a disagreement about when the integration of work on number and work on geometry occurred. Indeed, all parties agree that this is the work of the early decades of the fourth century, and it is here that formal proof and all the features of "Greek formal-deductive mathematics"

2 For more on the debate on Plato and the Academy, and also on the claim about the stability of the dating of the rise of theoretical mathematics, see Zhmud 2006 and his bibliography. Particularly relevant are Dillon (2003, 16–25), who addresses the role of mathematics in cementing an intellectual legacy in the early Academy, and Horky (2013, 3–35, 201–260), who offers a discussion on how appropriate the moniker 'mathematical Pythagorean(ism)' is with reference to Plato, colleagues of Plato and members of the early Academy after Plato.

truly take hold.[3] Rather, as we will see in some detail just below, Szabó 1978 insists that this theoretical integration and advance was built on the basis of a prior theoretical achievement within music theory that developed out of musical practice and was already mature in the first half of the fifth century. We do not hope to decisively determine anything in particular about Szabó's case for the priority of music theory, but we do hope to show it is plausible that there was a theoretical understanding of interdisciplinary issues in mathematics at work in the Parthenon's design.

How does Szabó hope to substantiate the claim that theoretical advance occurred by 430 BC, some decades before the conventional account allows? He hopes to show that even if nothing like the formal (i.e., 'axiomatic-and-deductive') proof, and surely nothing like 'proof-theoretical truth' had developed in the mid-fifth century, certainly it is the case that by the time of Philolaus and Hippocrates there was already a rich tradition of truly theoretical mathematics. Such a truly theoretical mathematics is one whose practitioners possessed a kind of familiarity with the interdisciplinary relevance of incommensurability that transcends a mere 'working' or practical knowledge within the mathematical disciplines of harmonics, geometry, or number and thus merits appreciation as a deep theoretical engagement with this issue.[4] But why does Szabó's account focus on the development of the three central mathematical disciplines named here? His whole historiographical framework, elaborated just below, is based on articulating a timeline that makes sense of how these disciplines came to be identified, together with astronomy, as the core mathematical disciplines in the education of philosopher-kings in *Republic* 7, and canonized soon thereafter as the center of mathematical, and thereby liberal, learning. For Szabó's overarching picture to hold, much must have been known in the middle of the fifth century. Corroboration for such a dating, earlier than allowed by Knorr, is provided by Waschkies 2004, who however sceptical he might be of Szabo's account, does argue that theorem sets are likely to have been "composed from about 475 BC onward," with "an order induced by stating some [theorems] to be principles and deducing all others logically."[5]

Szabo's central claim hinges on three subsidiary claims. First, owing in part to the fact that Propositions II.14 and VI.13 in *Elements* are more or less the same, and in part to the fact that II.14 is demonstrated so as to avoid any mention of propositionality, Szabó is confident that the construction of the mean proportional on a straight line

3 See Netz 1999, and the discussion of same below.
4 Hahn 2017 argues that already Thales was presenting such a theoretical understanding of incommensurability, complete with proofs that transcend the ostensive character of a diagram. If that's right, then the chronology debate at issue here becomes moot with respect to establishing plausibility for the notion that such a theoretical awareness can plausibly be attributed to mathematicians working on the Parthenon in 447-432 BC.
5 Waschkies 2004, 16. This article provides an in-depth analysis of the Szabo-Knorr debate. The juxtaposition of both the latter authors' view with Mittelstrass 2004 is highly relevant for the consideration of the continuity or discontinuity between Greek and earlier near Eastern mathematical practice.

(Proposition VI.13) was already known by the time of Hippocrates, whom he dates to having been active in Athens around 430 BC.[6] Second, that the oldest demonstrative reasoning in proving mathematical truth is Epicharmus' theory of odd and even, dated fairly accurately to ca. 500 BC, which is crucial because "this theory clearly culminated in the proof of the incommensurability of the diagonal and sides of a square".[7] Third, that provided sufficient attention to the integration of the three mathematical disciplines, a chronology can be established that proceeds in four stages: First, the musical theory of proportion develops, from which the terms of Eudoxean proportionality were borrowed; this occurs in two phases, with experiments with the monochord, giving rise to terminology for 2:1, 3:2, 4:3, followed by the development, by this means, of the technique of 'reciprocal subtraction' (*anthyphairesis*; Szabó: "successive subtraction").[8] Second, this musical theory of proportions is applied to arithmetic, in a manner later discernable in the presentation of numerical ratios in Book VII and especially the discussion of continuous proportionality in Book VIII.[9] Only then, Szabó holds, was this proportion theory then applied to geometry, at "the time of the early Pythagoreans", working with the construction of the mean proportional.[10] Finally, "mathematics within a deductive framework" developed; here speaking of *Elements* and its concomitant theory of proof.[11]

Szabó stresses two points about this chronology:[12] first, if things did proceed this way, then quadratic incommensurability "must have been known well before the time of Archytas"; second, "the discovery of incommensurability is due to a problem which arose originally in the theory of music". Of course 'knowing well' can mean a number of things, as critics of Szabó have pointed out.[13] Even if Szabó himself was not explicit, we think we can take his meaning here to coincide with the gloss provided for what is meant by a truly theoretical mathematics above; by 'knowing well' we mean being theoretically aware: the practitioners of mathematical knowledge, such as those at work in the design of the Parthenon, possessed a kind of familiarity with the interdisciplinary relevance of incommensurability that transcends a mere practical knowledge within the mathematical disciplines of harmonics, geometry, or arithmetic.

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6 Ibid., 14, 48–9; in the latter argument, Szabo points out (in conversation with Heiberg) that II.14 seems to be a later elaboration that expressly excludes the mean proportional from consideration, in the way done in VI.13: "Indeed the proof of Proposition II.14 gives the impression that knowledge of the mean proportional is perhaps unnecessary for carrying out *tetragonismos*".
7 Ibid., 25.
8 Ibid., 26–27.
9 Ibid., 27–28.
10 Ibid., 28–29.
11 Ibid., 29.
12 Ibid., 29.
13 We have Berggren 1984 and Bowen 1984 in mind.

Szabó's chronology, concerning the development of a proof of incommensurability that it entails, involves him in an extended argument to the effect that it is a mistake to attribute to Theodorus and to Theaetetus the decisive role in the proof of incommensurability in the period 410-370 BC.[14] Szabó's claim rests primarily on two grounds: (1) a philological argument focused on his understanding of the use of the term *dynamis* (as both 'square' and 'power'), in mathematical texts, from the earliest surviving fragments down to the fourth century; (2) an interpretive argument concerning Plato's complicated intentions in associating the discovery of a *proof* of incommensurability with these two men in the *Theaetetus*. Here, we briefly state the decisive moment in each of these arguments for Szabó, and then discuss the basis of criticism thereof, and why we think we ought to consider this an open question.

(1) Szabó's philological argument against dating development of a theory of incommensurability to after 410 BC. Szabó summarizes the philological argument as follows: [15]

> "Thus my previous conjecture to the effect that *dynamis* and *tetragonismos* originated at the same time inevitably leads to the conclusion that the creation of the concept of *dynamis* must have coincided with the discovery of how to construct a mean proportional between any two line segments".

This is the crucial link for Szabó, insofar as he holds that Hippocrates (and not Archytas) originated the proofs about mean proportionality. Szabó argues that the term 'dynamis' began signifying extent-in-square at the same time that it was learned that one could *practically* construct a square while using the mean proportional between two line segments.[16] If this is true, and if it is also true that Hippocrates first circulated proofs of some kind concerning the mean proportional prior to 420 BC, then we can be confident that a crucial step in *proving* incommensurability dates to around 430 BC and not 410-370 BC.[17]

(2) Szabó's hermeneutical arguments against dating development of a theory of incommensurability to after 410 BC were not especially well-received by Knorr himself, nor by Bowen 1984, for instance. This results from differing attitudes to interpreting the dialogues, however, and not at all to matters directly historical or historiographical. If, for the purposes of historical discussion, we can keep an open mind on how to interpret Plato—Griswold 2002, 1–18 and Nikulin 2010, 1–22 can help us to see why this is necessary—, then we find that this is an open matter. Szabó cannot be said to have proven that Theodorus and Theaetetus are not crucial for the development of

14 Szabo 1978, 33–84.
15 Ibid., 48.
16 Ibid., 50–54.
17 Bowen 1984 is particularly unconvinced by the *dynamis* argument and, hence, by the view that incommensurability was understood theoretically by the period of 450-430 BC. To the best of our knowledge, however, this matter has not received much scholarly attention.

a proper *theory* of incommensurability, that is, but he provides good reasons to doubt their centrality in this story. First, Szabó deploys a reading of the pun on '*dunamei dipous*' in the *Statesman* 266a5-b7 to confirm that what Theaetetus describes working through with young Socrates was common knowledge.[18] Second, Szabó (1978, 76) argues that the mistaken attribution of significance mostly derives from two sources, which he spends five pages trying to debunk: first, an ancient Scholium on *Elements* X.9; second, "a report which probably stems from Pappus' commentary on Book X and survives only in an Arabic translation".[19] Since both of these sources themselves rely mostly on a wrong minded reading of the mathematics section (147c-148b) of *Theaetetus*, there is no reason to see this attribution as anything other than a 'false tradition'.[20] Finally, Szabó presents a hermeneutical and philological analysis of the dialogue in the service of this answer: Plato presents Theaetetus as a very talented, but also young and naive, and over-eager, researcher. The relevant section about the discovery of a theory of irrationals must be read in this light.[21]

Holding these arguments to be clearly interpretive, but possibly right and in any case not disproven, we move to the second of the relevant points in Szabó's analysis: the central importance and chronological priority of musical theory, which inspired an analytical program in number theory, which in turn inspired a reflection on the (im) possibility of fully integrating number theory and geometry—the latter of which had developed separately, without prior integration with the other two disciplines. Szabó (1978, 108–78) argues that those working in music theory arrived at the three means and realized there is no geometrical mean for the fourth, fifth and octave, and on this point his view is corroborated in the literature on Greek music.[22] From this tradition, mathematicians approached the double square and discovered mean proportionality between two lengths, numbers or not. Thus: music then arithmetic, then geometry.

Against Szabó, Knorr argues that "by the time of Hippocrates both [the theory of congruence and the theory of similarity (based on proportion)] were already well developed", but that the combination of these two traditions began "only about fifty years later", with the contributions of Eudoxus.[23] Knorr insists that "evidence in the pre-Socratic literature discourages dating [the] discovery [of the theory of incommensurability] before ca. 430".[24] One may recognize only after that time signs of a dialectical interest in the problem of incommensurability. The center of the debate

18 Szabo 1978, 68–71.
19 See Szabo 1978, 76, nn. 57–59, for full bibliographical detail on this matter. For the Greek of the Scholium in question, see ibid., 76n59.
20 Ibid., 79.
21 Ibid., 79–84.
22 See, especially, Barbera 1991 and Barker 2007.
23 Knorr 1975, 7–9.
24 Knorr 1975, 49.

is in the interpretation of Plato's *Theaetetus*.[25] Knorr here provides three arguments against Szabó. First, that even if something like the statement to be proven in Euclid VIII.18 is at work here, this only shows that "there is no *integer* which is the mean proportional between two terms that are not similar numbers", which is not enough to establish incommensurability.[26] Second, Knorr claims that Szabó's "view of the antiquity of the mathematics of the dialogue is an assumption not well supportable by the available documentary evidence, against which he has offered an argument.[27] And, finally, given Knorr's own view of the relative novelty of any theoretical understanding (and certainly proof) of incommensurability, it is "at least a reasonable counter-assumption that the number-theoretic foundation, upon which Theaetetus and his successors built their theory of incommensurability, had not yet achieved an advanced form at Theodorus' time".[28] Knorr here underscores what Szabó himself acknowledges: the relative novelty or antiquity of the discovery within the range of 430-360 BC is a matter of speculation that existing sources will never definitively settle. Thus, we are dealing with the relative merit of a later or earlier date within this half century timeframe.

Given that the participants in the debate both acknowledge that definitive knowledge is impossible here, and that the matter is really about who knew what when, within a fairly narrow research program in a relatively short time period, this question might seem pointless, especially as the alternatives do not strongly differ. Nevertheless, determining which, if either, is correct would be very telling for the argument we make about the Parthenon. For, if Knorr is right that the kind of integration Szabó sees in Hippocrates was only done in the time of Eudoxus, then it becomes difficult-to-impossible to believe that this integration, which we argue is integral to the design of the temple, would have been thinkable—let alone achievable—for its designers in the period 447-432 BC.

In order to advance this deadlock, we turn now to the role of *anthyphairesis*. Fowler (1999, 1–24), asking different questions for different reasons but responding to the issues at stake in the Szabó-Knorr debate, focuses on this very knowledge procedure. Fowler notes that Knorr agrees with Szabó that "the theory of incommensurability will be perceived as contributing to important aspects of every part of the *Elements*, save for the oldest geometrical materials contained in Books I and III", because they are both committed to the standard story about the history of Greek

25 It is notable here that even though this debate proceeded and was received as a debate in the history of mathematics, a careful consideration of Knorr's argument—see Knorr 1975, nn. 71, 74—shows that, again, the decision comes down to how one interprets Plato's work, and not historical knowledge of independent sources. Thus, if one finds Szabo's reading of Plato more convincing than Knorr's, then there is no reason yet given *not* to adopt Szabo's historical hypothesis.

26 Knorr 1975, 116–117.

27 Ibid., 82–86.

28 Ibid., 117.

mathematics. This traditional account, Fowler (1999, 2) continues, holds that the "early Pythagoreans based their mathematics on commensurable magnitudes (or on rational numbers, or on common fractions *m/n*), but their discovery of [...] incommensurability (or the irrationality of √2) showed that this was inadequate". This, as Fowler concludes his retelling of this view, "provoked problems in the foundation of mathematics that were not resolved before the discovery of proportion theory that we find in Book V of Euclid's *Elements*".[29] Fowler, for his part, "disagrees with everything in this line of interpretation".[30] Fowler, thought indebted to Knorr, provides grounds for arguing that Szabó is right to identify the sophistication belonging to Greek mathematics already before 430 BC. This is so, even though Fowler does not agree with Szabó that theoretical advances occurred earlier than Knorr allows, rather insisting that mathematical *practice* was well "ahead" of its theorization. Especially relevant for us as we approach the mathematics of the Parthenon is Fowler's central proposal that *anthyphairesis* and its insights were familiar to and fascinating for mathematicians of the mid-fifth century. It may be that Fowler is entirely correct that a lot of what we—following the ancient and neo-Platonic commentaries—call 'Pythagorean' had nothing to do with Pythagoras and his followers.[31] It may also be that nothing like (i) the formal theory of proportion in *Elements*, V[32] or (ii) the formal system of deductive proof presented in *Elements* as a whole had been seriously developed during the fourth century, let alone the fifth.[33] All the same, by underscoring the centrality of "reciprocal subtraction" (*anthyphairesis*) as a *practice*—especially in addressing problems in geometry (Theaetetus), in music theory (Archytas), and in astronomy (Eudoxus)—Fowler provides the welcome corroboration of our suggestion that the practicing mathematicians and mathematically informed artisans of the generations working at the time of the Parthenon's construction used reciprocal subtraction as a means by which to test, geometrically, solutions to problems in harmonics.

The chronology debate we have reviewed here points to a more fundamental debate about the very nature of Greek mathematics and its logical and methodological (not to say ontological) foundations. In the next section, we will argue on this basis that Greek mathematics shares much more with its near Eastern predecessors than is often believed.

29 Fowler 1999, 4.

30 Ibid., 4.

31 See especially Fowler 1999, 289–302. Fowler 1999, 356–401 is also telling for how he hopes his work to serve as a corrective to the standard story. More recently, Netz 2014 has reviewed the serious difficulties involved in the historiography of 'Pythagorean' mathematics.

32 Fowler 1999, 15–20, but also Chapter 10.1.

33 Ibid., 204–217, but also Chapter 10.4.

III Beyond the 'Greek Miracle'

This section adduces one contextual argument against the idea of a Greek Miracle in which Greek thinkers introduced *sui generis* and from their own genius, as it were, patterns of thought utterly unknown prior to the "awakening" of (Greek) science, as Bartel L. van der Waerden (1961, 5) famously referred to the manner in which Greek mathematicians "took their start from Babylonian mathematics but gave it a very different, a specifically Greek character". The work of Walter Burkert is of central importance in the academic interrogation of the self-sufficiency of Greek culture, while also just as clear as those who found his arguments and evidence so unwelcome regarding the magnificence of the Greeks. In a late synoptic work that reflects his whole body of work over the previous 40 years, Burkert (2004, 12) paradigmatically concludes that "although Greeks had been on the receiving side for a long time, there is no doubt that the result is Greek. It is Greek art and architecture that have become classical, and Greek literature that has become world literature". More recently, Josiah Ober has offered a comprehensive account of how the 'Greek Miracle' arose as a way of describing the rise of Greek-speaking city-states from peripheral backwaters to centers of thought and of economic and political power in the period between 800 and 400 BC, which he calls "the efflorescence of Ancient Greece". Ober (2015, 15) argues that, rather than a miracle, the primary cause is "the powerful role of specialization, innovation, and creative destruction in the decentralized world of ancient Greece".

With this critique of the Greek Miracle and the established need for the contextualization of Greek culture that has occurred with respect to literature and religion in mind, we pose the wider of the two revisionist claims outlined in the introduction. Namely, what can the practice of *anthyphairesis* as developed from the earliest (scantily) recorded sources of the sixth century through to its presentation in Euclid's *Elements* tell us about the supposedly revolutionary character of classical Greek mathematics? In answering this question, it is worth noting that while much is controversial about the degree of novelty in Greek mathematics, no one denies that there are some indications of contacts between Greek mathematicians (especially astronomers) and near Eastern counterparts, certainly by the fifth century (Meton), maybe even earlier (Hesiod).[34] Waschkies (2004, 15) shows that rather than some logical or philosophical principles being at work—as Szabo had argued—it is perfectly possible that a problematic approach to an arithmetical issue such as the *psephoi*-arithmeticians used would lead to the recognition that 'the side and diagonal of a single square are

34 As pointed out to me by Jeanette Fricke and Lis Brack-Bernsen, one possible point of direct influence that has not yet been thoroughly checked would be an analysis of the 'Works and Days' section of *Works and Days* with Column 6 of the 'Astronomical Diary Text' known as BM47762/BM49107. In general, direct influence by at least the sixth and fifth centuries, perhaps as early as the eighth century, is attested by Høyrup 2010 and Rowe 2013.

a pair of incommensurable magnitudes, and therefore the method of proof by *reductio ad absurdum* might be an invention by early Greek mathematicians, if it had to be invented at all'. In other words, the algorithmic-problematic style of mathematics might very well have generated the practice of indirect proof quite independently of any philosophical interest, and this might just as well have been done by the early Greek mathematicians operating in this style as by their near Eastern predecessors, for all we know.

To our best knowledge, the most firmly established early point of contact between near Eastern and Greek mathematics is in the discipline of astronomy, particularly the dates of appearance of stars and constellation during the year, comparable to material in MUL.APIN, and already present in Greek sources of the 8th century. As Evans (1998, 15–16) points out, "the data gathered during the reign of Nabonassar (747-733 BC) is especially important from the viewpoint of later Greek astronomy".[35] Given the evidence of overlap of 'research problems' and particularly of calculative techniques/ knowledge procedures, we suggest it seems more likely than not that early theoretical advances in Greek mathematics developed with no small degree of continuity with the earlier work in near Eastern mathematics.

All the same, we quite agree with Netz (1999, 272–5) that the formal-deductive framework of Greek mathematics does in fact give every appearance of having been *sui generis* and becomes canonical only with and after Aristotle, with a handful of proleptic attempts (now lost) such as that of Oenopides, discussed above, preceding. Based on this overview, Netz concludes that since "mathematics appears suddenly, in full force" in Aristotle's works, one should believe that "as a recognisable scientific activity", it "started somewhere after the middle of the fifth century", and "since dates are a useful tool", he proposed 440 BC as the point of emergence. In keeping with Netz's conclusion, it seems to us that there are two significant respects in which slightly earlier, specifically mid-fifth century, Greek mathematics needs to be *read backward* in conversation with the non-formal and non-deductive procedures of mathematical knowledge from other traditions. First, we must see that both theoretical objects of mathematical knowledge (such as mean proportionality and periodicity) and intricate mathematical procedures (such as reciprocal subtraction) of great importance for the Parthenon were studied to a high degree of comprehension already by the mid-fifth century. In this respect, it is valuable to note that Proclus's account of Oenopides as the first to distinguish between theorems and problems, to insist on geometry done only with a compass and a straight edge, and to draw a perpendicular straight line from a given point to a given straight line is generally accepted. If this is true, it certainly signals a mature context of theoretical mathematical knowledge production *in Athens*—i.e., knowledge pursued solely for its own interest and

35 We thank Alexander Jones for the reference.

without relation to physical and practical production—by the time of the design of the Parthenon.[36]

Second, we must notice that (1) *if* this first claim is true, *then* there must have been some basis for such a (relatively) advanced state of development, and (2) it seems *prima facie* unlikely that such development would occur prior to the development of Greek-style mathematics as a largely fourth-century phenomenon. Thus, Waschkies 2004 seems right when, returning to an argument made at greater length in Waschkies (1989, 312–26), he argues that these objects and methods might well have been introduced not because of a burgeoning influence of Greek-style systematic, proof-theoretical knowledge procedures, but rather through the reception of near Eastern antecedents executed with an eye toward generalization. As he argues, both the near Eastern antecedents and their generalization were interesting for Greek mathematicians in their own non-deductive and not logically rigorous practice.

This second suggestion is, we know, controversial. The possibility of near Eastern influences on Greek mathematics of the Euclid type is both very difficult to establish and controversial to explore in the first place. Establishing connections is difficult for two main reasons, first chronology (most of the evidence for Old Babylonian mathematics is early second millennium BC), and secondly the manifest difference between the deductive-demonstrative style of mathematics of classical Greek mathematics (from the fourth century BC onward) and the algorithmic problem-oriented style of mathematicians in ancient Iraq, whether Old or Late. Indeed, if Fabio Acerbi (2010, 153) is right that writings in the Greek mathematical tradition of 'a foundational character' show us that 'the Greek way' of doing mathematics is very much through the establishment of a 'literary genre', which he attempts to delineate and interpret through the works of Apollonius and Geminus, then it would be hard to find a tradition more unlike the tradition of court-based astral science in ancient Iraq of the same period, as the latter is characterized by Eleanor Robson (2008, 214–62).

But more than just difficult to establish, possible links are suspect in advance as they seem both to belie the differences noted and to be driven by an *a priori* teleology where classical Greek mathematics must have been the direction that mathematical disciplines were headed, wherever practiced for whatever reasons. Indeed, in discussing one of the central foci on which the debate concerning the influence of near Eastern mathematics on Greek mathematics has focused—the right triangle problem at the heart of what came to be called the Pythagorean Theorem—Robson (2008, 110) concludes that the famous 'showings' of the relationship between rectangles and their diagonals with specific examples that express quantities later readers focus on as 'irrational' in Plimpton 322 and other early second millennium sources are "a student exercise in calculating lines and areas, not an anachronistic anticipation of the supposed

36 Thanks to Michalis Sialaros and Dmitri Nikulin for independently calling our attention to this piece of 'possible corroboration'.

classical Greek obsession with irrationality and incommensurability as is sometimes implied".

Robson is surely right that there is much suspect in twentieth-century attempts to place Old Babylonian sources in a specific kind of relation with classical Greek mathematics, with incommensurability at the heart of the story. Yet, as Høyrup (2009, 67) notes in his overwhelmingly positive review of the book, when Robson discusses

> "possible links to Greek mathematics, Robson restricts herself, on one hand, to sweeping arguments [and] on the other, to rather unspecific references to renowned publications that take the pertinence of these for granted in a way that unwittingly supports the myth of the Greek genius that invented everything on its own without interaction with other cultures".

Thus, given the current state of knowledge and the norms about historiographical methodology and the challenges of changing orthodoxy, it remains the case that what is largely a nineteenth-century prejudice holds firm in place of a better and firmer hypothesis. This prejudice has it that while Egyptian and Mesopotamian cultures developed practical mathematical understanding much earlier than the Greeks, it was the Greeks alone who sought to develop a proper theoretical understanding of these mathematical objects. This prejudice, as Høyrup has extensively argued, is quite hard to overcome; Steele 2006 has also discussed the ways in which cultural expectations can inform historical analysis, especially when original sources are scarce and questions of influence are involved. And, as Robson (2008, 274) notes, even as "(some) Old Babylonian mathematics was written into the Western tradition", anything from the 3000 year history of mathematics in ancient Iraq that "was irrelevant to the standard teleological narrative by virtue of being contemporaneous with, or even later than, Classical Greek mathematics" went ignored.

If we heed Robson's advice, as well as the work of Waschkies noted above, however, there are other reasons to be less than certain that the manifest difference in style between Greek mathematics, once it comes to be well-attested in the fourth and third centuries BC, and its near Eastern predecessors preponderates against the possibility of meaningful influence of the latter on the former. Robson herself finds that work, like that of Høyrup and Netz referred to elsewhere herein, which tries to uncover the workings of ancient mathematics in either the Greek or the near Eastern traditions in their own contexts and with less 'conforming' translations and explications "reveals three very different mathematical cultures", meaning Old Babylonian, Late Babylonian and Greek.[37] Surely there are profound and meaningful differences, and surely too, as Robson (2008, 272–4) argues, there has been much 'orientalizing' in the historiography of ancient mathematics. Yet, a granular and context-aware cognitive history of late first

37 Robson 2008, 275, 281.

millennium near Eastern and archaic (just pre-Classical) Greek mathematics is neither impossible nor without promise for those two reasons.

Philolaus (ca. 475-385 BC) is a central figure for the questions raised in this section, and thus makes for an excellent case study in attempting to address them.[38] First, his intellectual career closely coincides with, if largely slightly postdates, the design of the Parthenon, and shows a concern for precisely the kind of integration of the mathematical disciplines organized thematically around 'harmony' that we find there. Still more important, whenever exactly he published his one treatise on the cosmos, as Graham (2014, 48) notes, he did so by entering into a "larger conversation" that was already ongoing; a strong indication that the specifically mathematical elaboration of principles about nature already was directed at an audience sufficiently trained and large enough to receive such an account. While we will focus on another discovery/invention attributed to Philolaus, the Pythagorean tuning preserved in what Huffman 1993 refers to as Fragments 6/6A is integral to the mathematics of the Parthenon. Philolaus' work carries a distinct trace of what near Eastern mathematics did best, astral science, specifically the theory of the moon and the sun, and the position of the ecliptic.

Take the case of Philolaus' great year. Huffman provides a thorough account of what is known about this intellectual achievement, which purports to name the period within which solar years coincide with lunar months: namely, 59 solar years (where each solar year has a value of 364 1/2 days), or 729 lunar months (where each lunar month has a value of 29 1/2 days).[39] As Huffman pointedly asks: "The crucial question is: how did Philolaus arrive at this set of numbers?"[40] Huffman considers two main alternatives, one in which Philolaus is adopting and adapting the value that Oenopides (mentioned above, and a slightly older contemporary of Philolaus) had arrived at: 730; the other in which he derives it from recorded observations directly. If he revised an earlier value, the supposition goes, this could be either because of some preference in working with the observations—like the value it gives for the solar year (of 364 1/2 days, rather than 365 22/55 days—or because of the inherent appeal of 729 as a cube of the cube of 3.[41] The 'numerological' reading seems to gain credence from the tradition, going back to Aristotle (*Metaph.* 986a3-12, *Cael.*293a23-7), that holds Philolaus introduced the counter-earth solely to get a perfect number of ten heavenly bodies. As Graham (2014, 57–60) shows, however, Philolaus has good observational

38 For what little information there is on Philolaus' life, including his dates, see Huffman 1993, 5–6. See also Graham 2014, who builds on and updates this account.
39 Huffman 1993, 276–279.
40 Huffman 1993, 277.
41 This appeal was clear to Plato, who chooses to make the number crucial for the central problem of *Republic*—showing that the life of the just is better than the life of the unjust—by having Socrates claim that the life of the just man is 729 times more pleasant than that of an unjust man, and expressly citing Philolaus' great year in the process (*Resp.* IX, 587b-588a).

reasons to introduce the counter-earth—this would explain the greater occurrence of lunar than solar eclipses within his "central hearth" structured cosmos.

Building on Graham's work with the counter-earth, we would extend this way of reading the interplay of a priori reasoning and observational awareness to the enterprise that would lead one to posit a great year, a topic which Graham 2014 does not address. Huffman identifies this as "an attempt to harmonize two important ways of measuring time, the lunar month and the solar year"—and as not *sui generis* within Philolaus' cosmology, or within Pythagoreanism.[42] Both the observational data and the values with which Philolaus and his Greek contemporaries were working came to them from Egypt and Mesopotamia. Given this fact, Huffman's analysis—where the interpreter is left to decide between the possibility that Philolaus, as a mid-fifth century Greek mathematician/cosmologist, would base a certain key finding on either (a) fit with an overarching theoretical commitment or (b) fit with observational data—leaves aside a possibility worth serious consideration, even if it is difficult to substantiate fully. Namely, is it not possible that solar year and lunar month periodicity presses itself on these fifth-century Greek sources through the determination of the same issue in the recent Babylonian astronomical work of the seventh and sixth centuries BC?

While it is difficult to establish connections for the reasons stated above, here is an instance where we have Babylonian source materials—specifically 'Goal-year tablets', which make predictions of where certain heavenly bodies will be at certain times in a given year, and 'Lunar prediction tablets', which focus on the moon over long periods—from the time in question. Since the 1990s, a great deal of work has been done on exemplars of this tradition determinately dated to the period (e.g., tablets from ca. 642-640 BC, 593 BC, 523 BC) and others whose dating is not precisely known but date from some time not earlier than the fifth century BC and not later than the third century BC.[43] These texts display two features that relate directly to the question Huffman raises concerning Philolaus' process in arriving at his value for the great year.

The first point of close contact between near Eastern astral science of the mid-first millennium and Philolaus' interest in the great year rests in the fact that, as Huffman (1993, 277) notes, the latter "is an attempt to harmonize two important ways of measuring time, the lunar month and the solar year". For our comparative purposes, it is crucial to note with Robson (2008, 214) that a "significant quantity of mathematics survives from the Old Babylonian period" through the Persian (Achaemenid) period, during the fifth and fourth centuries BC, and down to Alexander's conquest and its

42 Huffman 1993, 276.
43 Britton, Brack-Bernsen, Huber, and Steele have published on these sources. Especially relevant for the consideration of a possible near Eastern influence on Philolaus' great year are: Brack-Bernsen 2014, Britton 2002, Huber & Britton 2007, and Huber & Steele 2007. For a more complete bibliography of works in this field, see Brack-Bernsen 2014.

aftermath. This transmission history includes a long tradition of interest in precisely this sort of solar year/lunar month harmonization at issue with Philolaus, among other topics of interest for reasons both mathematical and practical.[44] It is true that the underlying motivation in Late Babylonian mathematical practice displays a significant difference from what we can glean as being Philolaus' motivation, namely the political and religious importance of the eclipse in the near Eastern case as opposed to a more ontological and cosmological set of considerations for Philolaus.[45]

All the same, the observational data with which Philolaus and his contemporaries could seek out significant number patterns is owed to this tradition. But Greek astronomers and mathematicians likely received a good deal more than just the data. For instance, bearing in mind the work of Huber and Steele 2007 on the so-called 'Saros function'—an 18-year solar cycle—that is transmitted in lunar prediction tables dated to 642-640 BC, and also the work of Britton 2002 on a lunar prediction table (dated to ca. 620 BC) that uses a 27-year solar cycle, one detects a strong consonance between Philolaus' interest in powers of three and the repeated thematic and methodological use of powers of three in these near Eastern antecedents of which Philolaus and/or those with whom he worked might well have been aware. Unfortunately one cannot corroborate this claim more closely with comparison to Philolaus' work, for (as Huffman 2014, 276–7 notes) we have only a few scant later testimonies with which to work. But it can help to recall two facts concerning Plato's reference to Philolaus' great year at *Resp.* 587d-588a. First, we are told there that the number 729 is both 'true' and 'appropriate to lives' both because of days and nights (729 days and nights in one 364 ½ day long year) *and* months and years (729 lunar months coincides with 59 years having 21 intercalary months). Second, this passage precisely proceeds by means of taking the number three in square and then in cube. We can see, then, that both in the near Eastern astral science of the relevant time period and in the work of Philolaus, the harmonization of the solar year and the lunar month is of primary interest and best understood through the power series of the number three.

The second close point of contact between the great year as a paradigmatic instance of Philolaus' interest in observational astronomy and near Eastern practices of the first millennium is the way in which he directly interprets data, in this case charts of the positions of the sun and moon, as themselves objects of observation. This appears to have been the hallmark of Babylonian astral science in particular: already by the seventh century, we see the development of what is called the 'linear zigzag function', which describes the pattern that emerges on a table inscribed on these tablets, given the values for mean speed (for moon and/or sun) through the

44 Our account here would not have been possible without the assistance of Lis Brack-Bernsen. We are solely responsible for what errors remain despite her patient guidance and assistance.
45 For a discussion of the ontological and epistemological focus of Philolaus' work in cosmology and observational astronomy, see Graham 2014, 64–68.

phases of the moon over a period of solar years.[46] From especially the fifth century on, this sort of function is used in order to give sequences of numbers tabulated for equidistant intervals of time, allowing for the determination of the relevant period and thus the prediction (for instance) of eclipses. This back-and-forth procedure, from tabular data to worldly phenomena, became integral to Greek mathematics, especially astronomy. One striking, albeit later, example of this is the work of Hypsicles on the mean speed of the sun and the moon. Hypsicles used Greek mathematical rhetoric of a Euclidian kind to present precisely Babylonian values of the 'linear zigzag function' for the mean speed of the sun and the moon. As Evans (1998, 124) notes, Hypsicles' "whole scheme of using an arithmetic progression to represent the rising times of the signs is of Babylonian origin". We do not have a record of when precisely Greek mathematicians began working with the periodic functions of near Eastern astronomy. Given the contacts, known to have been established between the seventh and fifth centuries, however, there is no reason to believe that Hypsicles or his contemporaries were receiving the zigzag function for the first time; in any case, the interest in periodicity, and the habit of noticing periodicity in data directly, and then bringing it to objects of observation or construction is shared by the near Eastern astral science of the seventh century and Philolaus.

This brief investigation of Philolaus' great year can at best provide a plausibility test for our claim that fifth-century Greek mathematical practices were continuous with and expressly borrowed from near Eastern predecessors. This comparison of the great year with goal-year tablets does not *prove* anything. We would insist, though, that it shows questions concerning the development of Greek mathematics at the time of the design of the Parthenon remain open and scholarship could benefit from a more extended comparative analysis of the kind we have initiated here. What's more, it should at least be clear—however much we might doubt a direct reception of near Eastern astral science by Philolaus in developing his great year—that the kind of knowledge procedures employed in the 'algorithmic problem-oriented' style of near Eastern mathematics in this time period is quite similar to the ever-more-advanced, but still algorithmic and problem-based knowledge procedures in Greek mathematics prior to formalization and the introduction of deductive proof in the fourth century.

This brings us back to the Parthenon and the nature of mathematical theory and practice in Athens in the mid-fifth century. We see in the temple's design precisely the kind of algorithmic problem-oriented style we know to be the hallmark of near Eastern approaches. Analyzing the mathematically-informed features of the Parthenon's design, it seems quite likely that the *process* of coming to these design features is through the *reflective* deployment of a variety of instruments derived from just this (putatively near Eastern) style. If that was happening in Athens at the

46 Brack-Bernsen 2014, 37.

time of the temple's design, it seems natural to ask: how did they know how to do this? Why did they choose to do so? This article concludes with a first approximation of an answer to this question, which serves as the basis for the sustained analysis in the forthcoming publication of the findings of the research we have conducted on the Parthenon. In that further and deeper elaboration, we hope to show in detail how the continuity with, but also innovation when measured against, the sixth and earlier fifth-century tradition of Doric temple architecture indicates the underlying continuity of a relatively advanced fifth-century Greek mathematics with its near Eastern precedents.

IV The Parthenon: Continuity with Near Eastern Mathematical Knowledge Procedures

This closing section focuses on three related mathematical features manifestly present in the design of the Parthenon: periodicity, the application of the unit, and the constructed harmony of the whole. Each of these features displays significant connections with emerging themes known to be crucial for the agenda of fourth-century Greek mathematics. At the same time, each expresses the algorithmic-problematic style long associated with near Eastern mathematical traditions in contrast to the formal-deductive style of classical Greek mathematics. Noticing these two facts about these three features, we conclude by suggesting that their integral presence in the Parthenon's design shows how advanced the interdisciplinary practice of mathematics, theoretical as well as practical, was by the mid-fifth century. We further show that this advance neither depends on nor correlates with the development of formalization or deductive apparatus.

In order to see why the presence of these features could be telling in this way for our conceptualization of the relationship between theoretical and practical mathematics on the one hand and the two styles of mathematical practice discussed here on the other, let us begin by recalling the Plato and the mathematicians discussion, in which the use of the difference in methodology between one kind of mathematics and another is key for understanding Plato's critique of the (Greek) mathematicians (of his time) in *Republic* 6 and 7 (510c-d, 528b-d, 529b-c, 530a-b, 531b-c).[47] Plato here seems to be critical of mathematicians who simply apply their procedures, which they treat as granted setting-stones (hypotheses), and do not question their principles. In acting this way, Plato argues, they deprive mathematical practice of its

47 The literature here is very extensive. A good beginning might be Barbera 1981, Benson 2012, and Burnyeat 1987, 2000. For a more detailed discussion, see Cornford 1932, Cherniss 1951, Lloyd 1978, 1992, Mueller 1992, Taylor 1967, Zhmud 1998.

greatest potential: they make it impossible for mathematics to lead us to the forms. If we bear in mind the central suggestion of the previous section of this article concerning possible near Eastern influence on the Greek mathematical practices before ca. 375 BC, this critique takes on a new dimension. Might it not be that the nature of Plato's concern here is precisely the prevalence of the algorithmic-problematic style of mathematics argued here to be integral to the design of the Parthenon? Further interpretive attention is surely needed here. Yet should it prove true that there are substantial continuities between the (pre-Euclidian Greek) mathematicians Plato is criticizing and the near Eastern mathematicians that the second section has tried to show at least might have had an influence on them, this might help us to thicken our understanding of a critique that remains opaque to many of Plato's readers right down through controversies in the contemporary secondary literature.

By teasing out the background of Plato's critique, we can see how his worry about certain algorithmic-problematic practices points toward the development of formal-deductive practices that are more consistent with anti-materialistic ontology and with the forms in general. Nikulin 2012, discussing 'indivisible lines', and Negrepontis (in this volume), discussing the periodic *anthyphairesis*, make clear how constructions produce the tension that arises when trying to define and use a unit both arithmetically and geometrically. This tension, as we will see in a moment, correlates strongly with the central problems the designers of the Parthenon were attempting to resolve related to inevitable remainders that arise when trying to maintain strictly numerical ratios in constructing a three-dimensional figure (a temple in fact) according to the canon of proportions used in the Doric order.

This vindicates Negrepontis's finding—itself an echo of Szabó 1978 and Fowler 1999—about Aristotle's claim from *Topics* (158b-159a) that there was a pre-Eudoxan approach to proportions in Greek mathematics defined with reference to finite and infinite *anthyphairesis*. Such a pre-Eudoxan approach to proportions by an algorithmic procedure rather than a definition and demonstration suggests that there already existed by the time of the Parthenon's design a robust, *theoretical* reflection on (for instance) why some recursive expansions in square yield continuous proportions of whole numbers, while others arrive at the 'infinite *anthyphairesis*' Plato is worried about in dialogues like the *Theaetetus*, *Philebus,* and *Statesman*. A synoptic discussion of three crucial elements of the Parthenon's design mentioned above—periodicity, the application of the unit, and the constructed harmony of the whole—will show this quite clearly.

The first is periodicity. The Parthenon's construction seems to have begun with a literal, physical given: the column drums of 1.905m in diameter. However, another equally important given, or at least a starting point, for the Parthenon's exterior design was the 3:2 ratio of metope width to triglyph width towards which the aesthetic modifications and adjustments made to the Doric order over the sixth and early fifth centuries had been converging: indeed, by the time of the Parthenon's immediate predecessors—the Temple of Zeus at Olympia, the Temple of Hera Lacinia at Akragas, and a few others—the 3:2 ratio seems to have become canonical.[48] With these two starting points in mind, one can imagine a design process for the flanks and façades (figure 1) in which an initial continuous proportion is constructed to yield the building's governing ratio of 9:4. Beginning with the lower column diameter of 1.905m, the average or 'ideal' metope width is then established at 2/3 the column diameter (1.27m). Then applying the recently canonized 3:2 ratio of metope to triglyph, the triglyph width becomes 0.8467m (corresponding extremely closely to the actual standard triglyph width of 0.845m), creating a continuous proportion of column diameter to metope width to triglyph width of 9:6:4, with the 9:4 ratio—so central to the Parthenon in all of its measurements—holding between the column widths and their visually analogous form at the level of the frieze, the triglyph. Moving in the other direction, and applying this newly constructed 9:4 ratio to the next larger principal visual element of the exterior (whether façade or flank), the intercolumniation, in its relation to the columns (the 1.905m column drums), gives a standard intercolumniation of 4.29m, i.e., with a 9:4 ratio of intercolumniation to column diameter. At this point, the further extension of the façade's continuous proportionality appears, the one that, as we shall see, will ultimately define the cubic dimensions of the building as a whole: 81:36:16, here the ratio of intercolumniation to column diameter to triglyph. Thus, the design of the Parthenon's façades and flanks suggests the way these proportions are unfolded out of each other, and are in fact constructed, both in the literal sense—in terms of the measuring and cutting of individual stones, the making of the building itself, i.e., in terms of *technē* as craft—and in a musical or mathematical sense, viz., in a manner similar to the construction of a musical scale out of series of related small number ratios.

Because all of these elements are so prominently visible—indeed, they provide the principal visible horizontal articulation of the rectangular field (that is, the main divisions of its left-to-right extension)—they give the whole ensemble the character of a manifest demonstration of a simple mathematical law, the law of continuous proportionality. In addition, on a more profound if less explicit level, they collectively contribute to the aesthetic quality of harmony and of unity that so strongly characterizes the building as a whole. In other words, the pre-existing and recently canonized forms that defined the proportions of the Doric order have been pressed into service to make visible, in geometric form, some of the same harmonic principles, specifically the relationships among small integer ratios, by which the fifth-century Greek musical scale was constructed. Likewise, those ratios together produce an ineffable aesthetic effect parallel to the one produced by the same combinations of small integer ratios in music. This 'aesthetic' experience, it is worth stressing once again, could be more fully described as an embodied one—again, parallel to music and to its effects on the body. Thus the predominance of continuous proportions can be understood to

48 See Waddell 2002, 2–3 and Mertens 2006, 270–271, 389.

engage the viewer in a bodily as well as an intellectual fashion. Approaching one of the façades, passing between columns, and circumambulating just outside or within the outer peristyle, one may experience, even if pre-consciously, one's body as a '4' in relation to the '9' of the intercolumniations (since the width of the column drums, 1.905m, corresponds to the scale of the human body), just as one is encouraged to feel one's body scaled as a 9 in relation to the 4 of the triglyphs, or a 3 in relation to the 2 of the metopes, when visually following the line of the columns to the entablature above.

If this construction of *symmetria* (the proportionality of the whole) in the Parthenon can indeed be understood as analogous to the constructive methods, the aesthetics, and the ontology (the mathematical being) of the fifth-century Greek musical scale, one would expect the *symmetria* of the building's interior as a whole to relate to that of the exterior (the stylobate, flanks, and façades) in a significant way. And indeed it does: the overall ratio of principal exterior dimensions to principal cella dimensions is precisely 13:9 (figure 2). This is a small integer ratio, though not an obviously musical one, and as such it raises interesting new questions relevant for appreciating the crucial importance of *anthyphairesis* in the building's design. Just as subtraction of the *diapason* (octave) from 9:4 yields the *tonos* (whole tone); just as, in the musical scale, subtraction of the *diapente* (fifth) from the *diapason* produces the *diatesseron* (fourth) (and the *lemma* is produced by subtracting two *tonoi* from the *diatessaron*, etc.); so in the case of the 13:9 ratio, a single application of *anthyphairesis*, subtracting the smaller measure from the larger, produces a remainder of 4 and a new ratio of 4:9, the ratio of this remainder to the smaller of the original elements. One can imagine, and this is of course purely speculative, that the designers could have arrived at 13:9 as a desirable ratio quite easily, given the predominance of *anthyphairesis* as a working method in the Parthenon's construction, as well as the tendency to work from "both sides" of a problem, i.e., not just to start from a given, but to start from dimensions that, through a known procedure, will arrive at a given.

In addition, however, *anthyphairesis* not only has an arguably central importance, just as it does in the case of the musical scale, as a constructive method for the Parthenon, whose design employs the same algorithms and engages the same problems as musical scale construction; its use also has a self-reflexive quality. Indeed, the problems raised by *anthyphairesis*, *qua* problems, are made evident in various parts of the building, most notably in the treatment of the well-known 'corner problem' (discussed below), in ways that give the building a pedagogical character. This appears, arguably, in its simplest form in the superimposed Doric orders of the cella interior (figure 3), where the lower order relates to the upper one in a ratio of 13:9. Certainly there may have been aesthetic reasons for this particular choice of proportions, as well as structural ones (for instance, if it provided the appropriate reduction of mass of the upper order to avoid its being top-heavy but still strong enough to hold up the roof). However, one cannot help but notice that precisely here in the building, one finds two visible elements that are proportionally and formally identical but at two different scales, inviting comparison: two unequal but analogous magnitudes, emphasizing a single dimension of extension, inviting the viewer to size one up against the other, i.e., to perform a kind of intuitive *anthyphairesis* in thought. Walking around the Parthenon interior, as one can still experience it today in the twentieth-century reproduction of the building—begun in the 1920s, based on Dinsmoor's specifications— in Nashville, Tennessee, one feels (pre-consciously or otherwise) the simultaneous presence of the 9:4 proportions of the central space's ground plan and the 13:9 ratio of the two orders of Doric columns, along with the invitation to compare, and perhaps to calculate (figure 4).

To return to the corner problem, i.e., to the question of how to negotiate the necessary adjustments to proportions in the Doric entablature in order to ensure that a triglyph and not a partial metope (that is, a remainder) is placed at the building's corner (figure 5): the remarkable solution to this problem in the Parthenon seems to involve a particularly sophisticated use of *anthyphairesis* as a construction method. In the Parthenon, in contrast to earlier buildings, there is an astonishing variation in the widths of metopes on all four sides of the building, an apparent irregularity that is unprecedented in Doric design, and that suggests a complex process of reciprocal small adjustments. Furthermore, the adjustments both to the intercolumniations and to the frieze were coordinated in a way that implies orientation towards a larger goal, that of *harmonia* (harmony) in the corner articulation, and in the building as a whole. With this in mind, the sense of repeated reciprocal adjustments producing what seems like a slight but unaccountable irregularity in the dimensions and positions of elements in the Parthenon's entablature becomes intelligible. And those adjustments have the character of a procedure by which progressively smaller intervals are produced using *anthyphairesis*, as in the construction of the musical scale, paired with the recognition that such a process can only be an incomplete solution, given the existence of irrational magnitudes, but that, nevertheless, such incompleteness is preferable to ignoring the problem altogether. Thus, the complex process of interrelated adjustments to the Parthenon's 'ideal' proportions that seems to characterize the approach of the Parthenon's designers and builders to the corner problem—evidently an ongoing working out of an unsolved(/unsolvable) problem at the time the building was being constructed—[49]could be understood as parallel in method to the building of a musical scale, where the component numerical ratios do not *perfectly* fit together, in the sense that they cannot measure (i.e., subdivide) each other, requiring small fractional adjustments (resulting, among other things, in the *lemma*, the *apotome*, and the *comma*) generated by a method of reciprocal subtraction that inevitably leaves a remainder. This 'imperfection'—the impossibility of subdividing the octave into any number of equal units (whether *diapente*, *tonos*, or any other interval, however small)—involves a conscious acknowledgement of the existence of the irrational. And the use of *anthyphairesis*, both in the musical scale and in

49 Cf. Yeroulanou 1998, 414–416, on the evidence for modifications to the building's design during the course of construction, based on the variations of metope widths on the west and east façades.

the Parthenon's entablature, constitutes an algorithmic-problematic approach to the negotiation of this problem.

To give just one example: early analyses of the building's refinements, specifically by Guido Hauck 1879 and William Goodyear 1912, revealed minute adjustments not only to metope widths but to the guttae underneath the triglyphs and to the centering of the abaci over the columns in counterbalancing ways.[50] For instance, considering the south side of the entablature on the Parthenon's east façade (figure 6), if a triglyph is decentered to the left with respect to the intercolumniation beneath it (as is the case with the second triglyph from the south), the guttae underneath the triglyph are decentered a still smaller amount to the right to compensate, i.e., to produce a slightly more centered overall ensemble. Or, since the triglyph over the corner intercolumniation appears noticeably closer to one column than another, the triglyph over the adjacent intercolumniation is shifted off-center in the same direction, but by a smaller amount (and, correspondingly, the abaci of the columns below are shifted the opposite way to further, slightly, reinforce this decentering), in order to reduce the difference between the degrees of decentering in the two adjacent sections of the entablature. Just as the use of a 13:9 proportion on the largest scale invites one, in comparing them, to subtract the smaller from the larger and arrive at the normative 9:4 ratio, so too on this microcosmic scale the most subtle of the adjustments to the entablature proportions imply a constructive method that has the specific, procedural characteristics of *anthyphairesis*.

The approach taken on the west façade in fact differs from that of the east façade, and that of the northern half of the east front differs from that of its southern half, indicating an ongoing working-out process in which different algorithms were tried, but also (given the likely design chronology of east to west)[51] demonstrating an increasing tendency towards a harmonization of the corner intercolumniation with the corner of the frieze, with both being visibly contracted—an approach most fully realized on the west façade.[52] This progression towards greater harmony at the corners, culminating in the west façade (see figure 1)—with peristyle and entablature refinements coordinated, as well as the visible strengthening of the corners as a whole through contraction—seems consonant with the Parthenon's overall prioritization of *harmonia* as a value (cf. the 'third principle' discussed above). It is worth emphasizing the importance of corner definition for the impression of unity in the building, and both the harmonizing of elements and the greater density and strength at the corners contribute to this definition of the corner *qua* corner. However, a similar kind of experimental, working-out process could certainly be reconstructed with an alternative chronology, going from

50 Hauck 1879, 130–133 and Goodyear 1912, 200–202.
51 See Korres 1994, 115, Yeroulanou 1998, 423, and Goodyear 1912, 191–192. For an alternative chronology, see Wesenberg 1983, 60–62.
52 On corner column contraction in the Parthenon, see Coulton 1974, 66, 76, Bundgaard 1976, 63–64, and Wesenberg 1982.

west to east. What is important is not the specific reconstruction, but the sense of an experimentation through reciprocal adjustments to create the most satisfying solution to the corner problem, based on the interdependence of *all* elements in the design, and not only on the modification of an isolated few.

The second key feature is how modular construction relied on the practice of 'the application of the unit'. The achievement of continuous proportion (part of a larger goal to achieve *symmetria*, i.e., commensurability) both macrocosmically and microcosmically on the exterior of the Parthenon seems, indeed, an overriding concern of the building's designers, and makes a compelling case for the idea that the unusual octastyle façade was chosen to make these proportions possible and not, as has often been argued, for other reasons, such as to accommodate a larger cult statue in the cella.[53] One consequence of the unique method used to determine the Parthenon's stylobate dimensions, combined with the similarly atypical choice of octastyle fronts—and indeed as a likely motivation for such—is that the building's overall proportions of 81:36:16 (length:width:height) has as its modular unit the actual, visible triglyph width (figure 7). The mainland rule of design—five triglyphs per intercolumniation plus one additional triglyph defining the measure of each side of the stylobate—produces, for an 8 x 17 temple, a stylobate dimension of 36 x 81 triglyph widths. And with the height of the order (4/9 of the stylobate width) thus measuring 16 triglyph widths, the unit for the 81:36:16 cubic volume of the building becomes the basic formal unit of the entablature: the triglyph, so prominently visible at each of the building's corners, that articulates the horizontal extension of the building, in 2:3 rhythm with the metopes, at the level of the frieze. In effect, the Parthenon's design not only produces a rigorously commensurable building; it also foregrounds the unit of measure that defines it, seeming to invite the viewer to consider those very measurements reflectively, as well as the whole problem of *symmetria* and its significance.

However, the way in which the Parthenon uses the inward inclination of the columns of the peristyle to mediate between the proportions of the stylobate, based on an 'ideal' triglyph module, and those of the entablature, based on the actual triglyph module, indicates that the unity of the building as a whole, the joining together of its different parts (its *harmonia*), takes precedence even over *symmetria*. In other words, the designers created small deviations from the pure *symmetria* of stylobate and entablature for the sake of balancing and adjusting multiple elements and of harmonizing slightly different proportional systems, as well as to bring the corner into harmony with the rest of the building. This refinement—producing a slight discrepancy between the actual triglyph width and that of an 'ideal' triglyph module—allows the Parthenon's designers to bring the continuous proportionality of the facades and flanks (81:36:16, discussed above) into harmony with the traditional method for modular construction of the stylobate dimensions.

53 Cf. Bundgaard 1976, 69–70.

As Coulton 1974 has discussed at some length, in the middle of the fifth century there were effectively two rules for determining the dimensions of the stylobate, one based on the intercolumniation module, prevalent in mainland Greece and another based on the number of columns in the peristyle, prevalent in the western colonies, particularly Sicily. With its unusual design of an 8 column by 17 column peristyle—unprecedented in mainland Greece, where Doric temples invariably had 6 columns on the façades—the Parthenon, though based primarily on the 'mainland' rule, effectively harmonizes the two approaches by also conforming to the 'Sicilian' rule (where an 8:17 peristyle yields a 9:18, or 4:9, stylobate); at the same time this approach to the design achieves a precise 9:4 ratio for the stylobate (measuring 69.50m by 30.88m).[54] As Büsing 1988 has shown, the Parthenon's unusual 8 x 17 peristyle yields a stylobate, when following the mainland rule, of precisely a 9:4 proportion ([17+1/5]:[8+1/5] = 81:36 = 9:4). This rule assumes a triglyph module 1/5 the width of the intercolumniation, increasingly standardized as a modular basis for calculating the dimensions of the Doric stylobate, a problem discussed in illuminating fashion by Gene Waddell 2002 and Mark Wilson Jones 2001, 2006.[55] In the Parthenon, however, as we have seen, intercolumniation and triglyph are instead in an 81:16 ratio. What the designers seem to have done to reconcile these two givens is to construct the stylobate as if the triglyph width were 1/5 that of the intercolumniation (0.858m, a kind of 'ideal' triglyph module), but then account for the difference from the actual average triglyph width in the entablature, 16/81 of an intercolumniation (0.845m), through the inclination of the columns, leaning inward from the stylobate to the slightly smaller entablature. If this is calculated based on the flanks, which are 81 ideal triglyph modules in length, with the column inclination equal to half a triglyph width on each side, then the entablature, when 'compressed' by one triglyph module, would be 80 ideal triglyph modules in length, and at the same time of course, 81 actual triglyphs in length. And this in fact corresponds exactly to the 81:80 ratio of the ideal triglyph module—the one that determines the stylobate dimensions and the large scale *symmetria* of the building —to the actual average triglyph width of 0.845 (0.858:0.845 ≃ 81:80), while this difference between the ideal module of the stylobate and the real triglyph width in the entablature precisely corresponds to,

54 Coulton 1974, 76.

55 The reconstruction of Doric design methods in Waddell 2002 suggests that first the krepis and stylobate dimensions were determined in this fashion, and from there the dimensions of the module (the triglyph) were derived, while Wilson Jones 2001 proposes a somewhat different, but related, procedure for the determination of the proportions in Doric temples, emphasizing a modular design that begins not with the overall krepis dimensions but with the physical module, i.e., the triglyph. For his more recent work on mathematical problems relating to Doric design, see Wilson Jones 2006. In the broader context of Doric temple design, however, 1/5 is only one of a number of different small fractions that could be added to the number of intercolumniations to determine stylobate dimensions, as Coulton 1974, 76, 83–84 points out.

and counteracts, the difference between the 5:1 (=80:16) ratio of intercolumniation to triglyph module and the 81:16 ratio between actual intercolumniation and actual triglyph width with which the Parthenon was built, with its overriding concern for 3:2 and 9:4 continuous proportionality. Furthermore, it suggests that the holding together, indeed the very being, of the building, as *one thing*—what the refinements seem to address—takes precedence over proportion, per se: as we see here with the refinement of column inclination, the refinements are not just added onto pre-existing proportions, but instead engender adjustments to the building's overall *symmetria*—in fact, a reciprocal back and forth of adjustments—[56] for the sake of the unity of the whole.

One could look to the brief but challenging discussion of 'the one' in Resp. 7 (probably a later theoretical reflection on a mid-fifth-century mathematical problem), as an indication of the ontological depths of this problem of unity with respect to harmonics, and of the particular character of what is at stake in the question of the unit, or module, in the Parthenon. Just as Socrates and Glaucon, in *Resp*. 7, 524d-526c, discuss 'the one' as holding together as a unit despite the apparent divisibility of any specific 'one' into innumerable parts, so the Parthenon is a building made up of carefully coordinated parts involving progressively smaller adjustments (suggesting the possibility of an *anthyphairesis* without limit) that also has a powerful presence as a single, harmonious whole. Creating unity out of disparate parts is, of course, a fundamental aesthetic value in many contexts and is even, one might argue, the defining value of aesthetics, from the point of view of the object (as opposed to that of the subject, of subjective judgment). However, the construction of a unified whole in the Parthenon engages a much more specific set of concerns, namely, the consideration of the relationship of *harmonia* (the wholeness, or oneness, of the object) to a constituent module, or unit, made visible in the triglyph that measures the building—and, in effect, defines and anchors its corners at the level of the frieze.

In additional to being a practical issue of constructive design, this is a theoretical issue of a mathematical sort, and, in the case of the Parthenon, a question of direct experience as well. A viewer or worshipper in the presence of the Parthenon experiences the building as an ontological and aesthetic whole, and at the same time is engaged, directly and intuitively, by an interplay of related parts (see figure 7) whose proportional relationships function as a kind of visual analogue for the effects of harmony in music, i.e., the relationships among pitches, built from some of the very same numerical ratios present in the Parthenon, with respect to a whole. The tension between wholeness and multiplicity remains a tension, as it does in the experience of music, precisely because the multiplicity of proportional parts is ultimately irreducible to the aesthetic/ontological whole, since that whole necessarily incorporates

56 Consider, in this context, the analysis that Korres 1999 and Haselberger 2005, 127–133, 142–146 provide of the concept of 'refinements of refinements'.

the irrational. It is this tension that one experiences as harmony, that underlies what various scholars have identified as the building's uniquely organic quality,[57] and that shapes the intuitive experience of the building.

The third and last principle, which is mathematical but not merely mathematical, already alluded to above, is the harmony of the whole. As in the construction of the ancient Greek musical scale some decades before the Parthenon was built, and as Nikulin 2012 details in his discussion of the divided line and its importance to Plato some decades after it was built, so too in the temple's construction, the consideration of a geometric object in arithmetic terms as a means of forging an aesthetic (and ontological) whole gave rise to an irreducible tension between magnitude and multitude. The analogy between the architecture of the Parthenon and musical harmonics is based not only, or even primarily, on the simple presence of musically significant ratios throughout the building; rather, it is the coordination and, even more so, the construction of interdependent ratios to form the *symmetria* of the whole that gives the building its musical quality. We can properly speak of the harmonics of the building here. In terms of harmonics, however, *symmetria* is really only a starting point for a much more complex engagement of the problems raised by musical harmonics, including the necessary acknowledgement of the irrational, of magnitudes that cannot be reduced to multitudes. It is the notion of *harmonia*, in the fullest and most rigorous sense, that encompasses the irrational—as well as joining ratios and *symmetriai*—in order to make a whole that is central to understanding the Parthenon and its invitation to dialectical thought ('dialectical' in the sense discussed in *Resp*. 7, and which we will discuss further briefly below). However, just as with the construction of the musical scale, the deepest engagement with these problems in harmonics begins with the attempt to construct *symmetria*—itself already more than just a series of proportions, or even continuous proportionality, but rather the resonance of each part with all the others and with the whole. Indeed, as Pollitt 1964 has observed, *symmetria* for the Greeks was not only a fundamental aesthetic concept, but arguably the predominant term of value in all of classical Greek and Roman aesthetics.[58] *Symmetria* not only forms the basis of the Parthenon's aesthetics, but, as we have begun to outline here, was the starting point of the inquiry into the nature of the building's being itself.

However, as we have argued above, *symmetria* alone could not fully account for the building's being, as its designers understood it: the principle of *harmonia* was even more crucial, and ultimately given priority within an ontological/aesthetic understanding that explicitly and self-reflexively incorporated the irrational. In the inclination of the peristyle columns (which by the time of the Parthenon were a standard

57 See Haselberger 2005, 105, Gruben 2001, 187–188, Pollitt 1972, 74–78, Goodyear 1912, 81–103, and Hoffer 1838, esp. 370.
58 Pollitt 1964, 14–23, 256–258.

refinement in Attic Doric temples)[59] and the slight discrepancy that they introduce between the dimensions of the stylobate and the dimensions of the entablature, the Parthenon in fact strove to bring into harmony two different and not entirely compatible principles of *symmetria*. These two *symmetriai* were, once again, the 5:1 intercolumniation to triglyph ratio used in the modular design of the Doric stylobate in the mid-fifth century and the 81:16 ratio between intercolumniation and triglyph (intercolumniation: column diameter: triglyph :: 81:36:16) on which the continuous proportions specific to the Parthenon are based. Thus, the refinement of column inclination was thoroughly integrated into the design of the building: as a mediating factor, it effectively 'solved' the incompatibility of these two *symmetriai*, both so crucial to the temple's design. Or rather, since this 'solution' also introduced approximations and adjustments that departed from perfect measure (for instance, the visible triglyph was now very close to, but not exactly, the width of the module used for the stylobate), it was clearly part of the larger working-out process in response to the irreducible tension between magnitude and multitude, the process of joining the parts of the building as much as possible into a geometric whole defined by numerical ratios—that is, into a harmonic unity. As should be clear, this raises issues quite similar to those at stake in the Parthenon's approach to the corner problem, and in ways that make evident how the treatment of column inclination—with the adjustments it necessarily introduces into the entablature—is integrally bound up with the negotiation of the corner problem itself. This harmony shows, for instance, in the way the contraction of metopes at the corners, virtually unique to the Parthenon, relates both to the excessive column contractions (another unusual feature of the building) and to the now standard refinement of thickening of the corner columns: all three now work together to create the impression of a strengthened corner (see figure 5). And it is crucial to note that the emphasis on corners has the effect of reinforcing the unity and definition of the building as a three-dimensional object, i.e., as a harmonious whole, since the corners are what articulate the most general contours of the building's three-dimensional geometric form. Indeed, the method, discussed in this section, of joining slightly incompatible *symmetriai* and of integrating different but related problems (column inclinations, the corner problem) suggests an overriding concern with *harmonia*, even at the expense of perfect *symmetria*.

It is a defining characteristic of art objects, as opposed to ordinary objects of manufacture, not only to relate formal—and even material—qualities to the transcendent realm of thought and meaning, but also to reflect upon themselves, upon their own conditions of being (or, more precisely, to explicitly provoke such reflection in viewers).

59 On the integration of column inclinations with the other refinements, as an emerging canonical part of Doric design, in one of the Parthenon's most important predecessors, the Temple of Aphaia at Aegina, see Haselberger in Neils, ed. 2005, 119, 122, 124 and Haselberger in Haselberger, ed. 1999, 16, 32.

This self-reflection, embedded within the work through a kind of dialogue between the artist and the artwork, becomes, in the afterlife of the work, a dialogue between artwork and viewer. The Parthenon creates such a dialogue, simultaneously in the way it engages the senses—and, indeed, the whole body—and in the way it engages thought. Consider, for instance, the refinement of curvature: the curvature of all the apparently straight lines in the Parthenon, a feature without any evident constructive necessity, seems designed to introduce a discrepancy between the (straight) mathematical idea of the temple and its (curved) physical reality, and thus raises ontological questions in an entirely self-reflexive manner. Indeed, the introduction of this discrepancy, the product of extensive and precise technical labor, makes questions of (mathematical) ontology and of the relationship of mathematical to physical being explicitly, and self-reflexively, present in the building. Furthermore, there is a sense in which the beautiful, in its representation of a perfection that cannot be perfectly described—i.e., that contains a mysterious, irreducible remainder—both engages the viewer directly through the presentness of the object and leads her or him beyond it. In this sense, there is a resonance between the Parthenon's pedagogical function—its invitation to (what is described in *Resp*. as) dialectical thought and, to put it this way, to an education in the liberal arts—and its aesthetic function as a work of art: both involve an open-ended dialogue with the viewer or interlocutor, inviting him or her to engage with what is present in order also to go beyond it—affectively, interpretively, imaginatively, intellectually. And naturally, as we have indicated above, both the pedagogical and the aesthetic modes resonate meaningfully with the Parthenon's principal, religious function, its call to worship and to piety. Furthermore, both the building's beauty—and by extension, its power as a work of art—and its pedagogical character arise, in some fundamental way, from the dialogue between arithmetic and geometry, the kind of dialogue between mathematical arts discussed in *Resp*. as the basis for dialectic. In the Parthenon, these two mathematical arts are brought together with a third art, musical harmonics, and all are pressed into the service of trans-disciplinary questions and aesthetic expression alike—that is, the three arts together work to engage a viewer in philosophical and value-oriented questions (the latter with respect to beauty)—and in that sense could more properly be described in this context as liberal arts.

V Conclusion

The sophistication with which the Parthenon manifests an algorithmic approach to constructive problems makes clear that there is a robust reflective (theoretical) awareness of the mathematical principles involved, and no mere rules of thumb being arbitrarily instituted. Yet, this theoretical treatment is neither formal nor systematic, and it surely is not based on concerns related to proofs or the desire to offer them. If it is true both that (a) the Parthenon's design displays a theoretical sophistication in its mathematical principles and (b) there is clearly not the concern with formal deductive proof distinctive of fourth-century and later Greek mathematics, then we submit that our reading of the Parthenon substantiates both of the two revisionist claims in the historiography of Greek mathematics advanced above.

The first relates to dating the development of an understanding of key mathematical notions, such as incommensurability and continuous proportionality, within Greek mathematical thought. Here, given the dates of the Parthenon's construction (confidently set at 447-432 BC), we can conclude that if the Parthenon's design displays a reflective awareness of the knowledge procedures related to continuous proportion and the relationship of magnitudes and multitudes, then there must have been advances of this kind made before 430 BC, as Szabó argued. Second, with respect to the wider claim about how far Greek mathematics is continuous with earlier thought and practice, we must take note that the reflective awareness at work in the Parthenon is *not* formal or deductive, but rather algorithmic and problematic. As such we surely notice (at a minimum) the close analogy with Mesopotamian astral science of the seventh to fifth centuries. This, we conclude, is good reason to believe that, at least up to the time of the Parthenon, Greek mathematics was borrowing from and surely extending earlier mathematics. Much will remain undecidable in what we have presented, but interpreting the mathematical features of the Parthenon gives us reason to believe that the development of Greek mathematics in the first decades of the second half of the fifth century is, at least in part, a story of continuity with earlier mathematical traditions. If one is clear about the relatively advanced state of *practical* mathematics in mid-fifth century Athens, and agrees that these advances are a function of the algorithm, a development of recursive practice and not of theoretical reflection, then we have good reason to think again about the possibility that there was real continuity between these practicing mathematicians and their near Eastern predecessors. We must, that is, develop a deeper appreciation of the role that the algorithmic problem-oriented style was playing for pre-Euclidian mathematicians. Our monograph on the Parthenon tries to demonstrate that there is a conscious attempt to introduce this mathematics into the building in a thematized and programmatic way, and the intellectual background of that attempt is precisely the problem-based approach to ontological questions about the misfit between magnitude and multitude that arises from the reception of near Eastern mathematics in the Greek world by theorists and practitioners understood to be Pythagoreans.

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Illustrations

Fig. 1: The Parthenon, Athens, 447-432 BC: west façade.

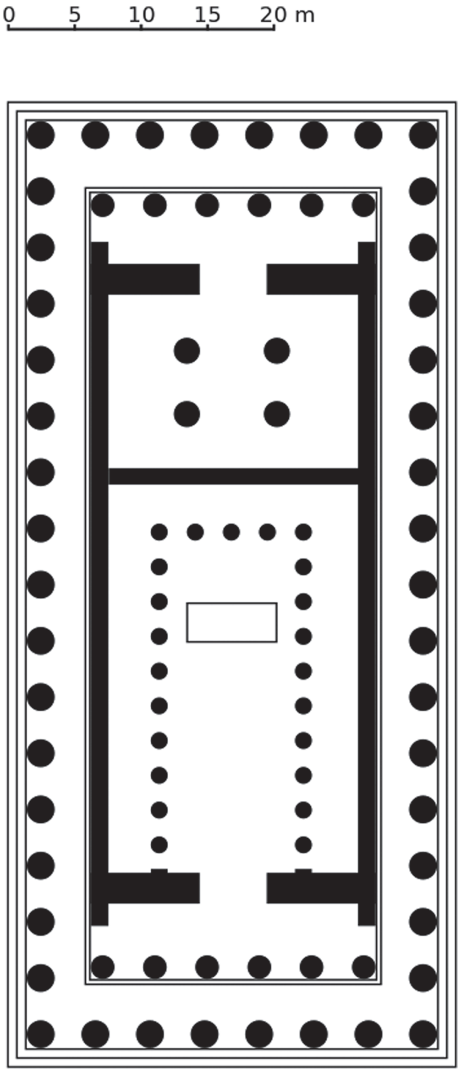

Fig. 2: The Parthenon: ground plan.

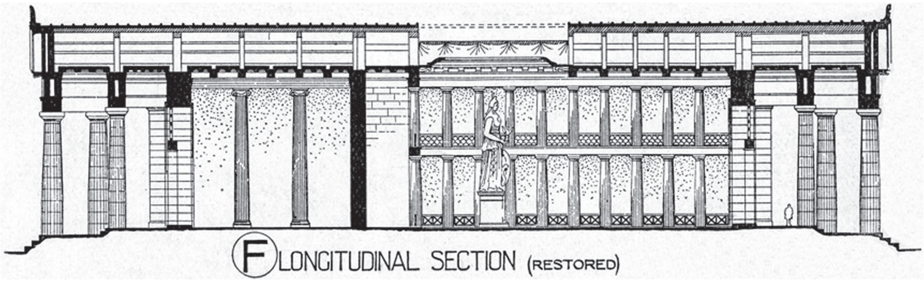

Fig. 3: The Parthenon: section, showing the Doric and Ionic orders of the interior.

Fig. 4: Parthenon replica, Nashville (begun 1920s): interior of the naos.

Fig. 5: Joseph-Philibert Girault de Prangey, *Façade and North Colonnade of the Parthenon on the Acropolis, Athens*, daguerreotype, 1842.

Fig. 6: The Parthenon: east façade, south side of the entablature.

Fig. 7: The Parthenon: view from the northwest.