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The Babylonian Cellar Text BM 85200 + VAT 6599 Retranslation and Analysis

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Contents

1 Babylonian "algebra"

In a number of earlier publications, I have proposed a new understanding of the Old Babylonian mathematical technique known as "algebra", concentrating on problems of the second and to some extent of the first degree.[1] As a background to the following investigation of a particular text dealing in part with problems of the third degree I shall need a summary of my methods and my main results.

The Babylonian interest in apparently algebraic problems of the second degree was discovered around 1930.[2] As natural, and as a first approximation, the texts were interpreted through the conceptual framework of more recent algebra and arithmetic, with the result that the operations involved were understood as purely arithmetical operations, and the obviously geometrical vocabulary ("length", "width", "area", etc.) was interpreted as nothing but a set of convenient labels (for "the first unknown", "the second unknown", "the product of the unknowns", etc.).

My reinterpretation was based on two methodological principles. One of these may be described as a "structural analysis", the other as "close reading".

It had been observed already at an early moment that the Babylonians employed a whole set of distinct terms for addition, another set for subtraction, and a third for multiplication. *Grosso modo*, the terms from each set were supposed to be synonymous, and no particular effort was spent to find possible differences between them.

This comfortable creed was undermined by the structural analysis. It turned out that two different groups of supposedly additive terms are sharply distinguished. One of them, e.g., will normally not be used when (the measuring numbers of) a length and an area are added; the other is never employed for the operation of quadratic completion. The distinction between the two groups is so sharp that two different operations and two different concepts must be involved. Similarly, two different subtractive operations exist, and no less than four "multiplications".

[1] The most thorough presentation of my methods and results will be found in [HØYRUP 1990]. A more concise exposition has been made in German [HØYRUP 1989], even this dedicated to HANS WUSSING as a delayed homage on the occasion of his 60th anniversary.

[2] A brief history of the historiography of Babylonian mathematics, taking precisely this discovery as its starting point, is [HØYRUP 1991].

These differentiations make no sense within the received arithmetical interpretation, but suggest instead a more literal understanding of the geometrical vocabulary, which they fit. The "close reading" — close observation of the variable contexts in which each term occurs and of the organization of procedures compared to alternative possibilities which are not used[3] — confirms this.

The main outcome of the analysis is that Babylonian "second degree algebra" was organized around a pivotal technique which may be characterized as "cut-and-paste geometry". This geometry is not critically reflective as, e.g., EUCLID's *Data*. Nevertheless, the correctness of its procedures is intuitively obvious to anybody following the transformations (in the same way, say, as the correctness of the transformations of $2x + 4 = 6x - 24$ successively into $6x - 2x - 24 = 4$, $4x = 4 + 24 = 28$, $x = \frac{28}{4} = 7$ is obvious to anybody trained in elementary algebra); we might speak of "naive" geometry.

Together with the geometrical technique goes a geometrical conceptualization. While Medieval and present-day elementary algebra can be understood as the art of finding unknown *numbers* entangled in complex relations, the basic concern of Babylonian "algebra" is the disentanglement of *measurable but unknown lines and areas*. In both cases, this basic conceptualization may serve as a model for other structurally similar problems: The modern abstract numbers may stand for monetary values, geometrical lengths and areas, etc.; the Babylonian lines and areas, on the other hand, are used to represent prices, pure numbers, etc.

Cut-and-paste techniques constitute the pivot of Babylonian second-degree "algebra" but do not carry very far on their own. More complex problems therefore involve two auxiliary techniques, both of which are related to familiar procedures employed since long by Babylonian calculators: An "accounting technique" used, e.g., to find "how much there is of lengths" — in other words, the coefficient of the "length"; and a "scaling technique", which can be assimilated to a change of measuring scale or unit, and which is used to "reduce coefficients to *1*".

2 Essential terms and operations

In the following I shall present and discuss the tablet BM 85200 + VAT 6599 in the light of this reinterpretation of techniques and mode of thought. However, this will be most conveniently done if certain basic aspects of the terminology are presented in advance.

"Algebraic" interest appears to have arisen in a new Akkadian scribe school during the earlier Old Babylonian epoch (which in total covers the period c. 2000 B.C. to 1600 B.C.): "Algebraic" problem texts form, like omen texts, a *new* literary *genre*, and early specimens (18th c. B.C.?) tend

[3] A similar principle has recently been advocated by KARINE CHEMLA [CHEMLA 1991] as a tool for analyzing the methods of Ancient Chinese mathematics.

to be formulated in Akkadian, not in Sumerian, with the exception of some five terms which have their origin in traditional Sumerian mensuration and computation (length, width, area, part/reciprocal and "square root"/side of the square). As time went on, Sumerian word signs were increasingly used as "scholarly" abbreviations for the Akkadian words, but this was clearly a secondary development. As a result, however, each originally Akkadian term will have at least one Sumerographic equivalent[4] — at times several.

The terms and operations used in our tablet can be categorized and explained as follows:

2.1 Additive procedures

Of these there are two of major importance, both of which are used in our tablet. One is represented by the sign group UL.GAR, which is used logographically for *kamārum*, "to amass in a heap". It appears to designate a genuinely numerical addition, for which reason it can also be used to add the measuring numbers of, e.g., lengths and areas. It is symmetrical, connecting its addends by *u*, "and"; concomitantly, it conserves the identity of neither addend. I shall translate it "to accumulate".

The other is designated by the Sumerian word daḫ (Akkadian *waṣābum*), "to append". It operates on concrete entities, for which reason it only joins entities of the same kind and dimension. It is asymmetrical, connecting with the preposition *ana*, "to", and conserves the identity of that entity to which something else is appended while increasing its numerical value (as the identity of *my* bank account is unchanged when interest is added).

The habitual terms for the sum by accumulation are absent from the text dealt with below, except for one plausible occurrence of UL.GAR as a logogram for *kumurrum*, "accumulation"; instead, the term nigin, the "total" of accounting texts, is used on two occasions.

2.2 Subtractive procedures

Even these form a couple. One is a comparison, stating (in its Sumerographic version) that "X u-gù Y D dirig", "X exceeds Y by D" or, in a word for word translation which I shall use in the following, "X over Y D goes beyond". The other is the reversal of appending. Our tablet uses the Sumerogram zi, which corresponds to Akkadian *nasāḫum*, "to tear out".[5]

The palpably-concrete character of the operations of "appending" and "tearing out" is highlighted by the more complete phrase in which they are often embedded: a is not simply "appended to" or "torn out from B"

[4]In order to facilitate identification and linguistic classification, syllabically written Akkadian is ordinarily rendered as *italics*, while identified Sumerian words are given in spaced or normal writing. Signs with unidentified reading are written with their sign names (normally one of several possible readings) in small capitals.

[5]A few texts tend to distinguish *nasāḫum*, "to tear out", from *ḫarāṣum*, "to cut off", using the former preferentially when surfaces are involved and the latter for linear entities — cf. [Høyrup 1992]. Our present tablet, however, exhibits nothing similar.

but "to/from the inside of B" (*libbum*, literally "heart" or "bowels", but in mathematical texts apparently worn down to a bare indication that B is something possessing bulk or body).

2.3 Multiplicative procedures

Our text only makes use of two of the four multiplications. One of these, furthermore, is only obliquely present. It is referred to by the Sumerogram ì-kú-kú, which normally corresponds to Akkadian *šutākulum*, used when a "length" and a "width" are put in place so as to "span" a rectangle, entailing the creation of an area equal to the product of the two measuring numbers. In the present text, however, it stands as a logographic equivalence for Akkadian *šutamḫurum*, "to make confront its counterpart", i.e., to position a single line together with its equal or "counterpart" as sides of a square.[6]

The other operation is referred to by the Akkadian term *našûm*, "to raise" — the Sumerogram íl is not used in the present text, nor are certain synonymous possibilities. It is used for the "scaling" technique mentioned above, and generally in all cases where considerations of proportionality lead to a multiplication; for the calculation of areas when this calculation is not the tacit by-product of a construction (e.g., for the computation of triangular and trapezoidal areas as average length times average width); for metrological conversions and similar multiplications by technical constants; and when divisions are performed through multiplication by a reciprocal. Below we shall discuss its use in the computation of volumes, which may provide us with the key to the etymology of the term and to the conceptualization of the operation.[7]

Connected in the present text to "raising" is a specific term bal. It designates a transformation factor, necessitated by the discrepancy between horizontal and vertical metrology (see below). THUREAU-DANGIN ([TMB], 232, followed by [MEA], 45) reads the sign as a logogram for *nabalkutum*/*nabalkattum*, "to escalate"/"transgression". However, the alternative connection to the verb *enûm*, "to change", appears much more suggestive; this implies that bal be understood as a "factor of change" or "conversion" (I shall use the latter translation).

2.4 Squaring and square-root

"Squaring" as a specific operation only occurs as a geometric operation, and is then designated by the verb *šutamḫurum* just mentioned. Related

[6] In still another text (YBC 4675, published in [MCT], 44f), the Sumerogram is used where "length and [a different] length" are made span, i.e., where a non-rectangular quadrangle is laid out. The semantic span of the term is obviously large, and based upon the construction of a quadrangle and not upon the computation of the area.

[7] The other two multiplicative operations are designated by a-rá, a term derived from "going" and used in the tables of multiplications; and by *eṣēpum*/tab, which designates the concrete repetition of a palpable entity. None of them is used in the text to be treated below.

to this verb is the verbal noun *mitḫartum*, which designates a situation characterized by a confrontation of equals — i.e., a square geometric configuration. In numerical terms, the *mitḫartum* is equal to the length of the side of the square.[8]

On rare occasions the verb may be written (as in our present text) by means of the Sumerogram ì-kú-kú. Also rare is the use of íb-si$_8$, as is the employment of this term as an abbreviation for *mitḫartum*. Utterly common (unavoidable in fact, apart from minor variations of the expression)[9] is, on the other hand, its use in connections where the arithmetical interpretation sees a square root. Properly speaking, the term is originally a Sumerian verbal form, meaning "makes equal" or (since sides are spoken of) "makes equilateral". That "A makes r equilateral" means that the area A, when laid out as a square, makes r the side of this square figure — to which corresponds of course the numerical relation $r = \sqrt{A}$. Secondarily, the term is used as a noun (which I shall translate "equilateral") designating this side.

The originally geometrical character of the íb-si$_8$ is made clear by texts where the íb-si$_8$ is found and "posed" together with its "counterpart" (*meḫrum*, another verbal noun related to *šutamḫurum*), as two sides forming the angle of a square. But as we shall see below, the term may also be used in more generalized senses.

2.5 Division, parts, and bisection

Evidently, division *as a problem* was encountered regularly by Babylonian calculators. *As a technique*, however, division proper is absent from the texts. Instead, a problem of the type $d = n \cdot x$ is treated in one of two ways. If n is listed in the table of reciprocals, the text will ask that the "igi" of n be "detached" (du$_8$/*paṭārum*) and then "raise" this number to d. The term igi n is derived by abbreviation from the expression for the "n'th part [of something]" (igi n gál-bi), but so clearly kept apart from this original meaning that it must be regarded as a technical term for the reciprocal of n as tabulated in the table of reciprocals.[10] (I shall retain the original

[8]This may seem strange to us, who are accustomed to the idea that a square *is* its area (i.e., is identified by and hence with this characteristic parameter) and *has* a side. *A priori*, however, the Babylonian conception of a square figure as *being* (i.e., being identified by and hence with) its side and *possessing* an area is no worse. The Greek mathematical term *dynamis*, moreover, appears to correspond to a similar conceptual structure (cf. [HØYRUP 1990a], as does perhaps an ancient Chinese mathematical term (JEAN-CLAUDE MARTZLOFF, private communication).

[9]Some of these consist in homophonous substitution of syllables. More important is, however, the alternative ba-si$_8$. Traditionally it has been suggested that the latter term stood for "cube root" and íb-si$_8$ for square root, and the exceptions to this rule have been regarded as minor anomalies. As the number of exceptions has increased with the publication of further texts, this explanation of the difference between the terms must now be regarded as outdated.

[10]igi n is "detached" because unity is imagined to be split and one n'th part is taken out, as demonstrated by the use of the verb "to tear out" in a variant expression.

term in the translation in order to stress the specifically Sumero-Babylonian character of this concept, connected as it is both to mathematical inversion and to the table). If n is not listed (which of course must happen when n is not sexagesimally regular, i.e., not of the form $2^p \cdot 3^q \cdot 5^r$), the text takes note of this fact and then asks "what shall I pose to n which gives me d?" and give the answer immediately — easily done, in fact, since mathematical problems were constructed backwards and the solution thus both guaranteed and known in advance.

Halving and division by 2 are treated as division by any other regular number, through multiplication by "igi 2". In certain cases, however, where the arithmetical interpretation sees nothing but a halving the Babylonians operated differently. This is the case when a "natural" or "customary" half is to be found, e.g., the radius from the diameter of a circle. Below we shall encounter this specific bisection designated *ḫepûm*, "to break", at the crucial point in the solution of second-degree problems where, in the arithmetical understanding, the coefficient of the first-degree term is halved — indeed a case where *only* exact bisection makes sense.

2.6 Numbers

The Babylonian place value notation for numbers is well-known among historians of mathematics. It was not the only system in use, and could not possibly be, since it did not indicate absolute place. The mathematical texts, however, make use almost exclusively of this system; so does the text to be discussed below. I shall therefore bypass the systems used in economical and administrative texts.

The place value system was sexagesimal, i.e., its base was 60 — or, better perhaps, alternately 10 and 6. "Final zeroes" were never used, nor was any "sexagesimal point". A marker for intermediate empty places was occasionally used in a few texts from the outgoing Old Babylonian period, but mostly these were just indicated by increased distance between surrounding signs or left to contextual understanding.

In my transliteration, I shall render each sexagesimal place by a corresponding Arabic numeral (between 1 and 59); places are separated by commas. The translation introduces an indication of absolute place as derived from context. A number transliterated 21,15,23,6,19 and interpreted as $21\cdot60^2+15\cdot60^1+23\cdot60^\circ+6\cdot60^{-1}+19\cdot60^{-2}$ I shall thus translate as $21^{\backprime\backprime\backprime}$ $15^{\backprime\backprime}$ $23^\circ6'19''$ (when it is not needed for understanding or as a separator, I shall leave out $^\circ$).

The distinction between "n'th part" and "reciprocal of n" is normally made as here described; so also in the text under discussion below. Another way to distinguish is that the reciprocal is "detached" while the n'th part is "torn out" — see [HØYRUP 1990], 54 n. 69.

2.7 Metrology

The only parts of Babylonian metrology which concern us here are the units for distance, area, and volume.

The fundamental unit for ***horizontal*** extension is the nindan or "rod", equal to approximately 6 m. It is subdivided into 12 kùš or "cubits", each thus approximately 50 cm (fingertip to elbow), which is the fundamental unit for *vertical* extension (heights and depths). "Fundamental units" are almost invariably left implicit, which makes measures given in fundamental units look like pure numbers. In other cases (e.g., horizontal extensions measured in kùš) the unit will be explicit.[11]

The fundamental [horizontal] area unit is the sar, equal to a square nindan, i.e., a square with the fundamental unit for horizontal extension as its side. The corresponding fundamental volume unit is a block of 1 nindan times 1 nindan times 1 kùš, which, similarly, is called a sar.[12] The standard volume is thus to be understood as a standard area covered to a standard height; as we shall see below, calculating a volume implies "raising" this standard height to the real height.

2.8 "Variables" and metalanguage

Anything somehow "algebraic" in character must possess ways to designate "unknowns" or variables and devices to display the logical organization of problems and procedures. So also Babylonian "algebra".

Designations for variables were more or less standardized, and more or less bound to specific problem types. An example of the highly specific is the "beginning of the reed", the original length of a measuring reed which during a mensuration process looses specified sections; another, used in our present text, is the couple igi and igi-bi, "the reciprocal and its reciprocal", a couple of numbers occurring together in the table of reciprocals and thus with product 1 (or, indeed, 60^n, where $n = 1$ is attested in the tablet YBC 6967, [MCT], p. 129). I shall employ the Akkadian loanwords, *igûm* and *igibûm* in order to emphasize the connection to the term igi.

Of most general use, almost corresponding to our semiautomatic choice of x and y as labels for a pair of unknowns, is the set uš/sag, representing the "length" and the "width" of a rectangle and thus linked to the basic geometrical technique.[13]

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[11] Actually, matters are somewhat more complex, which may give (and has given) rise to misreading of texts. A horizontal extension told to be "5 kùš" will often have to be read "5′ [nindan, i.e., a] kùš", alternatively, the expression "5 1 kùš" is used, meaning "5′ [nindan, i.e.,] 1 kùš". Both possibilities are used in the text discussed below.

[12] Often translations refer to this latter unit as a volume-sar, in order to keep the two apart. I shall avoid this convention, because it obscures an important aspect of Babylonian mathematical thought — cf. the discussion of problem N° 12 below.

[13] The existence of this link is most clearly though paradoxically seen in a case where the mensurational rectangle serving as a pretext for the problem appears to be distinguished from the rectangle serving the procedure ([TMS] XVI A; cf. corrected text in [HØYRUP 1990], 300). In the proof, the "true width", the width of the imaginary *real*

The tablet to be discussed below deals with rectangular parallelepipedal excavations (túl-sag), the horizontal dimensions of which are also designated uš and sag. The depth is designated GAM, a term of more specific but not idiosyncratic use.

The area spanned by "length" and "width" is not designated in the present text by the usual term a-šà, originally "field". Instead, and corresponding to the character of the problems, the sign KI (for Akkadian *qaqqarum*) is used, meaning ground, foundation or (here) "floor". The volume of the excavation is referred to through the [amount of] "earth" (saḫar) which has been excavated.

The various terms indicating the structure of problems and procedure in the text below need not be listed since they are easily understood in context. At this place it should only be pointed out that terms like en-nam "what", *mala* "so much as", *tammar* "you see" etc. are highly standardized, in general or at least in widespread use. The procedure itself has two names, *epēšum* and *nēpešum*, used respectively to announce the procedure and to tell that it has been performed; the first may be translated "the making", the second more clumsily as the "having-been-made", as done below.

3 The actual tablet

The tablet to be scrutinized in the following is BM 85200 + VAT 6599 — which means that one part of it is conserved in the British Museum and another in the Berlin collection of Vorderasiatische Texte. Below, line numbers from the Berlin fragment will be labeled by an asterisk *, while unlabeled numbers refer to the BM fragment.

The exact provenience of the tablet is unknown. Basing himself on the *ductus*, NEUGEBAUER dated the tablet to the late Old Babylonian period, which was confirmed by GOETZE ([MCT], 150f), who showed its spellings to be characteristic of his "6th group", "Northern modernizations of southern (Larsa) originals". Certain writing errors in the tablet demonstrate, furthermore, that the tablet is not the original modernization but a copy (e.g., the characteristic copyist's omissions in obv. II, 14 and rev. II, 4).

The text contains 30 problems, all of which deal with a túl-sag, i.e. (as made clear by the mathematical context), a rectangular parallelepipedal excavation (I shall use the translation "cellar"). Some problems have the mathematical structure of second-degree equations, and are in fact solved by means of the characteristic second-degree cut-and-paste techniques; others are effectively of the third degree, and are correspondingly solved by other means (factorization and recourse to a table, as we shall see). It is thus obvious that the Babylonian calculators knew the practical difference between the two algebraic degrees. It is equally obvious, however, that the characteristic feature shared by all problems of the tablet is the geomet-

rectangle, is multiplied by *1* before it is multiplied by "as much as there is of widths" (i.e., the coefficient of the width).

ric configuration dealt with. As we shall see in chapter 7, this primacy of geometric constitution over algebraic structure holds even on lower levels, which shall provide us with clues to the technique of didactical exposition.

The text was published, translated and discussed by NEUGEBAUER in [MKT] I, 193ff, and [MKT] II, Tafeln 7+39, with corrections [MKT] III, 54f. Other (partial) discussions of interest are [VOGEL 1934, TH.-D. 1937, GANDZ 1937], and [TH.-D. 1940] (where further bibliographic information is found on p. 1). VOGEL's treatment of the cubic problems takes a geometric approach; the others are all based on the customary arithmetical interpretation.

4 The text

The following transliteration builds on NEUGEBAUER's ([MKT] I, 193ff) with corrections suggested by THUREAU-DANGIN and mostly accepted by NEUGEBAUER; many restitutions of damaged passages also go back to [MKT] or to [TMB]. Problems the text of which is too incomplete to allow any meaningful attempt at reconstruction (N^{os} 1–4, 10–11 and 29) have been omitted.

The translation is my own, building on the results explained in chapters 1–2, and following the principle of "conformal translation" as set forth in [HØYRUP 1990], 60–62 (with the exception that no typographic distinction is made between translations from syllabic Akkadian and Sumerograms, and with the extra feature that italics are used to indicate translation of reconstructed passages). The aim of conformality is to obtain a translation where it is clear what precisely is told in the original text and what not, and where the conceptual distinctions of the original (e.g. between different additive procedures) are still visible. The basic tool is the use of "standard translations", where "all words except a few key terms are rendered by English words; a given expression is in principle always rendered by the same English expression, and different expressions are rendered differently with the only exception that well-established logographic equivalence is rendered by coinciding translation [...], while possibly mere ideographic equivalence is rendered by translational differentiation. Terms of different word class derived from the same root are rendered (when the result is not too awkward) by derivations from the same root [...]. Furthermore, syntactical structure and grammatical forms are rendered as far as possible by corresponding structure and grammatical forms; the simple style of the mathematical texts make this feasible" ([HØYRUP 1990], 61; the standard translations used in the present paper are, a couple of newcomers apart, those of this earlier publication).

As a preliminary philological commentary, two features concerning the way the text is written may be mentioned: Firstly, certain Akkadian words are written occasionally in abbreviated form, e.g. *šu-tam*⟨*-ḫir*⟩, *ta*⟨*-mar*⟩, *i*⟨*-ši*⟩ (the same holds for the Sumerian terms ba⟨-zi⟩ and íb⟨-si$_8$⟩ in obv.

II, 30 and rev. I, 6). Secondly, Sumerograms are written either with an Akkadian phonetico-grammatical complement (KI^{ri}) or, more often, with a Sumerian complement (gar-ra, daḫ-ḫa, sum-mu).

Specific commentary is given in footnotes to the text.

Obv. I

N^{o} 5

14*. [túl-sag *ma-la* uš GAM-*ma* saḫar-ḫi-a ba-zi KI^{ri} *ù* saḫar-ḫi-a UL.GAR 1,10]
A cellar. So much as the length: The depth. The earth I have torn out. My floor and the earth I have accumulated, 1°10′

15*. [...]

16*. [...][14]

1. [...uš sa]g en-nam
...*length and wid*th, what?

2. ...⌈3 *ta-mar*⌋ $\frac{1}{2}$ 3 *ḫe-pé* 1,30 *ta-mar*
...*3 you see.* $\frac{1}{2}$ of 3 break. 1°30′ you see,

3. ...[igi 1,30 du$_8$-a] 40 *ta-mar* bal sag igi 12 bal GAM du$_8$-a
...*the igi of 1°30′ detach*, 40′ you see, the conversion of the width. The igi of 12, the conversion of the depth, detach;

4. [5 *ta-mar a-na* 1⌉ *i-ši* 5 *ta-mar a-na* 40 *i-ši* 3,20 *ta-mar*
5′ you see. To 1 raise, 5′ you see. To 40′ raise, 3′20″ you see.

5. ⌈3,20⌉ *a-na* 5 *i-ši* 16,40 *ta-mar* igi 16,40 du$_8$-a 3,36 *ta-mar* 3,36
3′20″ to 5′ raise, 16″40‴ you see. The igi of 16″40‴ detach, 3‵ 36 you see. 3‵ 36

6. *a-na* 1,10 *i-ši* 4,12 *ta-mar* 6 íb-si$_8$ 6 *a-na* 5 *i-ši* 30 *ta*⟨*-mar*⟩ 6 *a-na* 3,20 *i*⟨*-ši*⟩
to 1°10′ raise, 4‵ 12 you see, 6 the equilateral. 6 to 5′ raise, 30′ you see. 6 to 3′20″ raise,

7. 20 sag 6 *a-na* 1 *i-ši* 6 *ta-mar* GAM *ki-a-am*
20′, the width. 6 to 1 raise, 6 you see, the depth. So

8. *ne-pé-šum*
the having-been-made.

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[14] It is not quite clear whether problem N^{o} 5 begins in line 14*, as suggested by NEUGEBAUER, or only in line 15* or even 16*, as suggested by THUREAU-DANGIN ([TMB], p. 11). Traces suggesting the end of the term [*ne-pé-šu*]*m* in line 13* support NEUGEBAUER's assumption; no other statements, on the other hand, extend over more than two lines, which supports THUREAU-DANGIN.

N° 6

9. túl-sag *ma-la* uš GAM-*ma* 1 saḫar-ḫi-a ba-zi KI^{ri} *ù* saḫar-ḫi-a UL.GAR 1,10 uš *ù* sag 50[15] uš sag en⟨-nam⟩
A cellar. So much as the length:[16] the depth. 1 the earth[17] I have torn out.[18] My floor and the earth I have accumulated, 1°10′. Length and width, 50′. Length, width, what?

10. za-e 50 *a-na* 1 bal *i-ši* 50 *ta-mar* 50 *a-na* 12 *i-ši* 10 *ta-mar*
You, 50′ to 1, the conversion, raise, 50′ you see. 50′ to 12 raise, 10 you see.

11. 50 *šu-tam⟨-ḫir⟩* 41,40 *ta-mar a-na* 10 *i-ši* 6,56,40 *ta-mar* igi-*šu du*$_8$*-a* 8,38,24 *ta⟨-mar⟩*
Make 50′ confront itself, 41′40″ you see; to 10 raise, 6°56′40″ you see. Its igi detach, 8′38″24‴ you see;

12. *a-na* 1,10 *i-ši* 10,4,48[19] *ta-mar* 36 24 42 íb-si$_8$
to 1°10′ raise, 10′4″48‴ you see, 36′, 24′, 42′ the equilaterals.

13. 36 *a-na* 50 *i-ši* 30 uš 24 *a-na* 50 *i-ši* 20 sag 36 *a-na* 10 6 GAM
36′ to 50′ raise, 30′, the length. 24′ to 50′ raise, 20, the width; 36′ to 10 raise, 6, the depth.

14. [*n*]*e-pé-šum*
*The h*aving-been-made.

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[15] This additive use of a mere "and" is rare but not unprecedented — cf. also rev. I, 1. YBC 4714 ([MKT] I, 487–492) offers a number of analogous examples, together with parallels which suggest that we have to do with an abbreviation "⟨accumulation of⟩ *a* and *b*". The controversy between VAN DER WAERDEN and BRUINS (see [V.D. WAERDEN 1962], 74) over the philological possibility of an interpretation of AO 6770 N° 1 (originally proposed in [GANDZ 1948], 38f, a fact not noticed by any of the contestants) depending on the assumed additive use of *ù* could thus have been settled long before it arose.

[16] I read MA as the Akkadian particle *-ma*. If this reading is correct, the structure of the passage is rendered most clearly when the ":" translating *-ma* is put in this place. It is, however, possible that the sign is simply a phonetic complement indicating that the preceding GAM is to be read gam, not gúr. GAM-*ma* is then to be replaced throughout the tablet by gam-ma, and the translation ": the depth" by ", the depth".

I prefer the first reading because the affix is found invariably when GAM closes an expression beginning with *ma-la*, and never in the final section of the problem when its numerical value is stated, nor in questions for this value. Such systematics is not found in other cases where a Sumerian phonetic indicator is used — compare the use of daḫ-ḫa in obv. II, 6* with that of daḫ in obv. II, 13*.

[17] The volume of earth removed *is* in fact 1 volume sar. The fact, however, is not used to solve the problem, and if it is taken into account, the problem is over-determined. Similarly in N° 7. Cf. below, chapter 7.

[18] A more idiomatic translation would be "removed" or "dug out". It is, however, worthwhile observing that the text uses the same term for digging out earth as for mathematical "subtractions".

[19] Written with a conspicuous space between 10 and 4 to distinguish 10,4 from 14.

N° 7

15. túl-sag *ma-la* uš GAM-*ma* 1 saḫar-ḫi-a ba-zi [K]I^{ri} *ù* saḫar-ḫi-a UL⟨.GAR⟩ 1,10 uš u-gù sag 10 dirig
A cellar. So much as the length: the depth. 1 the earth[20] I have torn out. My *fl*oor and the earth I have accumulated, 1°10′. Length over width 10′ goes beyond.

16. za-e 1 *ù* 12 [b]al gar-ra 10 ⌈dirig *a-n*⌉*a* 1 *i-ši* 10 *ta-mar a-na* 12 *i-ši* 2 *ta-mar*
You, 1 and 12, the *conv*ersions, pose. 10′ the *going-beyond to* 1 raise, 10′ you see; to 12 raise, 2 you see.

17. 10 *šu-tam⟨-ḫir⟩* 1,40 *ta-mar a-na* 2 *i-ši* 3,2[0 *t*]*a-mar* igi 3,20 du$_8$-a 18 *ta-mar*
10′ make confront itself, 1′40″ you see; to 2 raise, 3′2*0*″ *you* see. The igi of 3′20″ detach, 18 you see;

18. *a-na* 1,10 *i-ši* 21 *ta-mar* 3 2 21$^{(sic)}$ íb-si$_8$ [10 *a-na* 3 *i*]*-ši* 30 uš
to 1°10′ raise, 21 you see, 3, 2, 21 ⟨error for 3°30′⟩ the equilaterals. *10′ to 3* raise, 30′, the length.

19. 10 *a-na* 2 *i-ši* 20 sag 3 *a-na* 2 *i-ši* ⌈6⌉ *ta-mar* [6] GAM
10′ to 2 raise, 20′, the width. 3 to 2 raise, 6 you see, *6,* the depth.

20. *ne-pé-šum*
The having-been-made.

N° 8

21. túl-sag *ma-la* uš GAM-*ma* saḫar-[ḫi]-a ba-zi KI^{ri} *ù* saḫar-ḫi-a UL.GAR-*ma* 1,10 30 uš sag e[n-nam]
A cellar. So much as the length: The depth. The ear*th* I have torn out. My floor and the earth I have accumulated: 1°10′. 30′, the length. The width, w*hat*?

22. za-e 30 uš *a-na* 12 *i-ši* 6 *ta-mar* GAM 1 *a-na* 6 daḫ-ḫa 7 *ta-mar*
You, 30′, the length, to 12 raise, 6 you see, the depth. 1 to 6 append, 7 you see.

23. igi 7 nu du$_8$-a en-nam *a-na* 7 gar-ra *ša* 1,10 sum-mu 10 gar-ra igi 30 uš du$_8$-a
The igi of 7 is not detached. What to 7 shall I pose which 1°10′ gives me? 10′ pose. The igi of 30′ detach,

24. 2 *ta-mar* 10 *a-na* 2 *i-ši* 20 sag *ta-mar*
2 you see. 10′ to 2 raise, 20′, the width, you see.

25. *ne-pé-šum*
The having-been-made.

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[20]Once more, a value which is correct but not used.

N° 9

26. túl-sag *ma-la* uš GAM-*ma* saḫar-ḫi-a ba-zi KI^{ri} *ù* saḫar-ḫi-a UL.GAR-*ma* 1,10 20 sag uš ⟨en-nam⟩
A cellar. So much as the length: the depth. The earth I have torn out. My floor and the earth I have accumulated: 1°10′. 20′, the width. The length, *what*?

27. za-e 20 *a-na* 12 *i-ši* 4 *ta-mar* 4 *a-na* 1,10 *i-ši* 4,40 *ta-mar*
You, 20′ to 12 raise, 4 you see. 4 to 1°10′ raise, 4°40′ you see.

28. $\frac{1}{2}$ 20 sag *ḫe-pé* 10 *ta-mar* 10 *šu-tam-ḫir* 1,40 *ta-mar a-na* 4,40 daḫ-ḫa
$\frac{1}{2}$ of 20, the width, break, 10′ you see. 10′ make confront itself, 1′40″ you see; to 4°40′ append,

29. 4,41,40 *ta-mar* 2,10 íb-si$_8$ 10 *ša* ì-kú-kú ba-zi-ma
4°41′40″ you see, 2°10′ the equilateral. 10′ which you have made span tear out:

30. 2 *ta-mar* igi 4 du$_8$-a 15 *ta-mar a-na* 2 *i-ši*
2 you see. The igi of 4 detach, 15′ you see; to 2 raise,

31. 30 *ta-mar* {erasure} uš
30′ you see, the length.

32. *ne-pé-šum*
The having-been-made.

Obv. II

N° 12

5*. túl-sag *ma-la* uš GAM-*ma* saḫar-ḫi-a ba-zi KI^{ri} *ù* saḫar-ḫi-a U[L.GAR]
A cellar. So much as the length: The depth. The earth I have torn out. My floor and the earth I have ac*cumulated,*

6*. igi 7 gál *él-qé a-na* KI^{ri} daḫ-ḫa-*ma* 20 *ta*⟨*-mar*⟩ 30 [uš]
the 7th part I have taken, to the floor I have appended: 20′ you see. 30′ *the length.*

7*. za-e 30 *a-na* 12 *i-ši* 6 *ta-mar* GAM 1*a-na* ⌊6 daḫ-ḫa]
You, 30′ to 12 raise, 6 you see, the depth. 1 to 6 *append,*

8*. 7 *ta-mar* igi 7 gál *le-qé* 1 *ta-mar* 1 *ù* 1 U[L.GAR]
7 you see. Its 7th part take, 1 you see. 1 and 1 ac*cumulate,*

9*. 2 *ta-mar* igi 2 du$_8$-a 30 *ta-mar* 30 *a-na* 20 UL.GAR-⌊*i*$^{!}$*-ši*]
2 you see. The igi of 2 detach, 30′ you see, 30′ to 20′ the accumulation r*aise,*

10*. 10 *ta-mar* igi 30 uš du$_8$-a 2 *ta-mar* 2 *a-na* 10 *i-š*[*i* 20 sag]
10′ you see. The igi of 30′, the length, detach, 2 you see. 2 to 10 rai*se, 20′ the width.*

11*. *ne-pé-šum*
The having-been-made.

N° 13

12*. túl-sag *ma-la* uš GAM-*ma* saḫar-ḫi-a ba-zi *qá-qá-ri ù* saḫar-ḫi-a UL.[GAR]
A cellar. So much as the length: the depth. The earth I have torn out. My floor and the earth I have accumulated,

13*. 1,10 igi 7 gál-*šu él-qé a-na* KI^{ri} daḫ 20 20 sag
1°10′.[21] Its 7th part I have taken, to my floor I have appended, 20′. 20′, the width.

14*. za-e 20 *a-na* 7 *i-ši* 2,20 *ta-mar* 20 sag *a-na* 12 *i-ši*
You, 20′ to 7 raise, 2°20′ you see. 20′, the width, to 12 raise,

15*. 4 *ta-mar 4* a-na *2,20 i-ši* 9,20 *ta-mar a-na* 7 1 daḫ-ḫ[a]
4 you see. 4 to 2°20′ raise, 9°20′ you see. To 7, 1 appen*d*,

16*. 8 *ta-mar* 20 *a-na* 8 *i-ši* 2,40 *ta-mar* $\frac{1}{2}$ 2,40 *ḫe-pé* [*šu-tam*⟨*-ḫir*⟩]
8 you see. 20′ to 8 raise, 2°40′ you see. $\frac{1}{2}$ of 2°40′ break, *make confront itself*,

17*. 1,46,40 *ta-mar a-na* 9,20 daḫ-ḫa 11,6,40 *t*[*a-mar*]
1°46′40″ you see, to 9°20′ append, 11°6′40″ yo*u see,*

18*. 3,20 íb-si$_8$ 1,20 *ša* ì-kú-kú ba-zi 2 *ta*[*-mar*]
3°20′ the equilateral. 1°20′ which you have made span tear out, 2 you *see.*

19*. igi 4 du$_8$-a 15 *ta-mar* 15 *a-na* 2 *i-ši* 30 [uš]
The igi of 4 detach, 15′ you see. 15′ to 2 raise, 30 *the length.*

20*. *ne-pé-š*[*um*]
The having-been-ma*de.*

N° 14

1. túl-sag *ma-la* igi uš *ma-la* igi-bi sag *ma-la* igi {u-gù igi-bi dirig}[22]
A cellar. So much as the igûm, the length. So much as the igibûm, the width. So much as the igûm {over the igibûm goes beyond}:

2. GAM-*ma* 1⌊6 saḫar-ḫi-a ba-z⌋i uš sag *ù* GAM en-nam

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[21]This value, again, is correct but not used.

[22]With this emendation, the following calculation (as reconstructed by NEUGEBAUER) is correct. The wrong formulation (which is not solvable in rational numbers, and from which the *ša* of problems 15 and 17 is absent) seems to be a contamination from the following problem.

The depth. 1*6 of earth I have torn* out. Length, width, and depth, what?

3. za-e igi 12 du$_8$-a [5 *ta-*]*mar* 5 [*a-na* 16] *i-š*[*i* 1,2]0 *ta*⟨*-mar*⟩
You, the igi of 12 detach, *5′ you* see. 5′ *to 16* rai*se, 1°2*0′ you see,

4. 1,20 igi igi 1,2[0 du$_8$-a 45 *ta-m*]*ar* 4⟨5⟩ igi-bi [16] GAM
1°20′ the igi. The igi of 1°2*0′ detach, 45′ you s*ee, 45′ the igibûm. *16* the depth.

5. *ne-*[*pé*]*-šum*
The having-*been*-made.

N° 15

6. túl-sag *ma-la* igi uš *ma-*[*la* igi-bi sa]g *ma-la ša* igi u-gù igi-bi dirig ⟨GAM-*ma*⟩
A cellar. So much as the igûm, the length. So muc*h as the igibûm, the wi*dth. So much as that which the igûm over the igibûm goes beyond: *The depth.*

7. 36 saḫar-ḫi-a ba-zi-*m*[*a* igi igi-bi *ù* GAM] en-nam
36 of earth I have torn out*: Igûm, igibûm and depth*, what?

8. za-e igi 12 du$_8$-a [5 *ta-mar* 36⌋ *a-na* 5 {⌊2 *¿*TI$^?$...⌋} *i*⟨*-ši*⟩
You, the igi of 12 detach, *5′ you see. 3*6 to 5′ {...!} ra*ise*,

9. 3 *ta-mar* $\frac{1}{2}$ 3 *ḫ*[*e-pé* 1,30 *ta-mar*] 1,30 igi [40 igi-bi 36⌋ GAM
3 you see. $\frac{1}{2}$ of 3 b*reak, 1°30′ you see,* 1°30′ the igûm. *40′ the igibûm, 36* the depth.

10. *ne-*[*p*]*é-š*[*um*]
The having-*b*een-m*ade*

N° 16

11. túl-sag *ma-la* igi uš ma-la [igi-bi sag] *ma-la* nigin[23] igi *ù* igi-bi GAM-*ma*
A cellar. So much as the igûm, the length. So much as *the igibûm, the width.* So much as the total of igûm and igibûm: the depth.

12. 26 saḫar-ḫi-a ba-zi igi igi-bi *ù* GAM en-nam
26 of earth I have torn out. Igûm, igibûm, and depth, what?

13. za-e igi 12 du$_8$-a 5 *ta-mar* 5 *a-na* 26 *i-ši*
You, the igi of 12 detach, 5′ you see; 5′ to 26 raise,

14. 2,10 *ta-mar* $\frac{1}{2}$ 2,10 *ḫe-pé šu-tam-*⟨*ḫir*⟩ 1,10,25 *ta-ma*[*r*] ⟨1 *i-na* 1,10,25 ba-zi 10,25 *ta-mar*⟩[24]

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[23]According to its use an abbreviation for šu-nigin, the "total" or "summa summarum" of accounts.

[24]The omission of this passage is one of several indications that the tablet is copied from another tablet, and is neither an original nor the direct reproduction of an oral

2°10′ you see. $\frac{1}{2}$ of 2°10′ break, make confront itself, 1°10′25″ you se*e.*
1 from 1°10′25″ tear out, 10′25″ you see,

15. 25 íb-si$_8$ *a-na* ⟨1,⟩5 daḫ-ḫa *ù* ba-zi 1,30 *ù*[25] 40 *t*[*a-mar*]
25, the equilateral, to *1*°5′ append and tear out. 1°30′ and 40′ yo*u* *see;*

16. 1,30 igi 40 igi-bi 26 GAM
1°30′ the igûm; 40′ the igibûm; 26 the depth.

17. *ne-pé-šum*
The having-been-made.

N° 17

18. túl-sag *ma-la* igi uš *ma-la* igi-bi sag *ma-la ša* igi u-gù igi-bi d[irig]
A cellar. So much as the igûm, the length. So much as the igibûm, the width. So much as that which the igûm over the igibûm go*es beyond*

19. *i-na* igi ba-zi GAM-*ma* 6 saḫar-ḫi-a ba-zi igi *ù* igi-b[i en-nam]
from the igûm I have torn out: the depth. 6 of earth I have torn out. Igûm and igib*ûm, what?*

20. za-e igi 12 du$_8$-a 5 *ta-mar a-na* 6 *i-ši* 30 *ta-mar*
You, the igi of 12 detach, 5′ you see; to 6 raise, 30′ you see.

21. [i]gi 3[0 d]u$_8$-a 2 *ta-mar* 2 igi 30 igi-bi 6 GAM
*The i*gi of 3*0′ de*tach, 2 you see. 2, the igûm, 30′, the igibûm. 6, the depth.

22. *ne-pé-šum*
The having-been-made.

N° 18

23. túl-sag *ma-la* igi uš *ma-la* igi-bi sag *ma-la* nigin igi igi-b[i GAM-*m*]*a* 30 s[aḫar-ḫi-a ba-zi]
A cellar. So much as the igûm, the length. So much as the igibûm, the width. So much as the total, igûm, igib*ûm: the depth*. 30 of ea*rth I have torn out.*

24. za-e igi 12 du$_8$-a 5 *t*[*a-ma*]*r* 5 *a-na* 30 saḫar-ḫi-a *i-ši*
You, the igi of 12 detach, 5′ y*ou s*ee. 5′ to 30, the earth, raise,

25. 2,30 *ta-mar* $\frac{1}{2}$ 2,30 *ḫe-pé šu*[*-tam-ḫir* 1,33,4]5 *ta*[*-mar*]
2°30′ you see. $\frac{1}{2}$ of 2°30′ break, make *confront* itself, *1°33′4*5″ you *see.*

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presentation. Cf. the corresponding omission in rev. II, 4, equally called forth by the presence of two identical sequences of signs close to each other.

[25]With some hesitation, I follow THUREAU-DANGIN's reading of the sign as the first part of an *ù*. The other possibility is a full igi (NEUGEBAUER's reading).

26. 1 *i-na* 1,33,45 ba-zi 3[3,4]5 *ta-mar* ⌊4⌋5 íb-si$_8$
1 from 1°33′45″ tear out, *3*3′*4*5″ you see, *4*5′ the equilateral.

27. *a-na* 1,15 daḫ-ḫa *ù* ba-zi 2 *ù* 30 *ta-m*[*ar*]
To 1°15′ append and tear out, 2 and 30′ you *see*.

28. *ne-pé-šum*
The having-been-made.

N^o^ 19

29. túl-sag *ma-la* igi uš *ma-la* igi-bi sag *ma-la* {erasure} igi-bi GAM-*ma*
A cellar. So much as the igûm, the length. So much as the igibûm, the width. So much as the igibûm: the depth.

30. 20 saḫar-ḫi-a ba⟨-zi⟩ igi igi-bi *ù* GAM en-nam
20 of earth I have torn out. Igûm igibûm, and depth, what?

31. za-e igi 12 du$_8$-a *a-na* 20 *i-ši* 1,40 *ta-*{*m..*}*mar*
You, the igi of 12 detach, to 20 raise, 1°40′ you see.

32. 1,40 igi 36 igi-bi 20 GAM
1°40′, the igûm. 36′, the igibûm. 20, the depth.

33. *ne-pé-šum*
The having-been-made

Rev. I

N^o^ 20

1. túl-sag *ma-la uš-tam-*⟨*ḫir*⟩ *ù* 7 kùš GAM-*ma* 3,20 saḫar-ḫi-a ba-zi
A cellar. So much as I have made confront itself, and 7 cubit: The depth. 3′20″ of earth I have torn out.

2. uš sag *ù* GAM en-nam
Length, width, and depth, what?

3. za-e igi 7 gál 7 *le-qé* 1 *ta-mar* igi 12 du$_8$-a 5 *ta-mar*
You, the 7th part of 7 take, 1 you see. The igi of 12 detach, 5′ you see.

4. 5 *a-na* 1 *i-ši* 5 *ta-mar* 5 *a-na* 12 *i-ši* 1 *ta-mar*
5′ to 1 raise, 5′ you see. 5′ to 12 raise, 1 you see.

5. 5 *šu-tam*⟨*-ḫir*⟩ 25 *a-na* 1 *i-ši* 25 *ta-mar* igi 25 du$_8$-a 2,24
5′ make confront itself, 25″ to 1 raise, 25″ you see. The igi of 25″ detach, 2′24

6. *ta-mar* 2,24 *a-na* 3,20 saḫar-ḫi-a *i-ši* 8 *ta-mar* en-nam íb⟨-si$_8$⟩
you see. 2′24 to 3′20″, the earth, raise, 8 you see. What the equilaterals?

7. 1 1 8 íb-si$_8$ 5 *a-na* 1 *i-ši* 5 *ta-mar* 5 kùš uš
1, 1, 8, the equilaterals. 5′ to 1 raise, 5′ you see, 5′, a cubit, the length.

8. 8 *a-na* 1 *i-ši* 8 kùš {erasure} GAM
8 to 1 raise, 8 cubits the depth.

9. *ne*-[*p*]*é-š*[*um*]
The having-*been*-mad*e*

N° 21[26]

10. túl-sag *ma-la uš-tam-ḫir ù*[27] 7 [kù]š GAM-*ma* 13 ⟨[3,15]⟩ saḫar ba-zi
A cellar. So much as I have made confront itself, and 7 *cub*its: the depth. *3°15′* of earth I have torn out.

11. uš sag *ù* GAM en-nam
Length, width, and depth, what?

12. za-e [*ki-m*]*a meḫ-ri-ma e-pu-uš* 4,48 ⟨[7,48]⟩ en-nam íb-si$_8$
You, *as mu*ch as the counterpart: make,[28] *7' 48*, what the equilaterals?

13. ⌈6⌉ 6 13 íb-si$_8$ 6 *im*⟨*-ta-ḫar*⟩[29] 13 GAM
6 6 13 the equilaterals. 6 confronts itself, 13 the depth.

14. *ne-pé-šum*
The having-been-made.

N° 22

15. túl-sag *ma-la uš-tam-ḫir* GAM-*ma* 1,30 saḫar-ḫi-a ba-zi uš sag [*ù*] GAM ⟨en-nam⟩
A cellar. So much as I have made confront itself, the depth. 1°30′ of earth I have torn out. Length, width, *and* depth, *what?*

16. za-e igi 12 du$_8$-a 5 *ta-mar* 5 *a-na* 1,30 *i-ši* ⌊7,30 *ta-mar*⌋
You, the igi of 12 detach, 5′ you see. 5′ to 1°30′ raise, *7′30″ you see*

17. 30 íb-si$_8$ 30 *a-na* 1 *i-ši* 30 *im-ta-ḫar* 30 *a*[*-na* 12⌉ *i*⟨*-ši*⟩ 6 GAM
30′ the equilateral. 30′ to 1 raise, 30′ confronts itself. 30′ t*o 12* raise, 6 the depth.

18. *ne-pé-šum*
The having-been-made

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[26]The text as it stands is corrupt. In ⟨[...]⟩ I give THUREAU-DANGIN's corrections as proposed in his [TH.-D. 1936], 181, from where the reading of line 12 is also taken.

[27]In this place, a GAM seems to have been written first. Afterwards, the scribe has discovered the mistake and covered it by the *ù*.

[28]This clumsy phrase results from the use of standard translations. A more idiomatic version would be "proceed as in the corresponding (i.e., the preceding) case".

[29]This change from íb-si$_8$ to *mitḫurum* and the differentiation between the two demonstrates clearly that the former is no logogram for the latter (as claimed consistently by THUREAU-DANGIN, even in his transcription of this passage).

N° 23

19. túl-sag *ma-la uš-tam-ḫir ù* 1 kùš dirig GAM-*ma* 1,45 saḫar-ḫi-a [ba]-zi
A cellar. So much as I have made confront itself, and 1 cubit, going beyond: The depth. 1°45′ of earth *I have* torn out.

20. za-e 5 dirig *a-na* 1 bal *i-ši* 5 *ta-mar a-na* 12 *i-š[i* 1⌋ *ta-mar*
You, 5′, going beyond, to 1, the conversion, raise, 5′ you see; to 12 rai*se, 1* you see.

21. 5 *šu-tam⟨-ḫir⟩* 25 *ta-mar* 25 *a-na* 1 *i-ši* 25 *ta-mar* igi ⌊25 du$_8$-a]
5′ make confront itself, 25″ you see. 25″ to 1 raise, 25″ you see. The igi of *25 detach,*

22. 2,24 *ta-mar* 2,24 *a-na* 1,45 *i-ši* 4,12 [*ta-mar*]
2` 24 you see. 2` 24 to 1°45′ raise, 4` 12 *you see.*

23. *i-na* íb-si$_8$ 1 daḫ-ḫa 6 *¿*1?[30] íb-s[i$_8$] ⌊6 *a-na 5*⌋ *i-*[*ši* 30] *ta⟨-mar⟩ im⟨-ta-ḫar⟩* 6$^{\text{sic}}$ GAM
from ⟨"in [the table]"? or an error for "to"⟩ the equilateral, 1 append. 6 *¿*1? the equil*aterals.* 6 *to 5′* ra*ise, 30′* you see, confronts itself. 6 ⟨error for 7⟩ the depth.

24. *ne-pé-š[um]*
The having-been-m*ade.*

N° 24

25. túl-sag 3,20 GAM-*ma* 27,46,40 saḫar-ḫi-a ba-zi uš u-gù sag 50 d[irig]
A cellar. 3°20′: The depth. 27°46′40 of earth I have torn out. The length over the width 50′ go*es beyond.*

26. za-e igi 3,20 GAM du$_8$-a 18 *ta-mar a-na* 27,46,40 saḫar-ḫi⟨-a⟩ *i-ši*
You, the igi of 3°20′, the depth, detach, 18′ you see; to 27°46′40″, the earth, you raise,

27. ⌊8⌋,20 *ta-mar* $\frac{1}{2}$ 50 *ḫe-pé šu-tam⟨-ḫir⟩* 10,25 *ta-mar*
8° 20′ you see. $\frac{1}{2}$ of 50′ break, make confront itself, 10′25″ you see;

28+1*. *a-na* 8,20 daḫ-ḫa [8,3⌋0,25 *ta-mar*
to 8°20′ append, *8°30′*25″ you see,

29+2*. 2,55 íb-si$_8$ *a-di* [2 gar-ra] *a-na* 1 daḫ-ḫa *i-na* 1 ba-zi
2°55′ the equilateral; until *2 pose*, (25′ which you have made span) to 1 append, from 1 tear out.

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[30] As possible alternative readings, NEUGEBAUER suggests "6 1 1" and "6 nindan", none of which make sense. "6 7" seems to be ruled out by the autography. [TMB] appears to regard the traces following "6" as an erasure, neglecting them entirely.

30+3*. 3,20 uš 2,30 sag *ta-mar*
3°20′ the length, 2°30′ the width you see.

31+4*. *ne-p*[*é-*]*šum*
The having-been-m*a*de.

N° 25

5*. túl-sag 3,20 GAM-*ma* 27,46,4[0 saḫar-ḫi-a ba-zi uš *ù* sag UL.GA]R 5,⌊50⌋
A cellar. 3°20′: the depth. 27°46′4*0″ of earth I have torn out. Length and width I have accumu*lated, 5°5*0′.*

6*. za-e igi 3,20 GAM du$_8$-a 18 *ta-mar* ⌊*a-na* 27,46,40 *i-ši*]
You, The igi of 3°20′, the depth, detach, 18′ you see; *to 27°46′40″ raise,*

7*. 8,20 *ta-mar* $\frac{1}{2}$ 5,50 *ḫe-pé šu-tam*⟨*-ḫir*⟩ ⌈8,30,25 *ta-mar*]
8°20′ you see. $\frac{1}{2}$ of 5°50′ break, make confront itself, *8°30′25″ you see.*

8*. 8,20 *i-na lìb-ba* ba-zi 10,2[5 *ta-mar* 25 íb-si$_8$]
8°20′ from the inside tear out, 10′2*5″ you see, 25′ the equilateral;*

9*. *a-na* 2,55 daḫ-ḫa *ù* ba-zi 3,20 [uš 2,30 sag]
to 2°55′ append and tear out, 3°20′ *the length, 2°30′ the width.*

10*. *ne-pé*[*-šum*]
The having-been-*made.*

N° 26

11*. túl-sag 3,20 GAM-*ma* 27,46,40 saḫar-ḫi-a [ba-zi *ša* sag u-gù GAM dirig $\frac{2}{3}$ uš]
A cellar. 3°20′ the depth. 27°46′40″ of earth *I have torn out. That which the width over the depth goes beyond, $\frac{2}{3}$ of the length.*

12*. za-e igi 3,20 du$_8$-a 18 *ta-mar a-na* 2[7,46,40 *i-ši*]
You, the igi of 3°20′ detach, 18′ you see; to 2*7°46′40″ raise,*

13*. 8,20 *ta-mar* 8,20 *a-na* 40 *i-ši* 5,33,[20 *ta*⟨*-mar*⟩ 3,20 GAM *a-na* 5 *i-ši* 16,40]
8°20′ you see. 8°20′ to 40′ raise, 5°33′ *20″ you see. 3°20′, the depth, to 5′ raise, 16′40″.*

14*. nigín-na $\frac{1}{2}$ 16,40 *ḫe-pé* 8,20 *ta-mar šu-tam*⟨*-ḫir*⟩ 1,[9,26,40 *a-na* 5,33,20 daḫ-ḫa]
Go around.[31] $\frac{1}{2}$ of 16′40″ break, 8′20″ you see, make confront itself, 1′ *9″26‴ 40⁗ to 5°33′20″ append.*

[31] Traditionally, this phrase (nigín-na used logographically for the verb *saḫārum* ("to turn/go around") has been understood as indicating a shift from one section of the procedure to the next. As suggested to me by AAGE WESTENHOLZ (private communi-

15*. en-nam íb-si$_8$ 2,31,40[sic] *a-di* 2 gar-ra 8⟨,20⟩ da[ḫ-ḫa *ù* ba-zi]
What the equilateral? 2°31′40″ ⟨error for 2°21′40″⟩ until 2 pose; 8′ *20″* app*end and tear out*

16*. 2,30 sag 2,13,20 *ta-mar* igi 40 du$_8$-a 1,30 *ta-mar* [*a-na* 2,13,20 *i-ši*]
2°30′, the width; (and) 2°13′20″ you see. The igi of 40′ detach, 1°30′ you see. *To 2°13′20″ raise,*

17*. 3,20 uš *ta-m*[*ar*]
3°20′ the length you s*ee.*

18*. *ne-pé-šum*
The having-been-made.

N° 27

19*. túl-sag 1,40 uš igi 7 gál *ša* uš u-gù sag dirig GAM-*ma* 1,40 saḫar-ḫ[i-a ba-zi]
A cellar. 1°40′ the length. The 7th part of that which length over width goes beyond: The depth. 1°40′ of eart*h I have torn out;*

20*. uš sag *ù* GAM en[-nam]
Length,[32] width, and depth, wh*at?*

21*. za-e 1,40 uš *a-na* 12 bal GAM *i-ši* 20 *ta*[*-mar*]
You, 1°40′, the length, to 12, the conversion of depth, raise, 20 you *see.*

22*. igi 20 du$_8$-a 3 *ta-mar* 3 *a-na* 1,40 s[aḫar-ḫi-a[33] *i-ši* 5 *ta-mar*]
The igi of 20 detach, 3′ you see. 3′ to 1°40′, the e*arth, raise, 5′ you see.*

23*. 7 *a-na* 5 *i-ši* 35 *ta-m*[*ar* $\frac{1}{2}$ 1,40 *ḫe-pé šu-tam*⟨*-ḫir*⟩ 41,40]
7 to 5′ raise, 35′ you s*ee.* $\frac{1}{2}$ *of 1°40′ break, make confront itself, 41′40″.*

24*. 35 ⌊*i-na lìb*⌋-[*bi* ba-zi 6,40 *ta-mar* 20 íb-si$_8$]
35′ f*rom inside tear out, 6′40″ you see, 20′ the equilateral.*

25*. *a-n*[*a* 50 daḫ-ḫa *ù* ba-zi 1,10 *ù* 30 sag ...]
T*o 50′ append and tear out, 1°10′ and 30′, the width.*

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cation), the geometric interpretation allows a much more concrete explanation, *viz* as "going around" a field which is being/has been constructed. Evidently, the traditional reading does not fit the present case, while the concrete understanding seems to give an important hint concerning the procedure — cf. the mathematical commentary.

[32] Already given.

[33] I prefer this reconstruction (proposed in [TH.-D. 1937], 11 and [TMB]) to NEUGEBAUER's, both because its fits the autography best, and because of the parallel to the procedure in N° 29, rev. II, 3–4. (It also happens to make much better sense of the procedure.)

26* [igi 7 gál 1,10 *le-qé* 10 GAM][34]
The 7th part of 1°10′ take, 10′, the depth.

27* [*ne-pé-šum*]
The having-been-made

Rev. II

N° 29

1. túl-sag 1,40 uš igi 7 *ša* uš u-gù sag dirig *ù* 2 kùš GAM-*ma* 3,20 [saḫ]ar-ḫi⟨-a⟩ ⟨ba-zi⟩
A cellar. 1°40′ the length. The 7th part of that which the length over the width goes beyond, and 2 kùš: the depth. 3°20′ of *ear*th *I have torn out.*

2. sag *ù* GAM en-nam
Width and depth, what?

3. za-e 1,40 uš *a-na* 12 bal GAM *i-ši* 20 *ta-mar* igi 20 du$_8$-a 3 *ta-mar*
You, 1°40′, the length, to 12, the conversion of depth, raise, 20 you see. The igi of 20 detach, 3′ you see;

4. 3 *a-na* 3,20 *i-ši* 10 *ta-mar* ⟨10 *a-na* 7 *i-ši* 1,10 *ta-mar*⟩ 10 dirig *a-na* 7 *i-ši* 1[,10 *t*]*a-mar*
3′ to 3°20′ raise, 10′ you see. *10′ to 7 raise, 1°10′ you see*. 10′ going beyond[35] to 7 raise, 1°*10′ yo*u see.

5. 1,40 uš *a-na* 1,10 daḫ-ḫa 2,50 *ta-mar* $\frac{1}{2}$ 2,⌈50 *ḫe-pé šu-tam*⌉-*ḫir*
1°40′, the length, to 1°10′ append, 2°50′ you see. $\frac{1}{2}$ of 2°*50′ break, make con*front itself.

6. 2,25 *ta-mar i-na* 2,25 1,10 ba-zi 50,25 *ta-mar*
2°25″ you see. From 2°25″ 1°10′ tear out, 50′25″ you see,

7. 55 íb-si$_8$ *a-na* 1,25 daḫ-ḫa *ù* ba-zi-ma
55′ the equilateral; to 1°25′ append and tear out:

8. 2,20 *ù* 30 sag *ta-mar* igi 7 gál 2,20 *l*[*e-q*]*é* 20 GAM
2°20′ and 30′, the width, you see. The 7th part of 2°20′ t*ake*, 20′, the depth.

9. *ne-pé-šum*
The having-been-made.

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[34] Reconstruction suggested by rev. II, 8.
[35] I.e., the 2 kùš of line 1.

N° 30

10. túl-sag 1,40 uš igi 7 gál *ša* uš u-gù sag dirig *ù* 1 kùš ba-l[al] GAM-*ma*
A cellar. 1°40′ the length. The 7th part of that which the length over the width goes beyond, and 1 kùš diminishing: the depth.

11. 50 saḫar-ḫi-a ba-zi sag *ù* GAM en-nam
50′ of earth I have torn out. The width and the depth, what?

12. za-e 1,40 uš *a-na* 12 bal GAM *i-ši* 20 *ta-mar* igi 20 du$_8$-a 3 *ta*⟨*-mar*⟩
You, 1°40′, the length, to 12, the conversion of depth, raise, 20 you see. The igi of 20 detach, 3′ you see;

13. 3 *a-na* 50 *i-ši* 2,30 *ta-mar* 2,30 *a-na* 7 *i-ši* 17,30 *t*[*a-mar*]
3′ to 50′ raise, 2′30″ you see. 2′30″ to 7 raise, 17′30″ you see.

14. 7 *a-na* 5 1 kùš *i-ši* 35 *ta-mar* 35 *i-na* 1,40 uš ba-zi
7 to 5′, 1 kùš, raise, 35′ you see. 35′ from 1°40′, the length, tear out,

15. 1,5 *ta-mar* $\frac{1}{2}$ 1,5 *he-pé* 32,30 *šu-tam*⟨*-ḫir*⟩ 17,36,15 *ta*⟨*-mar*⟩
1°5′ you see. $\frac{1}{2}$ of 1°5′ break, 32′30″ make confront itself, 17′36″15‴ you see,

16. *i-na lìb-bi* 17,30 ba-zi 6,15 *ta-mar* 2,30 íb-si$_8$
from the inside 17′30″ tear out, 6″15‴ you see; 2′30″ the equilateral

17. *a-na* 32,30 daḫ-ḫa *ù* ba-zi 35 *ù* 30 sag *ta-mar* 7 35 5 GAM[36]
to 32′30″ append and tear out, 35′ and 30′, the width, you see. (The) 7(th of) 35′, 5′ the depth.

18. *ne-pé-šum*
The having-been-made.

5 The single problem types

All problems of our tablet share the "length", the "width", and the "depth", which determine the "cellar" and are thus silently supposed to be at right angles to each other.[37] The volume of the cellar is represented by the amount of "earth" dug out, while the area of its base is spoken of as the "floor". As always, length and width are supposed to be measured in nindan, depth in kùš, and volume as well as area in sar (nindan^2 and nindan$^2 \cdot$ kùš, respectively). When the depth is stated to be equal to (e.g.) the width, this is meant to concern "real" or "physical" extension, not measuring numbers. This holds even when the depth is equal to a width defined as *igibûm* (**N° 19**). Length and width spoken of as *igûm* and *igibûm*, and hence apparently as a pair of numbers from the table of reciprocals, are thus

[36]This sequence of numbers could be filled out as "⟨igi⟩ 7 ⟨gál⟩ 35 ⟨*le-qé*⟩ 5 GAM, but is remarkable enough to stand in its original formulation.

[37]In the sense opposing, so to speak, "right" to "wrong" angles, corresponding to the label "true length" distinguishing the side 1ˋ 20 from the other length (the hypotenuse) in a right triangle 1ˋ – 1ˋ 20 – 1ˋ 40 in the tablet YBC 8633, obv. 8, rev. 2 ([MCT], 53; Neugebauer and Sachs make a mistaken correction in note 150); the width and the "true length" are those sides whose semi-product gives the area.

As often observed (e.g., in [Gandz 1939], 415ff), the Babylonian mathematical texts exhibit no trace of a concept of quantifiable angle.

still to be understood as palpable extensions fulfilling a specific condition concerning the area they span, not as mere numbers.

The use of "length" and "width" as terms for unknowns was almost as standardized in Old Babylonian "algebra" as the use of x and y in modern school algebra. In symbolic representations of the structure of problems it is therefore fitting to make use of these letters, and not of l and w. "Depth" is no similarly standardized unknown, and I shall therefore use d to represent the depth measured in nindan, and G for the depth measured in kùš; if "the depth is as much as the length", we thus have $d = x$, $G = 12d$.

5.1 The third degree

The ordering of problems in the tablet is not derived from principles of mathematical structure, and there is thus no reason to follow it in the discussion. Instead, I shall group problems together which make use of the same characteristic technique; it is evidently no coincidence that this will also be a grouping according to algebraic degree.

Of greatest interest are probably the genuine third-degree problems, characterized by the application of a sophisticated version of the *makṣarum* or "bundling" method spoken of in certain other texts (cf. also [HØYRUP 1985], 105.11f).

In the tablet YBC 8633 ([MCT], 53), a triangle with length 1ˋ 40 and width 1ˋ is regarded as a "bundle" of 3–4–5–triangles, corresponding to a linear scaling factor 20 (1ˋ $= 20 \cdot 3$, 1ˋ 40 $= 20 \cdot 5$). The other ("true") length is therefore found as the product of "20 the *makṣarum*" and 4.

The tablet YBC 6295 ([MCT], 42) deals with the "*makṣarum* of a [cube] root", actually with the way to find the cube root of a cubic number ($3°22'30''$) not listed in the table of cube roots. The way, again, is to compare to a more familiar standard cube, *viz* with $7'\ 30'' = (30')^3$, finding the ratio to be 27 and the linear scaling factor thus $\sqrt[3]{27} = 3$.

Judging from these examples, "bundling" is nothing but (or at least closely related to) the method of a single false position applied in two or three dimensions. As we shall see, it is also the method used (though in sophisticated versions, and without any reference to the name) for most of the third-degree problems of the present tablet.

Let us first look at **N° 6**. We are told that the length equals the depth ($d = x$), that the accumulation of earth and floor equals $1°10'$ ($xyG + xy = 1°\ 10'$), and that length and width equal $50'$ ($x + y = 50'$).

According to the normal conceptualization of 2nd-degree problems of the type "surface + sides", we must expect the sum of "earth and floor" to be imagined as the volume of the cellar prolonged downwards by an extra kùš (cf. Figure 1). This *a priori* expectation is confirmed by **N° 8**, which "appends" an extra kùš to the depth (obv. I, 22).

The first step in the procedure is the computation of the volume of a cube. That a volume and no mere product is involved is made clear by the distinction between multiplications: length and width are "confronted" as

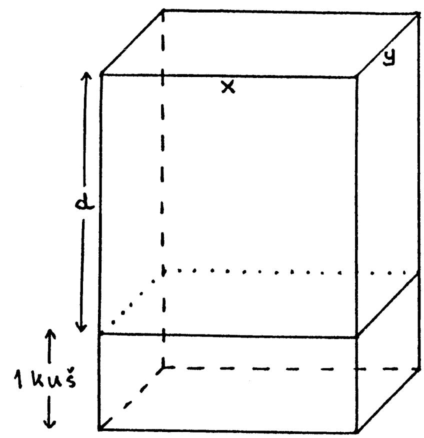

Figure 1

sides of a square, which is then "raised" to the height. The side of the cube is chosen as the sum of the length and the width of the cellar.

The treatment of the three dimensions is remarkably symmetric: all are found by a multiplication by the appropriate conversion factor: 1 for length and width (both thus 50′ [nindan]), and 12 for the depth (thus 10 kùš).

Next, the volume of the extended cellar is found by means of a customary "igi-division" to be $N = 10'4''48'''$ times the reference volume. This ordering of the computational steps is another indication that a concrete reference entity is involved; in the case of a mere normalization,[38] the volume of the cellar would (according to the habits known from other texts) have been divided by 50′, 50′ and 10 one after the other, not once and for all by their product. The "equilaterals" of the "quotient volume" N (actually the "sides" which are *not* equal) are given without explanation to be 36′, 24′ and 42′. What has to be looked for is, indeed, a factorization $N = p \cdot q \cdot r$ where $p + q = 1$, $r = p + 6'$ (6′ represents the extra kùš appended to the depth as measured by 10 kùš, i.e., by the depth of the reference volume).[39]

[38]As proposed by [Th.-D. 1940], 3, in an interpretation which otherwise seems to come close to the one presented here, apart from its arithmetical dressing (the formulation given in [TMB], xxxvff exhibits the difference more clearly). Even [MKT] I, 211 speaks of a "normal form".

[39]In his geometrical interpretation of this and the following problem, Vogel ([Vogel 1934], 91–93) does not build on the actual sequence of operations but rather

The length, finally, is found as 36′ times the length of the reference cube, i.e., as $36' \cdot 50'$ [nindan] = 30′ [nindan]; the width is found to be $24' \cdot 50'$ [nindan] = 20 [nindan], and the depth as $36' \cdot 10$ [kùš] = 6 kùš (while the extended depth would have been $42' \cdot 10$ kùš = 7 kùš).

N° 7 is a close parallel; this time, however, the excess of length over width is given (and equal to 10′ [nindan]). The reference volume is a cube with sides equal to this excess. It is constructed and found to be 3′20″ sar, yielding a quotient volume equal to 21; this is told without explanation to have the "equilaterals" 3, 2 and 21 (mistaken for 3°30′).[40] Since the side of the reference volume equals $x - y$ and is 2 kùs it is indeed required that $p - q = 1$, $r - p = 30'$.

The two factorizations into sets of "equilaterals" may have been found by systematic search — even though the number of possible factorizations is infinite (Babylonian sexagesimals made no distinction between integers and non-integers), start from the simplest possibilities combined with a bit of mathematical reflection would soon lead forward.[41] However, the complete absence of calculation (e.g., of the 6′ and 30′ representing $r - p$ in the two problems) and justification — as compared, e.g., to the careful multiplication with a factor *1* in lines 10 and 16 — suggests that they are drawn from the sleeves. Since the problems have been constructed backwards from known dimensions this will have been quite feasible. On the other hand, the fact that even the factor r for the extended depth is listed — though of no use further on — demonstrates that what may perhaps be drawn from the sleeves is still meant as a solution by factorization.

N° 23 is of a similar though simpler structure. It is told that the depth exceeds "as much as I have made confront itself" by 1 kùš, which means that length and width confront each other as sides of a square; thus $x = y$, $d = x + 1$ kùš. Furthermore, the volume is $xyG = x \cdot y \cdot 12d = 1°45'$; the same structure would have come about if we had added the base and a cubic volume.

This time, the reference volume is a cube with side equal to the excess of depth over length, i.e., to 1 kùš = 5′ [nindan]. Its volume is found to be

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on mathematical feasibility. It is thus not astonishing that his explanation differs from the one given here while being closely related.

The relation between original volume V, reference volume v and quotient volume N may be more clear to the modern reader if made explicit in symbols. In the present case, V represents the prolonged cellar, $V = x \cdot y \cdot d'$, $d' = d + 1$ kùš $= d + 5'$ nindan; $v = a \cdot b \cdot c = 50' \cdot 50' \cdot 50'$ nindan^3 $= 50' \cdot 50' \cdot 10$ nindan$^2 \cdot$kùš; $N = (\frac{x}{a}) \cdot (\frac{y}{b}) \cdot (\frac{d'}{c}) = p \cdot q \cdot r$. Thus, since $d = x$ and $a = c$, $r = \frac{d'}{c} = (x + 1\ \text{kùš})/c = p + (1\ \text{kùš})/(10\ \text{kùš}) = p + 6'$.

[40] While other copyist's mistakes in the tablet (jumps from one occurrence of a sequence of signs to another) could have been made by a scribe who copied word for word without understanding what goes on in the text, this one intimates that the copyist was aware of its mathematical content, and inserted by mistake a 21 which was still on his mind (the same cause seems to have produced the "13" of **N° 21**, l. 10).

[41] Cf. VOGEL's tabulations ([VOGEL 1934], 92f).

$5' \cdot 5' \cdot 1$ [nindan^2 · kùš] $= 25''$ [sar], the quotient volume being hence equal to 4` 12. This must be factorized as $p \cdot p \cdot (p+1)$, and if the habits from N^{os} **6** and **7** had been followed, the listing of three equilaterals 6, 6 and 7 would have been expected. Instead we are told "from ["to"?] the equilateral, 1 append, 6 ¿1? the equilateral[s]", which seems to mean, firstly, that one side should be obtained by adding 1 to the others (which are equal); and secondly that the resulting equilateral is 6.[42] A tabulation of $n^2 \cdot (n+1)$ is actually known (VAT 8492, [MKT] I, 76), which identifies only one number (n) as the equilateral; furthermore, the only other problem of the present tablet which might be solved by means of such a table (N^o **5**) also lists only one equilateral, while all others making use of a quotient volume indicate three. It is thus highly plausible that the phrase "from the equilateral, 1 append" refers either to the designation of such a table or to its content, and that a table has indeed been used for the solution of these (and only these) two problems.[43] Since *ina*, beyond "from", also means "in" and "by means of", the phrase should perhaps be interpreted "in/by means of [the table] 'equilaterals, [with] 1 appended', 6 [is found as the] equilateral".

We should now be ready to tackle N^o **5**. The beginning is lost, but it is clear from the following that the accumulation of earth and floor will have been given as 1°10′, and that depth equals length. A supplementary condition leads in lines 2–3 to the conclusion that the length is equal to 1°30′ widths, and the width hence equal to 40′ times the length;[44] thus, the "conversion of the width" — the factor converting the measuring number for the length into that of the width, if we are to believe the parallel to the "conversion of the depth" — is 40′.

The total configuration can thus be obtained from that of N^o **23** by a simple shrinking of the width by the factor 40′: whether 1 kùš is added to the depth or the "floor" to the "earth" makes no structural difference, and $40' \cdot 1^\circ 45' = 1^\circ 10'$.

It cannot be decided whether the author of the text has noticed this, even though a geometrical interpretation suggests so. In any case, even the reference volume of N^o **5** can be obtained from that of N^o **23** by a similar

[42]The dubious ¿1? might be another result of the copyist's thinking about the procedure while writing and perhaps attempting to stamp out a number *1* written by mistake — cf. notes 30 and 40.

[43]NEUGEBAUER, who already proposed (in [MKT] I, 210f) that N^{os} **5** and **23** were solved by means of the table $n^2 \cdot (n+1)$, also presumed N^{os} **6–7** to have made use of tables, confessing at the same time, however, that he was unable to imagine their make-up.

[44]The wording of the original condition is not obvious at all, but so much is clear at least that an intermediate step finds 3 widths to be equal to a double length, since 3 follows from a computation and is then "broken", the operation resulting in a "natural" half. One possibility (though unusual — but cf. N^o 26) might be that the two lengths are told to exceed the two widths by one width.

shrinking by the "conversion of the width", as 1 kùš length times 40′ kùš width times 1 kùš depth.[45]

Then everything goes as usually, and the quotient between the volumes is found again as 4` 12, which is said to have the (single) equilateral 6, corresponding to a factorization $p \cdot p \cdot (p+1)$.

Other variations on N^o^ **23** might have been produced where the excess of depth over length was a regular number. Arithmetically speaking, the system

$$x \cdot x \cdot (12x + a) = b$$

may be reduced to

$$\left(\frac{12}{a}x\right)^2 \cdot \left(\frac{12}{a}x + 1\right) = \left(\frac{12}{a}\right)^2 \cdot \frac{b}{a}\,.$$

Such problems, however, are not to be found in the conserved parts of the tablet. Instead, N^os^ **20** and (presumably) **21** demonstrate how to proceed if a is irregular (and its third power does not divide b).

In N^o^ **20** it is first observed that the 7th part of 7 is one, i.e., that a reference cube 1 kùš high divides the excess height 7 times. Next the reference volume is constructed and computed in painstaking detail: its height, 1 kùš and thus 5′ nindan, is reconverted into 1 kùš. The quotient volume is found to be 8, which has to be factorized as $p \cdot p \cdot (p+7)$, and which is indeed told to have the (three) equilaterals 1, 1, and 8.

N^o^ **21** as it stands is corrupt, but so much sense remains that THUREAU-DANGIN's emendations can probably be relied upon. It is then a close parallel to N^o^ **20**, jumping with the (most unusual) phrase "proceed as in the corresponding case" directly to the value of the quotient volume, and factorizing it into the equilaterals 6, 6 and 13. At this point is stops, having shown the essential step and omitting the conversions of the length and width from 6 lengths/widths of the reference volume into 30′ nindan.

The final third degree problem is N^o^ **22**, which is homogeneous and quite simple. All three dimensions of the cellar are told to be equal, and the method seems to be a simple conversion of the volume 1°30 [sar] into 7′30″ [nindan^3]. 7′30″ is found in the standard table of cube roots, and its

[45]So at least it looks. The multiplication by the 1 kùš depth, however, goes unmentioned, and that of length by width is an unexpected "raising", contrary to all other problems where a reference volume is used. The explanation might perhaps be that N^o^ **5** closes a sequence of gradually more complex problems (the tablet contains several series of that kind), and that an explicitly geometric technique introduced in the preceding problems is reduced here to the arithmetical essentials required for reducing the present problem to a preceding one (as happens in other places of the tablet, cf. below). Still, the absence of concrete information on the preceding problems and on the beginning of N^o^ **5** prevents us from knowing.

(single, and true) equilateral is told in agreement with this table to be 30′. Raising this number to 1, the "conversion" of horizontal extension, yields 30′ [nindan] as sides of the square base of the cellar; raising it to 12, the "conversion of depth", gives 6 [kùš] as the depth.

5.2 The second degree: Length-width, depth-width and length-depth

The tablet contains several groups of second-degree problems, which coincidentally and for convenience can be grouped according to their dress. Of greatest interest are the two sequences **24–25–26** and **27–[28?]–29–30**.

In N° **24**, the volume of the cellar (27°46′40″), the depth (3°20′) and the excess of length over width (50′) are given. Elimination of the depth leaves us with a problem which can be translated

$$x \cdot y = 8^\circ\, 20' \qquad x - y = 50' \ ,$$

and which is solved by ordinary cut-and-paste methods (cf. Figure 2), transforming the rectangle into a gnomon of the same area, completing it as a square, finding the "equilateral" of this square and posing it twice (along the directions of length and width), finally appending and tearing out that half-excess which was cut and pasted in these two directions in order to form the gnomon.

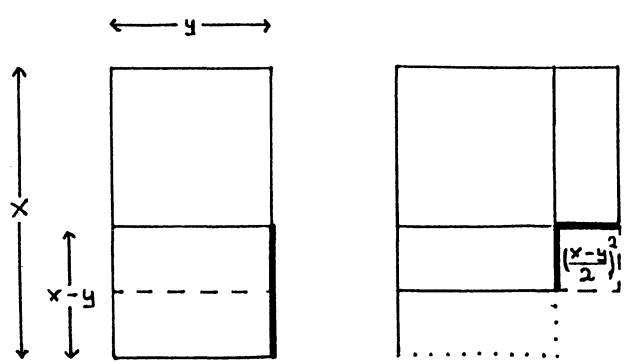

Figure 2

N° **25** is the usual companion-piece, giving the sum instead of the difference between length and width, and presents no noteworthy features apart from a more concise formulation, evidently a recurrent feature of our text when minor variations on already known patterns are presented. N°

26, however, though from the viewpoint of mathematical structure nothing but a slightly more complex variant, provides important information.

The depth is still 3°20′ [kùš] (transformed in line 13* into 16′40″ [nindan]), and the volume is given again as 27°46′40″ (all three problems have the same solution). We are informed, finally, that the excess of the width over the depth equals $\frac{2}{3}$ of the length. Division by the depth thus transforms the problem into one which in symbols (remembering that $x \cdot y$ represents a rectangle and no mere number) can be expressed

$$x \cdot y = A \qquad y = \frac{2}{3}x + D$$

(A = 8°20′, D = 16′40″)
or as

$$\frac{2}{3}x^2 + Dx = A \, .$$

A is multiplied by 40′, corresponding either to

$$\frac{2}{3}x \cdot y = 40' \cdot A \qquad y = \frac{2}{3}x + D$$

or to

$$\left(\frac{2}{3}x\right)^2 + D \cdot \left(\frac{2}{3}x\right) = 40' \cdot A \, .$$

Both of these versions are Babylonian standard problems: the former is similar to **N° 24** (rectangle with known area and excess of length over width); the second is an instance of the problem "sides added to square area", and both follow the same cut-and-paste procedure. If the latter interpretation of the procedure was correct, however, we should expect the solution to tell only the side ($\frac{2}{3}x$) of the square, and to find from there first x and next y. Instead, the text "appends and tears out" precisely as **N° 24**, and presents immediately the larger resulting number (2°30′) as the width, finding the length as $(40')^{-1}$ times the smaller resulting number. It has thus been kept in mind throughout that the longer side of the rectangle $40' \cdot A$ coincides with the original width, while the shorter side is 40′ times the original length. The nigín-na, "go around", appearing at the moment where both sides ($\frac{2}{3}x$) and y are ready for further operations, seems to tell that they should now be marked out "in the terrain". The details of the procedure hence leave no doubt that the transformed problem was thought of in terms of a "rectangle with known excess length" and not as a square with appended sides.[46]

[46] NEUGEBAUER's interpretation of the procedure ([MKT] I, 217) refers to a square area and sides (actually to a quadratic equation in one variable), while THUREAU-DANGIN supports the two-variable option. None of them give arguments for their choice.

The observation is interesting, not because it has general value for Old Babylonian mathematics but rather because it shows that even the opposite observation (following from similar close reading of other late Old Babylonian texts[47]) cannot be generalized (cf. also below on N^{os} **9** and **13**). Depending on expediency or personal preference, Babylonian calculators might conceptualize problems of this type one way or the other.

N^{os} **24–26** can be characterized as *length-width*-problems. Correspondingly, N^{os} **27–30** (with a proviso concerning the missing N^{o} **28**) can be seen as *depth-width*-problems. Their particular interest lies in their relation to the previous group.

In N^{o} **27**, the length is given to be 1°40′, the volume equally 1°40′, and the depth to equal $\frac{1}{7}$ of the excess of length over width.

The first step in the procedure is now to tip the cellar around mentally, putting the length in vertical position: The length is raised to 12, identified as "conversion of depth", and thus converted into 20 kùš.[48] It is then eliminated, and the rectangle spanned by the width y and the depth d is seen to be 5′ [*nindan*^{2}].

Since $d = \frac{1}{7}(1° \, 40' - y)$, the next step is to find the area $7 \cdot 5' = 35'$ of another rectangle with sides $7d$ and y. In this rectangle, the sum of length and width is indeed known, and we are thus brought back to the situation known from N^{o} **25**. The procedure is the same in the part of the text which is conserved, and according to the parallel passages in N^{o} **29** and **30** throughout. According to the parallels, one resulting side is identified immediately as the width, while the other side is divided by 7, and the outcome 10′ [nindan] stated to be the depth without being converted into kùš, in agreement with the reconceptualization of the depth as a horizontal dimension.

N^{o} **29** is strictly similar, containing the slight complication that $d = \frac{1}{7} \cdot (1° \, 40' - y) + 10'$ [nindan], and thus $7d = 1° \, 40' - y + 7 \cdot 10' = 2° \, 50' - y$. Apart from that everything is analogous. The same holds for N^{o} **30**, where the complication is a subtraction of 1 kùš. Together the three problems (and, we may suspect, N^{o} **28**) appear to present an attempt at systematic training of the mutual conversion between horizontal and vertical dimensions.

Comparison with another group of closely related problems (N^{os} **9** and **13**, *lenght-depth*-problems) shows that a particular and not the normal procedure is thought of in the sequence **27–30**. In N^{o} **9**, the accumulation

[47] See [HØYRUP 1990], 341, concerning IM 52301 N^{o} 2, and [HØYRUP 1985], 58 concerning BM 85194, rev. II, 7–21.

[48] Once again, that this is what goes on is demonstrated not only by the identification of the factor 12 but also by the exact ordering of steps. A mere elimination of x and a conversion of the resulting area from nindan · kùš into nindan^2 would indeed, according to Babylonian customs, have been performed through successive "divisions" by 1°40′ and 12, not through a single "division" by their product.

of floor and earth is given as 1°10′, the width is 20′, and the depth equals the length. In symbolic translation,

$$xyG + xy = 1^\circ\,10' \qquad d = x \qquad y = 20'\ .$$

The first step in the procedure is to multiply 20′ with the number 12, which is *not* presented as the conversion of depth or in any similar way. A simple arithmetical recalculation of $x \cdot y \cdot G$ as a certain number of squares with side x (*viz* 4 such squares) appears to be the best interpretation,

$$\begin{aligned} xyG + xy = xy \cdot 12d + xy = x \cdot 20' \cdot 12x + 20'x &= 12 \cdot 20' \cdot x^2 + 20'x \\ = \quad 4x^2 + 20'x \quad &= \quad 1^\circ\,10'\ . \end{aligned}$$

In the next step, this is transformed into a genuine square-area-and-sides problem with the side equal to $4x$,

$$(4x)^2 + 20' \cdot (4x) = 4 \cdot 1^\circ\,10' = 4^\circ\,40'\ ,$$

which is solved by the usual cut-and-paste technique, giving $4x = 2$, and hence $x = 15' \cdot 2 = 30'$ (the depth is not spoken about).

N° 13 is similar but more sophisticated. In symbolic translation

$$\frac{1}{7} \cdot (x \cdot y \cdot G + x \cdot y) + x \cdot y = 20' \qquad d = x \qquad y = 20'\ .$$

Once again, the initial steps may be explained in symbols (remembering, as always, that the "products" are areas and volumes, and not mere numbers):

$$(x \cdot y \cdot G + x \cdot y) + 7 \cdot x \cdot y = 7 \cdot 20' = 2^\circ\,20'\ ,$$

whence

$$12 \cdot 20'x^2 + x \cdot y + 7 \cdot x \cdot y = 4x^2 + x \cdot y + 7 \cdot x \cdot y = 2^\circ\,20'$$

and thus

$$(4x)^2 + (4x) \cdot y + 7 \cdot (4x) \cdot y = 4 \cdot 2^\circ\,20' = 9^\circ\,20'\ .$$

It is only at this point, when the problem has been transformed into one concerning $4x$, that the total number of sides to be added to the square area $(4x)^2$ is found, as $1 + 7$ raised to $y = 20'$, i.e., as $8 \cdot 20' = 2^\circ\,40'$. Then finally everything can go by cut-and-paste geometry, and $4x$ and eventually x be found. Once again, the depth goes unmentioned.

The delayed computation of the number of sides is a recurrent feature in similar problems.[49] It seems as if the primary aim is to reduce in principle to a configuration of square area plus sides, which geometrically is represented by a rectangle; only when this has been achieved is the question about

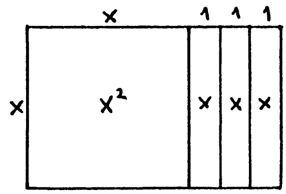

Figure 3

excess of length over width raised — i.e., about the number of sides to be added, cf. Figure 3.

The depth is not only absent from the answer but also from the question of both problems (while e.g. N° **22** asks for and gives the depth, even though it is told to equal the length). Inspection of the steps of the procedure show, furthermore, that they obliterate the very possibility of referring one side of the rectangle which is cut and pasted to the depth (while the other is easily identified as 4 lengths). We can thus be fairly confident, firstly, that even these two problems should be understood as training *a specific* technique; and secondly, that this technique is the use of the "square area and sides" model, as presupposed in my symbolic translation.

5.3 Second-degree igûm-igibûm-problems

A final cluster of second-degree problems (N^os^ **15, 16** and **18**) determine the length and the width as *igûm* and *igibûm*, i.e., as a pair of numbers from the table of reciprocals. In all cases, the volume is also given, implying that the depth follows trivially ($V = x \cdot y \cdot G = x \cdot x^{-1} \cdot G = G = 12d$). N^os^ **16** and **18** furthermore identify the depth with the total of *igûm* and *igibûm*, which leads to a problem of the same type as N° **25**: A rectangle with known area ($x \cdot y = 1$) and known sum of length and width ($x + y = d$). Their only specific interest lies in their use of the elliptic formula "append and tear out" which is shared by N^os^ **25, 26, 27, 29** and **30** and appears nowhere else in the tablet. Since N^os^ **16, 18, 25** and **26** are indubitable "rectangle-" and not "square-problems", this provides us with corroborative evidence that N^os^ **27, 29** and **30** should be understood in the same way.

Rectangle-problems with known sum of length and width normally go together with problems where the excess of length over width is given. So also here: N° **15** tells the depth to be equal to the excess of *igûm* over *igibûm*, while the volume is 36 and d thus 3 nindan. The interesting feature is that this problem has no rational (and hence no *Babylonian*) solution. None the less the text proceeds in a way which demonstrates that 36 is no

[49] E.g. BM 13901 N° 14 ([MKT] III, 3, cf. [Høyrup 1990], 306).

writing error, and proceeds as done in all similar problems until the point where the excess is bisected. Then suddenly it breaks off and states the result of the bisection to be the *igûm*, which is impossible whatever the area of the rectangle, as long as this area exceeds 0.

Evidently, either the text or the procedure of the problem is somehow corrupt. On the other hand the presence of a companion piece to N^os **16** and **18** with given excess instead of total is next to compulsory. A textual mixup which could produce as much sense as actually present is not very likely; it seems rather as if somebody (not necessarily, and probably not, the mathematically gifted author of the first version of the text[50]) has inserted a problem which for once was *not* constructed backwards from given results, and has then broken off and cheated at the point where the insolubility became evident: when 1°30′ follows from the bisection, even a moderately trained calculator will immediately know its square (2°15′) as well as the result of the quadratic completion (3°15′), and hence that this latter number does not appear in the table of square roots.

5.4 First-degree problems

The tablet contains two groups of first-degree problems, N^os **8+12** and N^os **14+17+19**, respectively. Both are quite simple as far as mathematical substance is concerned.

In N^os **8** and **12**, the length is given as 30′, and the depth is told to equal the length. In N^o **8**, furthermore, the accumulation of earth and floor is told to be 1°10′, while N^o **12** tells that $\frac{1}{7}$ of this accumulation appended to the floor gives 20′. The procedures are quite similar, and we shall only follow that of N^o **12** in the geometrical diagram (Figure 4) which is suggested by the "appending" in obv. II, 6.

As a first step, the length is multiplied by 12, resulting in "6 [kùš], the depth". To this 1 [kùš] is *appended*, giving 7 [kùš] — the depth of the figure representing "earth plus floor". Its 7th is found as 1 [kùš] — implying that the corresponding volume coincides with the floor. That this observation is in fact tacitly made is suggested by the next step: 1 and 1 are *accumulated*, i.e., the number 20′ is understood as two times the floor (which is probably the reason that it is regarded as an "accumulation", in spite of its origin in an appending process[51]), not as a volume 2 kùš high and with base equal

[50]Firstly, as noted, the present tablet is a somewhat error-ridden copy of an original; secondly we may remember GOETZE's statement that the tablet belongs to the group of "Northern modernizations of southern (Larsa) originals".

[51]NEUGEBAUER ([MKT] I, 196) as well as THUREAU-DANGIN ([TMB], 13) read the logogram UL.GAR in its function as a verb, "accumulate", take it to be an error for *i-ši*, "raise", and read traces of the ensuing sign as the beginning of a *-ma* ("and then" / "thus"; in mathematical texts to be translated simply as ":"). According to the autography (in particular the way *i* is written elsewhere), however, the reading ⌊*i-ši*⌋ is just as plausible while avoiding the (always unpleasant) hypothesis of a scribal blunder.

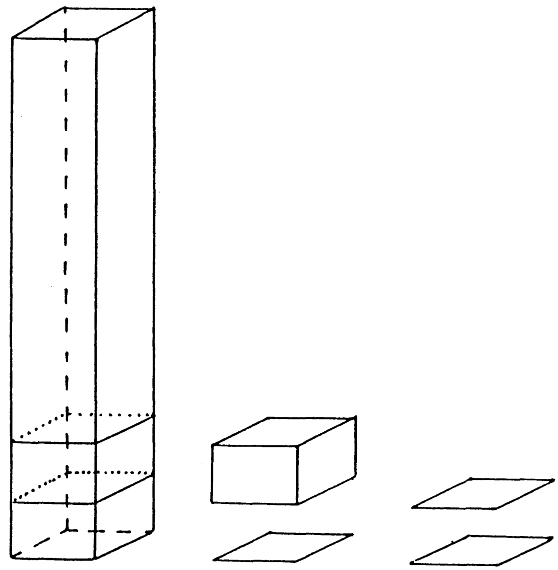

Figure 4

to the floor. 20′ is thus multiplied by the igi of 2, resulting in 10′ [the floor]. Division by "30′ the length" yields 20′, the width.

The shift between the two "additions" thus reveals something about the pattern of thought involved: Accumulation of earth and floor automatically produces a geometric interpretation, so that another "floor" can be *appended*. On the other hand, a height equal to one kùš calls forth an immediate identification of surface and volume (in perfect agreement, of course, with the coinciding metrologies and the coinciding values of the two in sar).

The other group of first-degree problems determine length and width as *igûm* and *igibûm* (whence G = volume). Furthermore, the volume is given. In **N° 19**, the depth is told to be identical with the *igibûm* (even though the results makes it coincide with the *igûm* instead); in **N° 17**, the depth results if the excess of *igûm* over *igibûm* is torn out from the *igûm* — a trivially complicated way to tell that it equals the *igibûm*; in **N° 14**, finally, the procedure and the solution forces us to believe that the depth should have been told to coincide with the *igûm*, even though the statement contains some extra words which might make us expect another companion piece to **N°s 16** and **18** in spite of a certain grammatical clumsiness. In all cases, the solution follows from a simple division of the volume (and thus of G) by 12, which yields either *igûm* or *igibûm*. In **N° 17**, no word is wasted upon the identification of $x-(x-y)$ with y; the problem looks more like a challenge or a puzzle than as a step in a didactical sequence.

6 Further observations on mathematical terminology and techniques

6.1 The third-degree technique

The genuine third-degree-problems made use, as we saw, of the *makṣarum* or "bundling" method. This is no staple method for the treatment of second-degree problems. Nor could it reasonably be, since the method of quadratic completion made both factorizations and tabulations of (e.g.) $n \cdot (n + 1)$ superfluous as techniques for solving mixed second-degree equations. In a few homogeneous problems, however, related ideas turn up. The triangle of YBC 8633 was mentioned above. In VAT 8390 and in BM 13901 N^{os} 10–11, moreover, a rectangle and two squares, respectively, are cut into smaller "reference squares" (cf. [HØYRUP 1990], 279–284). Even factorization was a familiar technique, as we know from various tablets (e.g., YBC 4704 and VAT 5457, in [MCT], 16). While it remains true that the Babylonians were unable to treat problems of the third degree in general (as already stated by THUREAU-DANGIN in his commentary to the third-degree problems from our present tablet im [TMB], xxxviii), the techniques displayed here must be recognized as not merely ingenious artifices but the very best that could be done by means of the mathematical techniques at hand.

6.2 Raising

"Raising" (*našûm*/íl) was presented in chapter 2 as one of the multiplicative operations. In the text we have encountered it in several functions: In connection with multiplication by "conversion" factors and with reciprocals, etc. Most striking was its role in the construction of reference volumes: Here, length and width were "confronted", a constructive procedure implying but not reducible to the computation of the product; in this context, the ensuing "raising" to the height must therefore also be considered constructive.

In all other connections the term appears to have no connotations beyond the calculation by means of multiplication. The double meaning in the computation of volumes, taken together with the rather obvious metaphor ("raising to n" means "raising from the standard height 1 kùš to the actual height n kùš"), can be taken as evidence if not as fully conclusive proof that the origin meaning of the term is indeed the multiplication by a height in the computation of volumes. Other applications of the term will then have been by analogy; as the period where the extension by analogy has taken place we may point to the Ur III period (21st c. B.C.), where the sexagesimal place value system and tables of reciprocals and metrological and technical constants were apparently introduced.

A look at the order of the factors in the raising multiplications contained in our tablet corroborates the conclusion. In general it is arbitrary, the main rule being the purely stylistic convention that the number which has just been calculated is raised to the other factor. In cases where this stylistic rule does not apply, no constraints can be found. If we compare the various multiplications of the "equilaterals" of quotient volumes by the corresponding side of the reference volume, the former are raised to the latter in N^{os} **5**, **6**, and **23**; but both N^{os} **7** and **20** exhibit alternating orders. Even the stylistic rule is nothing but a non-compulsory habit, as demonstrated by a comparison between rev. I, 23* (7 raised to 5′, against the rule) and the strictly parallel passage in rev. II, 13 (2′30″ raised to 7, in agreement with the rule). Similarly, the stylistic rule implies that the igi of a divisor will have to be raised to the number to be divided, which is indeed normally the case; none the less, obv. II, 8 follows the opposite pattern. In the construction of volumes, however, the base is invariably raised to the height (cf. also the tablet Haddad 104, passim, [RAWI/ROAF 1984]). It seems as if the imagery originally inherent in the term was still felt compulsory by Babylonian calculators.

6.3 "Subtractive numbers"

The question whether the Old Babylonian calculators understood the concept of negative numbers is rather meaningless as long as we have not told *which* concept. What is suggested by two passages of our text is that they possessed an idea not only of "subtraction" (which is evident) but also of "subtractive numbers".

The passages are to be found in the statements of N^{os} **29** and **30**. The former tells that the depth is "the 7th part of that which the length over the width goes beyond, *and 2 kùš*", the second that it is "the 7th part of that which the length over the width goes beyond, *and 1 kùš* ba-lal". In the first passage, the "and" is clearly additive. The lal of the second passage is certainly used logographically for a derived form of *maṭûm*, "to be(come) small(er)". If the evidence of the two passages is aggregated we may say that the "normal" role for a number brought into play by "and" is accumulative/additive; but an epithet may make the role diminishing or subtractive.

A related phrase can apparently be pointed out in another late Old Babylonian text. TMS XVI, 23 ([TMS], 92, cf. correction and commentary in [HØYRUP 1990], 301f) contains the phrase "45 *ta*⟨*-mar*⟩ *ki-ma* sag gar gar zi-*ma*", "45′ you see, as much as of widths pose. Pose to tear out", indicating that this coefficient should somehow be recorded as the number of widths to be subtracted.

The term ba-lal is also familiar from the highly systematic "series texts", long sequences of concisely formulated problems which do not tell the procedure. Its occurrences there have often been quoted (e.g., [MKT] I, 410f, 455f, etc.) as instances of negative numbers; the real function of the term, however, is simply to allow the reversal of the order of two magnitudes which are compared, mostly made for stylistic reasons (cd. [HØYRUP 1992]). Applying OCCAM's razor we should only claim that the Old Babylonian calculators had a categorization of additive and subtractive *roles* of numbers within a computation, perhaps even a way to record these roles; whether they would consider this as a categorization of *numbers* as either "positive" and "negative" is not only subjected to doubt but outright dubious.

6.4 The non-technical character of terminology

At an earlier occasion ([HØYRUP 1990], 331) I have claimed that only as a first approximation can Babylonian mathematical terminology

> be called "technical". It appears not to have been stripped completely of the connotations of everyday language, nor does it possess that stiffness which distinguishes a real technical terminology. We should rather comprehend the discourse of the mathematical texts as a highly standardized description in everyday language of standardized problem situations and procedures, and we should notice that the discourse is never more, but sometimes less standardized than the situation described.

This conclusion is corroborated by two interesting terminological details of the present text. One of them is the use of the term translated here "to tear out". As in so many other mathematical texts it is used for the "identity-conserving" subtractive process. But it is also used to tell how much earth has been dug out from the cellar. Moreover, in *both* functions the same logogram zi (provided with the same Sumerian prefix ba-) and not a syllabic Akkadian *nasāḫum* is employed. Clearly, the author of the text saw no point in distinguishing a technical mathematical terminology from the vocabulary of everyday.

The use of *meḫrum*, "counterpart", in **N° 21** (rev. I, 12) is similar. *meḫrum* is a well-known mathematical term. Where the present text tells (e.g., **N° 24**, rev. I, 29+2*) to "pose the equilateral until 2", i.e., to draw two sides of the square meeting in a corner, others ask us, e. g., to "lay down 8°30′ [the equilateral] and 8°30′ its counterpart" (YBC 6967, obv. 11, [MCT], 129). Once again, there is no clearcut boundary between technical-mathematical and everyday speech. No wonder, then, that a geometrical text concerned with triangles uses the word (written logographically TUḪ = gaba) in still another sense (IM 55357 l. 10, [BAQIR 1950], 42).

At the same time the text gives us a glimpse of what might be a grid of fine terminological distinctions, not as much according to mathematical meaning as depending on problem dress and thus perhaps historical origin. The singular use of the accounting term nigin, "total", in **N^os 16** and **18** was pointed out already. This could of course be another instance of floating terminological boundaries. Both occurrences, however, are found in connection with *igûm-igibûm*-problems, which might be no accident: as mentioned above, *igûm* and *igibûm* refer to tables of reciprocals, and thus to the same sphere of social activity as does nigin: scribal accounting and planning rather than surveying. According to the principle that recreational problems are to be considered as a "non-utilitarian" superstructure on mathematical practice (see [HØYRUP 1989]), this might point to an origin of *igûm-igibûm* problems within this specific orbit[52] and to a tendency to conserve a characteristic vocabulary.

7 Unexpected light on the organization of mathematics teaching

In two respects, our text looks primitive or clumsy from a modern mathematical point of view. At closer inspection, however, both apparent flaws turn out to be sound reflections of the technique of didactical exposition, and thus, reversely, strong supportive evidence for what could be guessed about this technique from weaker data.

7.1 Numbers used for identification

The first apparent weakness is what looks like a tendency to give destructively redundant numerical information. Indeed, **N^os^ 6, 7,** and **13** seem to be overdetermined. In **N^os^ 6** and **7** the earth is referred to as 1, and in **N^o^ 13** the accumulation of earth and floor is told to be 1°10′. In neither case are these data used — and the whole point would have been spoiled if they had been taken into account.

Evidently, these numbers were never meant to serve the solution. Nor can they be manifestations of ignorance on the part of the author of the text — everything else in these problems is perfectly clear and points to the goal. Instead, the presence of these numbers can be understood if we think of the purpose and use of the text as a tool for actual teaching. We should imagine the teacher explaining beforehand the total situation: the cellar, its dimensions, the earth and the floor, giving also their numerical values in as far as these may be useful as identifying labels; it is to be observed that **N^os^ 6** and **7** speak about "1 the earth" and do not use the expression "the earth: 1" found when data for the calculation are told. Afterwards, he shows how to extricate the dimensions from a specific set of data; in the oral exposition of the procedure he will have the possibility to identify, say, the original volume as "1 the earth", in contrast to the extended volume — just as a modern exposition will distinguish V from V'.

In the present case, the written text only conserves traces of this oral exposition technique. A couple of other late Old Babylonian texts, however, are more explicit and exhibit the use of numbers as identifiers beyond doubt.[53] What a modern mathematical reading tends to see as a manifestation of incompetence or deficient understanding is thus a rudiment of

[52]Though certainly *not* to an independent focus for the creation of second-degree "algebra" — as demonstrated by the formulation of YBC 6967, the unknown numbers of *igûm-igibûm*-problems were represented by the geometrical magnitudes of normal "surveying" cut-and-paste geometry ([MCT], 129, cf. [HØYRUP 1990], 263–266).

[53]TMS IX and XVI, cf. translation and interpretation in [HØYRUP 1990], 299ff, 320ff.

an oral technique achieving by other means what *we* are accustomed to achieve in writing by algebraic symbols.

7.2 Ordering determined by configuration

The observations just made on the method of exposition may also serve as a key to the seemingly disorderly arrangement of problems within the tablet. Admittedly, Chapter 5 referred to a number of brief sequences of a similar mathematical structure — yet all problems making use (e.g.) of a reference volume were not collected in one place. Mathematical structure and techniques are thus *not* the primary ordering principle.

Let us look instead at the statements. Firstly, of course, the uniting principle of the tablet as a whole is the cellar, and not the investigation of a specific mathematical structure or training of a particular technique. This was already pointed out in chapter 3. But there is more to it. N^os^ **5–9** all tell the accumulation of earth and floor to be 1°10′. Whatever the mathematical character of the problem, be it of the first, the second or the third degree, it will thus have to be discussed with reference to a cellar prolonged one kùš downwards. N^os^ **10** and **11** are missing. N^o^ **12**, which as far as mathematical substance is concerned is nothing but a slight variation on N^o^ **8**, starts from a corresponding variation of the configuration, as does N^o^ **13**, which regarding mathematical substance has the same relation to N^o^ **9**. Instead of exhausting first the possibilities of the *method* of N^o^ **8**, which would make N^o^ **12** follow it immediately, the possibilities *of the configuration* shown in Figure 1 are exhausted before further training of the various methods is undertaken.

N^os^ **14–19** are then *igûm-igibûm* problems; N^os^ **20–23** deal with cellars with a square floor; N^os^ **24–26** all have the same volume and depth given and a rectangular base; and N^os^ **27–30** all (with a proviso for the missing N^o^ **28**) have the length given as 1°40′ and make use of the entity $\frac{1}{7}(x-y)$.

While a categorization according to mathematical structure and techniques only suggests fragments of local order within a generally chaotic structure, the categorization according to configuration thus uncovers a genuine global order and explains the most striking examples of seeming disorder. There is thus no reasonable doubt that the global order of the tablet is determined by the way didactic exposition was organized, and that this organization was the one imagined above.

Below the level of global order, and subordinated to its principles, we find of course an ordering of shorter sequences according to mathematical principles and progression. Recognition of the important role of didactic exposition should not overshadow the fact that understanding of mathematical principles is *also* demonstrated by the tablet. There is certainly no reason to dismiss it as "merely didactic opportunism and *hence* no testimony of real mathematical thought".

8 Mathematics ?

A widespread joke runs as follows: A physicist and a mathematician are put in front of a cooker with two gas-rings, a match-box and an empty kettle standing on the left gas-ring. Asked how to cook water for tea they both tell that you fill the kettle with water and put it back; you turn on the gas, and then you use a match to light the gas. Asked what is to be done if the kettle is to the right, the physicist says "Act correspondingly". The mathematician has a different solution: You move the kettle to the left, reducing thus the situation to the previous case.

Our tablet shows traces of "the physicist" in N° **21** — cf. the reference to the "counterpart". This is not astonishing, widespread as this principle is in systematic yet practice-bound discourse. What *is* astonishing is that even "the mathematician" of the joke is visible; reduction to the previous case instead of direct use of the same method *mutatis mutandis* is in fact the principle used in N^os^ **27–30**, where the cellar is tipped around, changing the depth into a length.

Traditionally, our tablet has mostly been seen as a high point in Babylonian mathematics because it undertakes an attack on third-degree problems. Since the attack leads to no general breakthrough, the high point may be an illusion seen from this angle. Still, if the gauge is not mathematical subject-matter but rather the organization and progression of thought the tablet may still be closer to modern mathematics than many other Babylonian mathematical text, both according to the "kettle principle" and if the occasional tendency to give only the essentials of parallel cases (N^os^ **18, 21**) is taken into account. Both features, indeed, are portends of an incipient break with that casuistic principle which is otherwise so characteristic of Old Babylonian mathematical no less than legal texts.

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