

Numerical and Metrological Graphemes: From Cuneiform to Transliteration¹

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§1. Introduction

§1.1. The aim of this paper is two-fold: first, to analyze some normative aspects of metrological and numerical notations in mathematical cuneiform texts; second, to examine issues raised by modern conventions of transliterations.

§1.2. The argument presented in this paper relies mainly on Old Babylonian school tablets because these sources bear deep traces of normalization processes, and they serve as examples that elucidate the principles of notations used in mathematical texts. In the Old Babylonian period, metrology and place value notation were taught in scribal schools in which this knowledge made up the first level of the mathematical curriculum. School tablets provide us with valuable evidence of the elements that the teachers considered essential. Thus, they constitute a good source for understanding the new concepts involved in numeration and metrology that emerged at the end of the 3rd millennium BC.

§1.3. From a methodological point of view, the paper will for the most part depend on the visual properties of the tablets, and will examine closely the way in which the texts are displayed. This kind of analysis potentially yields a classification of graphemes most similar to that of ancient scribes. In another respect, this paper is based on the general principles and functional classification of graphemes developed by CDLI collaborators.² It contains, moreover, an attempt to import the descriptive system of graphemes used in the field of Mycenaean epigraphy.³

§1.4. This paper will first present a detailed analysis of texts used in scribal schools to teach metrological notations (§§2-3) and place value notation (§4). Problems raised by the distinction between positional and non-positional numbers will then be examined (§5). The last section (§6) advances some practical suggestions for a greater standardization of the transliteration of mathematical texts.

§2. School Tablets

§2.1. School tablets have been unearthed at almost all great sites of the ancient Near East, but the bulk of the extant corpus comes from Nippur.⁴ This city provided

¹ I wrote the first version of this paper for a meeting of the Cuneiform Digital Library Initiative (CDLI) held in Berlin in May of 2008, particularly to contribute to ongoing discussions about the transliteration of numerical and metrological signs used in the mathematical texts. The exchange of views during the meeting and in subsequent e-mails were very fruitful, and I am grateful to Jacob Dahl, Peter Damerow, Steve Tinney, Manfred Krebernik and Bob Englund, as well as to Madeleine Fitzgerald and the referees of CDLI, for their help, clarifications and comments. Abbreviations follow those of CDLI (<http://cdli.ucla.edu/wiki/doku.php/abbreviations_for_assyriology>), adding:

MMT Neugebauer and Sachs 1984

TMH 8 Proust 2008a

Needless to say, any remaining faults in this paper are my own.

² See the “white paper” posted by S. Tinney (2004), <<http://cdli.museum.upenn.edu/doc/ATF/wnm.html>>.

³ Bennett 1963; Bennett 1972; Olivier and Godart 1996: 12. My warmest thanks go to Françoise Rougemont and Maurizio Del Frio, who provided me with the bibliographic references and numerous helpful ideas from the field of Mycenaean epigraphy.

⁴ The numbers of mathematical school tablets found at some important sites are, for example, the following: ca. 900 tablets at Nippur; ca. 150 at Mari; 64 at Ur; and 62 at Kiš.

us with one of the few groups of school tablets that permit qualitative as well as statistical analysis. However, the striking homogeneity of content in school mathematical tablets found in Mesopotamia, as well as in its neighboring regions, gives to the Nippur documentation a relevance beyond the local scale.⁵ Recent studies have allowed the reconstruction of the Nippur curriculum.⁶ It has been established that the education started with a first level, called “elementary” by modern scholars. The young scribes had to memorize huge lists, and then reconstruct them in a given order as a written form, and probably also as an oral recitation. In the field of mathematics, these compositions include

- metrological lists (enumeration of an increasing progression of measures of capacity, weight, surface, and length, in that order);
- metrological tables (enumeration of the same items as in the metrological lists, but including, in front of each item, its correspondence with a number written in place value notation);
- numerical tables (tables of reciprocals, multiplications, squares, square roots, and cube roots).

§2.2. After this first level came a more advanced program dedicated to calculation, namely, algorithms for the calculation of multiplications, reciprocals, surfaces, and probably also volumes. A rough idea of the proportion of tablets containing these different texts can be gathered from the following distribution⁷:

Metrological lists:	187 tablets
Metrological tables:	161 tablets
Numerical tables:	417 tablets
Calculation exercises:	38 tablets

⁵ In this regard, the comparison between Nippur and Mari is enlightening. The contents of elementary mathematical tablets from both sites are highly similar, proving a strong uniformity of the knowledge transmitted. The differences in tablet typology merely indicate local variations in pedagogical methods.

⁶ See the study of N. Veldhuis on the lexical texts from Nippur (Veldhuis 1997); see also E. Robson (2001) on the tablets found in House F at Nippur, and my own work (2007) on the complete corpus of Nippur mathematical texts. The reconstruction of the curriculum is mainly based on the correlation of texts written on “type II” tablets, as initiated by N. Veldhuis (1997, ch. 2).

⁷ These data derive from tablets excavated in the course of archaeological campaigns funded by the University of Pennsylvania towards the end of the 19th century (Babylonian Expedition), which provided the bulk of Nippur sources. For other statistical data concerning mathematical tablets from Nippur, see Robson 2001; Proust 2007: 268-275.

§2.3. These data show that the first step of mathematical education focused on the notation of measures. The curriculum indicates quite clearly that metrology was not just an integral part of mathematics, but clearly an *essential* component, since metrological texts represented approximately half of all Nippur mathematical school texts. Where metrology constituted the first part of mathematical education, writing and using place value notation made up the second part. For the scribes, the memorization of an ordered set of elementary results (reciprocals, multiplications, etc.) was essential for the mastery of algorithms for calculation. School tablets are a coherent and strongly structured group of texts that focuses on notation of measures and calculation. Consequently, the extant corpus of such documents enables us to produce an exhaustive and methodical overview of metrological and numerical cuneiform notations, as well as an analysis of the broader systems into which these notations fit. It is precisely this ancient presentation made by the scribes themselves that I would like to re-examine in order to gain a better understanding of some basic principles applied by scribes in mathematical texts.

§3. *Metrological Lists*

§3.1. *Introduction*

§3.1.1. Following the natural progression of the curriculum, let us begin with metrological lists and examine the organization of information in these texts. Metrological lists are documented by a good number of duplicates (see, for example, the last column of figure 1 below). In the following, I will consider both the composite text and the individual tablets. The composite text given in §8 is the reconstructed list of items found at least once among the Nippur tablets. Individual tablets contain realizations of this “ideal” text. The complete set of metrological lists was preserved at Nippur in the so-called “type I” tablets, which include four lists (capacity, weight, surface, length).⁸

§3.1.2. The first column of figure 1 (below) indicates how the lists appear on the tablets. Though it is somewhat artificial (in particular, I have noted only the beginning for each list), this presentation is a faithful reproduction of the visual properties of these lists. These properties are particularly well illustrated by the tablets HS 249+1805 (= *TMH* 8, no. 3) and HS 1703, reverse (= *TMH* 8, no. 8).

⁸ Examples of metrological lists in Type I tablets: HS 249 + (= *TMH* 8, no. 3); Ist Ni 3515; Ashm 1931-137 (Robson 2004: 33-34).

	Cuneiform text	Transliteration	Sources
List of capacities		1(diš) gin ₂ še	Ist Ni 3238
		1(diš) 1/3 gin ₂	Ist Ni 3279
		1(diš) 1/2 gin ₂	Ist Ni 3772
		1(diš) 2/3 gin ₂	Ist Ni 3976+
		1(diš) 5/6 gin ₂	Ist Ni 4750
		2(diš) gin ₂	Ist Ni 5293
		2(diš) 1/3 gin ₂	Ist Ni 5339
		2(diš) 1/2 gin ₂	Ist Ni 10203 etc.
	...	=====	
List of weights		1/2 še ku ₃ -babbar	Ist Ni 3742
		1(diš) še	Ist Ni 3515
		1(diš) 1/2 še	Ist Ni 5196
		2(diš) še	
		2(diš) 1/2 še	
		3(diš) še	
		4(diš) še	
		5(diš) še	
	...	=====	
List of surfaces		1/3 sar a-ša ₃	Ist Ni 5263
		1/2 sar	Ist Ni 3814
		2/3 sar	Ist Ni 5295
		5/6 sar	
		1(diš) sar	
		1(diš) 1/3 sar	
		1(diš) 1/2 sar	
		1(diš) 2/3 sar	
	...	=====	
List of lengths		1(diš) šu-si	Ist Ni 3767
		2(diš) šu-si	Ist Ni 3991
		3(diš) šu-si	Ist Ni 5234
		4(diš) šu-si	Ist Ni 4715
		5(diš) šu-si	
		6(diš) šu-si	
		7(diš) šu-si	
		8(diš) šu-si	
	...	=====	

Figure 1: Extracts of metrological lists

§3.1.3. At first sight, three features attract our attention: the presence of double strokes, the clearly visible incipit, and the fact that signs are lined up in sub-columns on the tables. Let us have a closer look at these elements. Each list begins with an incipit and ends with a double stroke. The incipit gives, generally, the title of the list and the structure of the items.

- Capacities: the item 1 gin₂ še, literally “1 gin₂ of grain,” introduces the list of capacity measures. The term še (grain, barleycorn) takes here the generic meaning of “capacity.”
- Weights: the item 1/2 še ku₃-babbar, literally “1/2 grain of silver,” introduces the list of weight measures. The term ku₃-babbar (silver) takes here the generic meaning of “weight.” The term appears again later in the list, at the beginning of the section of gu₂ units (see §8.2, item “1(aš) gu₂ ku₃-babbar”).
- Surfaces: the item 1/3 sar a-ša₃, literally “field of 1/3 sar,” introduces the list of surface area measures. The term a-ša₃ (field) takes here the generic meaning of “surface.”
- Lengths: the first item does not contain the name of the magnitudes that measures are enumerated in the list. In fact, there is no generic Sumerian term in mathematical texts to designate linear magnitude, but rather a variety of terms depending on the context (length, width, diagonal, height, depth).

§3.1.4. Thus all incipits include a generic qualifier with the exception of lengths. How can we explain the absence of such a qualifier in the final case? It should be recalled that each metrological list of capacities, weights, and surfaces corresponds to one metrological table (see §4), but for the list of lengths, we have in fact two corresponding tables: one for the horizontal dimensions, and the other for the vertical ones (see §§9.4-9.5).⁹ Strictly speaking, the incipit for the list of lengths should announce measures both for length and height. Whatever solution the scribes chose (whether they mentioned the two magnitudes or neither of them), this incipit necessarily included an irregularity. It should be noted, however, that the words for length, diagonal, height, and depth do sometimes appear in metrological tables. These words are mentioned at the end of the table, as can be seen in exemplars from Ur, where we find tables both for horizontal dimensions (uš, dagal) and vertical measurements (sukud, bur₃); see for instance *UET* 7, 115 (Friberg 2000: 156).

⁹ The existence of an additional table for heights is linked to the methods of calculation for volumes (Friberg, 1987-1990; Proust 2007, §6.6).

§3.1.5. Through the display of graphemes on the tablet, we see clearly two sub-columns (that I designate as i and ii). These main sub-columns are occasionally further subdivided in some particular sequences in which the measures include the use of sub-units (for example, 1 uš 20 ninda / 1 uš 30 ninda / 1 uš 40 ninda / 1 uš 50 ninda). These short sequences, omitted in some sources, do not modify the general structure made up of two main sub-columns—the structure that is of interest to us. For example, the two main sub-columns are quite visible in Ist Ni 3913, reverse; Ist Ni 5196, obverse; Ist Ni 3352, obverse; HS 249+1805 obverse (*TMH* 8, no. 3); HS 247, reverse (*TMH* 8, no. 10).

§3.1.6. Let us analyze the content of the main sub-columns. The list below presents a selection of items that give an overview of the different measures listed. (I chose one item randomly for each unit of measure; for the complete list, see §8).

sub-columns	i	ii
<i>capacities</i>	1(diš) 1/3	gin ₂
	9(diš)	silā ₃
	1(ban ₂)	še
	1(barig)	še
	4(geš ₂)	gur
<i>weights</i>	1(diš) 1/2	še
	1(diš) 1/3	gin ₂
	1(diš) 5/6	ma-na
	4(aš)	gu ₂
<i>surfaces</i>	1(u) 9(diš)	sar
	2(eše ₃)	GAN ₂
<i>lengths</i>	3(diš)	šu-si
	1/3	kuš ₃
	8(diš) 1/2	ninda
	1(u) 4(diš)	uš
	5(u)	danna

§3.1.7. A cursory analysis shows that the sub-column i includes graphemes designating integer and fractional values; the sub-column ii includes ideograms designating units of measure. A closer analysis will show that this simple structure seems locally altered (see §§3.4-3.5). The fact that the sub-columns correspond to different classes of graphemes suggests a vertical reading of the metrological lists.¹⁰ The information conveyed by

¹⁰ As Veldhuis stressed in the case of late lexical texts, “vertical reading” of a list reveals important information about conceptual substrata: “Mesopotamian culture has no textual modes for abstract reasoning nor, in other terms, any meta-discourse. Abstract notions such as morpheme, polyvalency of graphemes, square, and square root are demonstrated by listing. First millen-

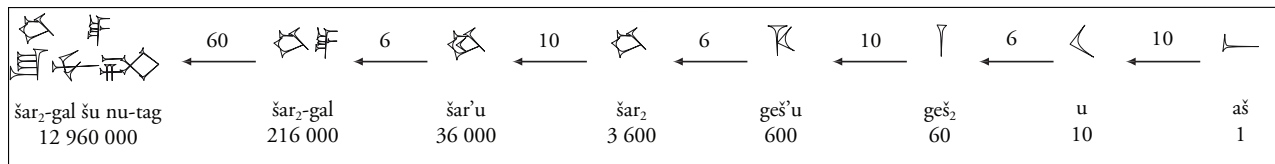


Figure 3: System S

this vertical reading is what I have termed a “system.” This term designates both the grapheme lists appearing in the different sub-columns and the ratios between the values represented by each sign. More precisely, according to our initial analysis given above, “numerical systems” are conveyed by the vertical reading of sub-column i (see §3.2) and “units systems” are conveyed by the vertical reading of sub-column ii (see §3.3).

§3.2. Numerical Systems

§3.2.1. As noted above, the reconstruction of numerical systems arises from a simple vertical reading of the sub-column i, where sequences of integer and fractional numerical values appear. For integer numerical values, three systems can be identified.

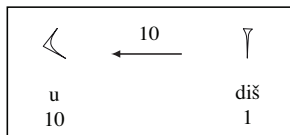


Figure 2: common system

§3.2.2. The most widely used system—and the simplest—is made up from signs “diš” (vertical wedges for which the numerical value is 1) and “u” (the “Winkelhaken,” for which the numerical value is 10); this “common system” is used for the majority of units of measure (capacities: gin₂, sila₃; weights: še, gin₂, ma-na; surfaces: sar; lengths: šu-si, kuš₃, ninda, UŠ, danna). The common system is an additive, decimal system and can be schematized by way of the diagram in figure 2.¹¹

§3.2.3. In order to express integers ranging from 1 to 59, the scribe needs to write the signs “diš” and “u” as many times as necessary. This minimal repertory of signs is sufficient to express the complete range of useful measures, since units of a superior order are generally used for values beyond 59 lower units. The use of

values above 60 is necessary for the largest units of each system; only then is it necessary to employ special numeration systems (see §§3.2.5-3.2.9). Note that in the case of the danna, which is the greatest length measure, values above 59 are rarely used. I am aware of only one attestation of such a value, found in a metrological list from Nippur (HS 249+1805 = *TMH* 8, no. 3, reverse v) which ends with the following sequence: 50 danna / 1(geš₂) danna (see §5 for more details about the numerical system 1(geš₂) belongs to). 1 danna represents a long distance—ca. 10 km—and the range from 1 to 59 danna, expressed in the common system, seems to have been generally sufficient.

§3.2.4. The system used to express measures in gur (the largest unit of capacity) and in gu₂ (the largest unit of weight) in metrological lists is represented by the diagram in figure 3.¹²

§3.2.5. The numeration called “system S” is already attested in Late Uruk texts.¹³ System S is based on a sexagesimal structure (hence its name) and an additive principle. System S as such appears in a literary text from Old Babylonian Nippur (CBS 11319+, figure 4; Sjöberg 1993). In this tablet, we find all the graphemes displayed in the part of sub-column ii of metrological lists concerning gur and gu₂ units; but here the numerical graphemes are isolated from the context of their use, and brought together in a systematic list.

§3.2.6. System S is, to my knowledge, the only numeration that has been presented as a system in a non-mathematical document. Numerical and metrological

nium lexical lists are to be read in two dimensions. The horizontal dimension is represented by the single item that clarifies the reading of one sign or the translation of one Sumerian word. The vertical dimension clarifies the abstract principles through the sequentiality of the items” (Veldhuis 1997: 134-135). This vertical structure is developed to great effect in some mathematical texts as “series texts” (Proust 2009).

¹¹ This “factor diagram” representation was introduced by J. Friberg (1978: 38).

¹² These numerical notations appear in many school tablets; I will limit myself here to a few quotations. Lists of capacities with gur: Ist Ni 5376, reverse; Ist Ni 3913, reverse; Ist Ni 5206, reverse; Ist Ni 3711, reverse; HS 249, obverse (*TMH* 8 no. 3); HS 236, reverse (*TMH* 8 no. 7); HS 1703, obverse (*TMH* 8 no. 8), and many others; lists of weights with gu₂: Ist Ni 5108 reverse; HS 247, reverse (*TMH* 8 no. 10); HS 249, obverse (*TMH* 8 no. 3).

¹³ See for example Friberg 1978; Damerow, and Englund 1987: 127, 165; Nissen, Damerow and Englund 1993: 28; Friberg 1999.

[...]	[1(aš)]		1(u)	1(aš)	1(u)
[...]	[2(u)]		3(u)	2(u)	3(u)
[...]	[4(u)]		5(u)	4(u)	5(u)
[...]	1(geš ₂)	2(geš ₂)	3(geš ₂)	1(geš ₂)	2(geš ₂)
[...]	4(geš ₂)		5(geš ₂)	4(geš ₂)	5(geš ₂)
[...]	6(geš ₂)		7(geš ₂)	6(geš ₂)	7(geš ₂)
[...]	8(geš ₂)		9(geš ₂)	8(geš ₂)	9(geš ₂)
[...]	1(geš'u)	2(geš'u)	3(geš'u)	1(geš'u)	2(geš'u)
[...]	4(geš'u)	5(geš'u)	1(šar ₂)	4(geš'u)	5(geš'u)
[...]		1(šargal) ^{gal}			1(šargal) ^{gal}
[...]	me-a-at		me-a-ta		
[...]	li-mu-um		li-ma-am		
[...]	LIL ₂ -e		mu-un-a		

Figure 4: CBS 11319+, first section¹⁴

signs are widely attested in lexical lists (namely Ea and Hh as well as their precursors),¹⁵ but they are generally classified according to acrographic principles. This organization entails a dislocation of the original coherent system. The special treatment given to system S in CBS 11319+ perhaps indicates its particular importance in an Old Babylonian conceptual framework. However that may be, this tablet, as well as the fact that the same system is associated with both gur and gu₂, indicates an autonomy of system S in relation to the unit systems and, more generally, implies an independence of system S in regard to the nature of the quantified items.

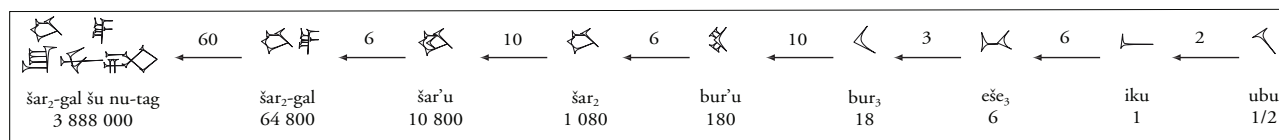


Figure 5: System G

§3.2.7. The notation used to express measures with the greatest unit of surface area has the same formal structure as the other metrological notations: numerical graphemes in sub-column i and unit graphemes in sub-column ii. The vertical reading of sub-column i leads to the diagram in figure 5.¹⁶

§3.2.8. We recognize here the well known system G or “GAN system”.¹⁷ Its structure is partially sexagesimal

and its principle is additive.¹⁸

§3.2.9. Both systems S and G have very ancient roots, but we can note some Old Babylonian innovations, such as the introduction of sexagesimal multiples of the šar₂ count unit (šargal^{gal} and šargal^{gal} šu nu-tag). These very large multiples are more theoretical than practical, since they rarely appear except in metrological lists and tables;¹⁹ in fact, as far as I know, the expression “šargal^{gal} šu nu-tag” is not mentioned elsewhere. The addition, in metrological lists, of the same great sexagesimal multiples to system S and G brings out a parallelism between the two systems. It also stresses the sexagesimal structure of system S and, partially, of the system G. Another innovation is the functional reorganization of the graphemes in notations of measures of surface area (see §3.5).

§3.3. Units Systems

§3.3.1. A vertical reading of the sub-column ii yields the diagrams in figure 6 below.

§3.3.2. The determination of ratios between units belonging to the same unit system results from the enumeration itself. This enumeration proceeds through a regularly increasing progression of the measurements

¹⁴ This transliteration is slightly different from that of Sjöberg (see more details in Proust 2008: 151-152).

¹⁵ Powell 1971.

¹⁶ Sources: Ist Ni 5295; HS 249+1805 reverse (TMH 8, no. 3); HS 240 (TMH 8, no. 28).

¹⁷ Thureau-Dangin 1900; Allotte de la Fuÿe 1930; Friberg 1978: 46; Damerow and Englund 1987: 142, 165; Powell 1987-1990; Nissen, Damerow and Englund 1993, ch. 10.

¹⁸ Allotte de la Fuÿe, in his study of the surface units in Jemdet Nasr texts, takes bur₃ as the basic unit and then “établit la nature sexagésimale de cette numération” (Allotte de la Fuÿe 1930: 70). In fact, from the sign bur₃ onwards, we can observe the same alternation of ratios 10 and 6 as in system S. However, if we look at the whole system both in its primitive form and in its Old Babylonian form, we can see that it is only partially sexagesimal. Another observation made by Allotte de la Fuÿe in the same study shows that the form of the graphemes is a reflection from numerical ratios: the sign eše₃ is made up from the ligature of a horizontal wedge and a Winkelhaken, a form which can be seen as the combination of a 60 and a 10, corresponding to the representation of the value 600; this brings us back to the ratio 1(eše₃) GAN₂ = 600 sar (Allotte de la Fuÿe 1930: 66).

¹⁹ Sources: Ist Ni 10135 + CBS 10181+10207 (TMH 2007); HS 249+1805 (TMH 8, no. 3).

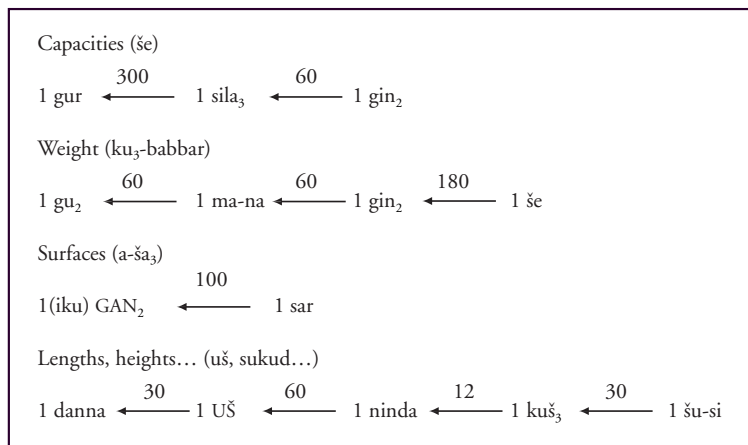


Figure 6: units systems

(see for instance the sequence 1 šu-si / 2 šu-si / ... / 9 šu-si / 1/3 kuš₃, which shows clearly that 1/3 kuš₃ = 10 šu-si, and thus, that 1 kuš₃ = 30 šu-si).

§3.3.3. Metrological lists contain not only a graphical repertory, but also, through their organization in sections and sub-columns, a clear structure for the metrology, a classification of the graphemes, and a fixed definition of the ratios between the quantities indicated by the graphemes. They provide us with information about the notations and—more importantly—the systems behind these notations.

§3.3.4. The absolute values of the standard metrological units are well known (1 sila₃ ≈ 1 liter; 1 gu₂ ≈ 30kg; 1 sar ≈ 36m²; 1 ninda ≈ 6m), but the relationship between written metrological systems and practical uses of metrology can be more complex locally. Differences may result from both the geographical location and the historical period. To take only one example, metrology in school and mathematical tablets is highly normalized, unlike the metrologies found in administrative and business documents. This disparity reflects both the variety of local practices²⁰ and the uniformity of teaching traditions. This issue, as well as the open question of the relationship between script and language, will not be dealt with in the limited framework of this paper.²¹

²⁰ For recent data from Syria and Ugarit, see for example Chambon 2006; Bordreuil 2007.

²¹ One aspect of this issue is the order in which the words were uttered. It seems that numbers, units and commodity names were probably not enumerated in the spoken language in the same order as they are recorded in the script, neither in Sumerian nor in Akkadian. There are also, potentially, historical variations (Powell, 1971: 2-5).

§3.3.5. Another aspect will also not be treated here. Figure 6 does not include the list for units of volume. This is because the list of surface measures is in fact also a list of volume measures, both in standard units and in “brick” units. A set of coefficients allowed the scribes to use a unique list for different systems (Proust 2007, §6.6).

§3.3.6. Let us now use a “horizontal reading” in order to analyze the complete notation of measures and the resulting classification of the graphemes. From a formal point of view, each measure includes an initial component

belonging to the sub-column i, and a second component belonging to the sub-column ii. I will designate these components as class (1) and class (2), respectively. The components of class (1) include integer and fractional numerical values; the components of class (2) include units of measure (the case of GAN₂, ban₂, and barig will be examined later). Graphically, numerical values and units of measure are designated by specialized signs, which have a precise function. It is therefore useful to assign names to classes of graphemes, using concepts developed in the field of Mycenaean studies around the time of the decipherment of the Linear B script (Greece, 1450-1200 BC): *arithmograms* are signs specialized for the designation of integer values; *klasmatograms* are signs specialized for the designation of fractional values; and *metrograms* are signs specialized for the designation of units of measurement.




§3.3.7. In the incipit of tablets, a third component, which is a substantive indicating the nature of the items quantified, appears. In some lists from sites other than Nippur, the third component is repeated for each item (e.g., Ashm 1931-137, from Kish, Robson, 2004: 31-34; see also a surface list in Nissen, Damerow and Englund 1993: 148, P235772). In these cases, three sub-columns i, ii and iii appear. In Ashm 1931-137, we can see for instance the following sequences:

<i>i</i>	<i>ii</i>	<i>iii</i>
1	gin ₂	še
2	gin ₂	še
3	gin ₂	še
<i>etc.</i>		
1/2	še	ku ₃ -babbar
1	še	ku ₃ -babbar
1 1/2	še	ku ₃ -babbar
<i>etc.</i>		
1/3	sar	a-ša ₃
1/2	sar	a-ša ₃
2/3	sar	a-ša ₃
<i>etc.</i>		

Graphically, the components of class (3) are represented by the usual ideograms found in Sumerian texts: še (capacity), ku₃-babbar (weight), and a-ša₃ (surface). These ideograms have the same function as the signs called “ktematograms” or “substantive symbols” by Bennett in his study of the Linear B script (Bennett 1963: 115).²² Finally, the notation of a quantity includes:

- a measure, which is made up from numerical values (1) and units of measure (2)
- a lexeme indicating the nature of the quantified items (3).

These three components are almost always present when metrological quantities are written down in mathematical texts.

§3.3.8. Looking at the graphical repertory of metrological lists, we see that the same grapheme has different functions dependent on the sequence in which it is written. For instance, the sign  (še) is used as a metrogram at the beginning of the list of weight units, but as a substantive grapheme in the list of capacities. Moreover, some metrograms represent different units of measure dependent on the system to which they belong. For instance, the sign  (gin₂) represents a unit of capacity (1/60 sila₃, ca. 1/60 liter) or of weight (1/60 ma-na, ca. 8g). Some arithmograms also represent different values dependent on the metrogram with which they are associated. For instance, the sign  (U) represents in general the value 10 (u), but, in association with the surface sign GAN₂ it represents the value 18 (bur₃). This phenomenon of polysemy is common in cuneiform writing. A sign does not have a meaning in itself, but only in reference to the system to which it belongs.²³

§3.3.9. Finally, for each component, the cuneiform notation of metrological quantities refers to three aspects that are in general closely linked: a dispositional aspect, a semantic aspect and a graphical aspect. The relation between the three aspects is quite stable in metrological lists and follows the pattern given in figure 7.

Components:	(1)	(2)	(3)
Aspects:			
Dispositional	sub-column i	sub-column ii	incipit or sub-column iii
Semantic	Integer or fractional numbers	Unit of measure	Commodity
Graphic	Arithmograms and /or klasmatograms	Metrogram	Substantive grapheme

Figure 7: components of metrological quantities²⁴

This pattern is relevant for the majority of measure units (gin₂, sila₃, še, gur, ma-na, gu₂, sar, šu-si, kuš₃, ninda, UŠ, danna); it is not clearly the case for capacity measures expressed in ban₂ and barig, nor, in a way, for surface measures expressed with the sign GAN₂. These units of measure belong to old systems that were in use in Mesopotamia for a long time before they were integrated into the normalized system. I will now examine more closely these cases in §3.4 and §3.5, and evoke the historical roots of these discrepancies with the dominant pattern summarized in figure 7.

§3.4. The Case of Capacity Measure in ban₂ and barig

§3.4.1. The Old Babylonian concept of quantities clearly distinguishes, as we have seen, three components, exactly as we do nowadays when we write ‘3kg of honey’ or ‘3m of rope’. In the more ancient systems, however, these components are sometimes amalgamated. In some cases, a unique sign serves at once as both arithmogram and metrogram (see, for example, the administrative texts from Kushim dated from the beginning of the 3rd millennium in Nissen, Damerow and Englund 1993: 36-37). In other cases, the arithmogram, metrogram and substantive grapheme are amalgamated (Nissen, Damerow and Englund 1993: 34, text b).²⁵

§3.4.2. In the Old Babylonian metrological lists, capacities expressed in ban₂ and barig present analogous features, as described in figure 8. Each sign of this fig-

²² In cuneiform texts these substantive graphemes can represent magnitudes (length, surface area, volume, capacity, weight), commodities (grains, oil, earth, stone...), or collections (persons, animals, years, tablets, lines or sections in a tablet, bricks ...).

²³ The discovery of this polysemy in the corpus of archaic texts has allowed J. Friberg to make considerable progress in the decipherment process of proto-cuneiform numerations (Friberg 1978; Nissen, Damerow and Englund 1993, 25).

²⁴ These classifications and the associated vocabulary are partially inspired by Mycenologists (Bennett 1963; Olivier and Godart 1996: 12), and correspond also to Tinney’s white paper (Tinney 2004): Semantic aspect = Formal Constituent; Graphic aspect = Written instantiation; Integer number = Count; Unit of measure = Unit; Measure (Integer and/or fractional number + Unit of measure) = Value (Count + Unit); Commodity = Commodity; Arithmogram = Count-grapheme; Metrogram = Unit-grapheme. Fractional number, klasmatogram and substantive grapheme have no counterpart in the white paper.

²⁵ This phenomenon is described by Ritter (1999).

𐎶 1(ban ₂) = 10 sila ₃	𐎶 1(barig) = 60 sila ₃
𐎷 2(ban ₂) = 20 sila ₃	𐎷 2(barig) = 120 sila ₃
𐎸 3(ban ₂) = 30 sila ₃	𐎸 3(barig) = 180 sila ₃
𐎹 4(ban ₂) = 40 sila ₃	𐎹 4(barig) = 240 sila ₃
𐎺 5(ban ₂) = 50 sila ₃	

Figure 8: measures in ban₂ and barig

ure represents a measure of capacity. What is the nature of these signs? Are they arithmograms, klasmatograms, metrograms, or something else? From a formal point of view, since these signs are written in the sub-column i, they should be considered arithmograms or klasmatograms. One could, for instance, consider the sequence of figure 8 as fractions of gur.²⁶ If the signs 𐎶, 𐎷, etc., were klasmatograms representing the fractions 1/30, 1/15, etc., of gur, we would expect the metrogram gur to appear in sub-column ii. In a certain way, it does for larger measures: the notation 𐎶 𐎶 𐎶 could be understood as 1 1/5 gur.²⁷ However, the metrogram gur does not appear for lower measures. It would not be consistent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs 𐎶 and 𐎷 cannot be considered klasmatograms.

§3.4.3. In the same way, the sequence of figure 8 may be considered as representing multiples of sila₃, and, in this case, the signs 𐎶, 𐎷, etc., would be arithmograms representing the values 10, 20, etc. According to this hypothesis, the metrogram sila₃ should appear in the sub-column ii, which is not the case.

§3.4.4. Is the sign še present in the sub-column ii a metrogram? This sign is frequently used in administrative texts as a unit of capacity, and, in this case, its value is 1/180 gin₂.²⁸ As a metrogram, its place would thus be at the beginning of the list of capacities and not after

the measures in sila₃. The sign še is obviously not used here as a metrogram, but most probably as a substantive grapheme, and so has the same function as the one it assumes in the incipit.

§3.4.5. In fact, the signs presented in figure 8 are at once both arithmograms and metrograms. These integrated signs could be dubbed “arithmo-metrograms.” It is interesting to note that their layout in the metrological lists is nevertheless identical to that of other measures. In the sequence of ban₂-barig, sub-column i contains the arithmo-metrograms, and sub-column ii, which is not of further interest, contains the substantive grapheme še. So, the relationship between sub-columns and classes of graphemes does not conform to the dominant pattern resumed in figure 7 above, but instead follows another one:

Components	(1)+(2)	(3)
Sub-columns	i	ii
Graphemes	𐎶	𐎶
	1(ban ₂)	še

§3.4.6. The presence of a substantive grapheme in sub-column ii is an anomaly, but it gives a formal regularity to the whole document. Thus, the semi-archaic capacity system in ban₂-barig with its integrated graphemes has been preserved. At the same time, from a formal point of view, it has been assimilated to the normalized system of measures.

§3.5. The Case of Surface Measures Using the Sign GAN₂

§3.5.1. The measures of surface area raise two problems: the first is related to the function of the graphemes of system G (ubu, iku, eše₃, etc.); the second to the function of the sign GAN₂.

§3.5.2. In Late Uruk texts, the metrological system for surface areas is based on the system G, but the function of the graphemes seems to have evolved in the time between the archaic texts and the Old Babylonian period. Metrological tablets from the end of the 4th millennium (Nissen, Damerow and Englund 1993, 55-59, to MSVO 1, nos. 2-3) contain a discrete set of numerical signs with specific surface area reference:

- 𐎶 1(iku) represents a surface of 3600m²
- 𐎶 1(eše₃) represents a surface of 21,600m²
- etc.

The signs iku and eše₃ constitute by themselves measures of surface areas. These measures are usually followed by the sign GAN₂, which means either surface or field and

²⁶ Such notations would be analogous to the peculiar fractional notations discovered by Laurent Colonna d’Istria in the *šakkanakku* texts from Mari (“Les šakkanakkû de Mari, nouvelles perspectives,” paper read by L. Colonna d’Istria at the workshop *Recherches récentes sur l’histoire et l’archéologie du Moyen Euphrate syrien*, University of Versailles Saint Quentin en Yvelines, in December 2007; paper read at a REHSEIS seminar, on 10 April 2008).

²⁷ The transliteration made according to the rules established by the CDLI is 1(aš) 1(barig) gur.

²⁸ It is also used as a small surface unit (1 še = 1/180 gin₂ and 1 gin₂ = 1/60 sar—see, for example, Ist Ni 18), as well as a small weight unit (1 še = 1/180 gin₂ and 1 gin₂ = 1/60 ma-na).

assumes the function of a substantive grapheme.

§3.5.3. In the rest of the corpus from the 3rd and 2nd millennia, the pronunciation, meaning, and function of the sign GAN₂ are far from clear. In some third millennium texts, GAN₂ can be interpreted as simple semantic indicator that was not supposed to be pronounced (Powell 1973). This is the reason why GAN₂ is sometimes transcribed in superscript (for instance, for the Old Babylonian mathematical texts, see Neugebauer 1945; Robson 2004: 34; Høyrup 2002: 204). From a semantic point of view, GAN₂ and a-ša₃ seem to have been in competition in the third millennium, which led M. Powell to suggest the value “aša_x” (in current sign lists aša₅) for GAN₂ in particular contexts (Powell 1973); this reading of GAN₂ has been accepted by Friberg (2000: 140). The pronunciation of GAN₂ in the Old Babylonian period in metrological contexts is unknown. These observations shed light on reasons why so many different transliterations of GAN₂ can be found today in editions of mathematical texts.²⁹

§3.5.4. What is the nature of graphemes in the system G (ubu, iku, etc.) and of the sign GAN₂ in Old Babylonian metrological lists? In these lists, the substantive grapheme for surfaces is a-ša₃, since this is the word indicated in the incipit (or in a third sub-column, as we have seen in the Kish text and in P235772). The sign GAN₂ is therefore not a substantive grapheme. This sign is systematically written in sub-column ii, as is the smaller surface area unit (sar). Formally, the sign GAN₂ assumes the function of a measure unit equivalent to 100 sar (see the sequence 2(u) sar / 3(u) sar / 4(u) sar / 1(ubu) GAN₂³⁰). Note that the competition between a-ša₃ and GAN₂ pointed out by Powell is here resolved by the attribution of a precise function to each grapheme: the sign GAN₂ as metrogram and the sign a-ša₃ as substantive grapheme. The graphemes of the system G are situated in sub-column i and formally have the function of arithmograms. The system G is thus presented in the Old Babylonian lists as a numerical system, just like system S. Consequently, the Late Uruk surface area system with its integrated graphemes has been reorganized

in order to be assimilated into the normalized writing of measures made up from two components.

§3.5.5. These remarks about the evolution of the systems of capacity and surface area measures demonstrate two distinct modalities of integration of ancient systems. The solution chosen by the scribes to make new things out of old ones seems to have been pragmatic: in the case of capacities, which are of very common use in Babylonian administrative and business practices, old habits have been preserved; in the case of surface area measures, however, it seems that notations have been rationalized, at least at a graphical level (we don't know what the linguistic counterpart for a notation such as 1(iku) GAN₂ is).

§3.5.6. To summarize, metrological lists indicate that the conception of the notation of quantities changed with the systematization of dissociation into three components (numerical value, unit of measure, nature of the quantified items), each with its graphical counterparts (arithmograms and klasmatograms, metrograms, and substantive graphemes).³¹ This conception is clearly shown by the layout of metrological lists (with sub-columns and incipit), for which the visual effect is particularly striking on the tablet HS 1703. We can observe how the formal rigidity of metrological lists reflects the coherence of the whole system, and how it hides irregularities. These results are reviewed in figure 9.

§4. *Calculation Tools: Metrological and Numerical Tables*

§4.1. Aside from the metrological lists, the sources from Nippur include a comparable number of metrological tables. These tables contain the same items as the lists, in the same order, but each item is mentioned side-by-side with a number in sexagesimal place value notation (see the composite text in §9 and, for example, the following sources: Ist Ni 5382, reverse, table of capacities; Ist Ni 5072, obverse, table of lengths). The issue of the respective pedagogical function of metrological lists and tables has not been exhaustively described (see discussion in Proust 2008b). Whatever their precise role in the curriculum may have been, metrological tables provided future scribes with two fundamental notions that are new in relation to the lists: the sexagesimal place value notation and a correspondence between the measures and these positional numbers.

²⁹ Consider these examples of transliterations of GAN₂ in the context of mathematical texts: Thureau-Dangin: gan; Neugebauer (1935): gan₂; (1945): iku in superscript; Friberg: aša₅; Høyrup: iku in superscript; Robson: gana₂ in superscript; CDLI: GAN₂.

³⁰ Sources: Ashm 1931-137 reverse ix (Robson 2004: 33-34); P235772 obverse i-ii (Nissen, Damerow and Englund 1993: 148)

³¹ See also Ritter 1999: 230 about the consequences of the introduction of phonetic script on the distinction between numbers and units of measure.

	(1) <i>Arithmograms belonging to</i>	(2) <i>Metrograms</i>	(3) <i>Substantive graphemes</i>	<i>Notes</i>
<i>capacities</i>	common system	gin ₂	še	
	common system	silā ₃	"	
		ban ₂	"	a)
		bariga	"	a)
<i>weights</i>	system S	gur	"	b)
	common system	še	ku ₃ -babbar	
	common system	gin ₂	"	
	common system	ma-na	"	
<i>surfaces</i>	system S	gu ₂	"	
	common system	še	a-ša ₃	c)
	common system	gin ₂	"	c)
	common system	sar	"	
<i>Lengths, heights, ...</i>	system G	GAN ₂	"	
	common system	šu-si	uš, sukud, ...	d)
	common system	kuš ₃	"	
	common system	ninda	"	
	common system	UŠ	"	
common system	danna	"		

Figure 9: Overview of metrological notations

Notes: a) Component (1) and (2) are amalgamated and the substantive grapheme appears in sub-column ii; b) In metrological lists from Nippur, the metrogram gur appears in sub-column ii, but in some cases the metrogram gur and the substantive še are switched; see lists in Ashm 1931-137 and P235772 quoted above and administrative texts; c) These sub-multiples of sar are not recorded in metrological lists, but they are used in some mathematical texts; see Ist Ni 18 (TMN), UM 29-15-192 (MMT 251); UM 55-21-76 (MMT 246); IM 57846 (MMT 246-8); d) Substantive graphemes for linear magnitudes are generally omitted in metrological lists (they are attested only in some tables from Ur), but they are present in mathematical texts in the notations of measures of length. Note also that the same klasmatograms are used for all the units of measure (except of course for ban₂ and bariga).

§4.2. Concerning the place value notation, which is treated in a number of publications, I will limit my discussion to a few crucial characteristics. First, positional numbers are written without any indication of their order of magnitude (that is, 1, 60 and 60² are written in the same way).³² Second, positional numbers are not associated with a unit of measure or any quantified item (such as magnitude, commodity, or collection). Lastly, from a graphical point of view, each of the 59 sexagesimal digits is written according to the “common system” presented above, i.e., with “diš” (1) and “u” (10) repeated as many times as necessary.

§4.3. Graphical variations, which sometimes distinguish the “common system” from the positional system, can be discerned. The notation of numbers in the positional system is strongly normalized: vertical wedges and Winkelhakens are arranged in groups of three elements

at most. Some exceptions can be found in the earliest of the Old Babylonian mathematical texts (Isin-Larsa period), but they are scarce. However, the notation is much more diverse in administrative texts, in which different arrangements of the wedges and Winkelhakens for the digits 4, 7, 8, 9, 40 and 50 are frequent (Oelsner 2001).

§4.4. I discussed the issue of the nature of the correspondence between measures and positional numbers in (Proust 2008c), where I argued that the nature of this correspondence can be understood only through a study of the entirety of the school curriculum documentation, from elementary to advanced levels. This documentation shows that each numeration plays a specific role. The numerations developed in metrological lists, all of which are additive, are used for quantifying (measures, and, as we will see, discrete counting). In the school tablets, positional numeration is used exclusively for arithmetic operations belonging to the field of multiplication: multiplication, inversion, power, square and cube roots. Metrological tables enabled the scribes to switch from measures to positional numbers and vice

³² Thus, the same numbers appear repeatedly in the right column of metrological tables. For scribes, reading metrological tables from left column to right column is easy, but reading them from right to left column requires a mental control over the orders of magnitude.

versa. These frequent transformations are the basis of methods for calculating surface areas and volumes.³³ The calculation of the surface area of a square in the text Ist Ni 18 is a good illustration of this mechanism: two distinct zones of the tablet contain, respectively, the positional numbers and the corresponding measures.

Line	Numerical signs	Transliteration	Translation
271		mu 5(u) 6(diš)	56 years
77		mu 2(geš ₂) 2(u)	140 years
183		mu 6(diš) šu-ši	360 years
79		mu 5(diš) šu-ši 6(diš)	306 years
175		mu 5(geš'u) 3(geš ₂) 1(u) 5(diš)	3 195 years
7		mu 1(šar'u) 8(šar ₂)	64 800 years

§4.5. The operations made with positional numbers refer to algorithms, some of which (including methods of factorization for the calculation of reciprocals, square or cube roots) have left traces in the texts, whereas others (such as the algorithm for multiplication³⁴) have left few, if any, traces. These traces as well as the lacunae indicate that the calculation was based on a perfect knowledge, probably completely memorized, of numerical tables. We know of these tables thanks to school tablets; at Nippur they were studied just after metrological lists and tables. Numerical tables are composed of the following sections: a reciprocal table (Ist Ni 10239), 38 multiplication tables (Ist Ni 2733), a squares table, a square roots table (Ist Ni 2739) and a cube roots table, all written in sexagesimal place value notation. Once these tables had been memorized, the young scribes were introduced to calculation by means of a small repertory of exercises bearing on multiplication (Ist Ni 10246), the calcula-

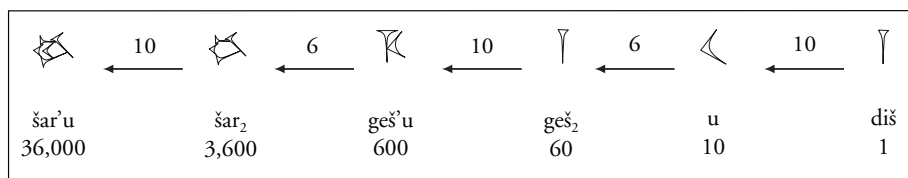


Figure 11: System used in SKL

³³ As Friberg wrote, all the Mesopotamian systems of measures are “sexagesimally adapted. What this means is that all the “conversion factors” appearing in the various factor diagrams [...] are small, sexagesimally regular numbers. Indeed, all the conversion factors are equal to one of the following numbers: 1, 1/2, 2, 3, 4, 5, 6, 10, 12, 30.” (Friberg 2007: 379). For this reason, since ninda corresponds to number 1, all other units of measure correspond also to a “sexagesimally regular number.” This property makes the system of calculation very powerful.

³⁴ Traces of an explicit algorithm for multiplication in Late Babylonian sources have been discovered by J. Friberg (2007: 456-460).

tion of reciprocals (Ist Ni 10241), the determination of surface areas (Ist Ni 18), and probably also of volumes.

§4.6. After this short overview of the contents of mathematical school tablets from Nippur, I would like to emphasize two important consequences for transliteration. Additive numerations were used by scribes specifically for measuring and counting, and place value notation was used for computing. So we have to distinguish in the clearest way the additive numerations from the positional ones, as did the ancient teachers. Moreover, the positional numeration was a powerful tool primarily for calculation, and this ability should be reflected in modern notations.

§4.7. Thus, it is crucial to determine, when faced with a number noted on a tablet, whether it belongs to a positional numeration or not. Generally, this identification is easy: in a non-positional number, the different orders of magnitude of the digits are indicated by the shape of the signs (e.g., 1(šar'u) 8(šar₂), quoted in figure 10) or by a special word (e.g., 5(diš) šu-ši 6(diš), also quoted in figure 10); in a positional number, the different orders of magnitude of the digits are not noted (as in 1.8, or 44.26.40). However, in some cases, the identification is not so simple. I will try, in the following, to analyze some of these ambiguous situations in order to

determine why confusion often arises.

§5. How Can We Distinguish Additive and Positional Notations?

§5.1. Sumerian King List

§5.1.1. The Sumerian King List (hereafter SKL), with its long series of reign durations for various kings and dynasties of Mesopotamian history prior to 2000 BC, provides us with particularly interesting material. The numbers quoted in this text cover a very large range, from a few years to hundreds of thousand years for the mythical, antediluvian periods. The SKL offers an exhaustive repertory of cardinal numbers. Moreover, though the majority of sources come from Nippur, the SKL has been found at many other sites (Larsa, Ur, Isin, Kish, Sippar, Tutub, Šaduppum, Šubat-Enlil, Susa).

§5.1.2. In current transliterations of SKL, the numbers are converted into our decimal system, which prevents the reader from restoring the ancient notations (see, e.g., ETCSL 2.1.1). I have made an inventory of cuneiform notations included in the SKL according to the copy made by Langdon from the Larsa prism W-B 1923-444 (Langdon 1923, pls. I-IV). A representative sample of these notations, classified in increasing order, is given in figure 10 (the line numbers are those from ETCSL). It must be pointed out, however, that the exact same cuneiform notations are found in other sources, thereby validating the following commentaries for all Old Babylonian versions of the SKL.³⁵ This small extract is enough to identify the numerical system to which the graphemes belong.

§5.1.3. The values 1 and 60 are represented by the same sign, a vertical wedge, with the same size. However, the scribes avoided possible confusions by including the name of the sixties (šu-ši) in ambiguous cases, as can be observed for example in lines 79 and 183. Values above 600 are represented by the same signs as

	Sources	Numerical signs	Transliteration	Translation
1	AO 8865 (MKT I, 72)		4(geš ₂) 1(u) 3(diš) mu-bi-im	There are 253 lines
2	YBC 4607 (MCT, O)		1(u) im-šu-me-eš	10 sections
3	YBC 4708 (MKT I, 389)		1(diš) šu-ši im-šu	1 sixty of sections
4	A 24194 (MCT, T)		4(diš) šu-ši im-šu	4 sixties of sections

Figure 12: Number of lines and sections

the ones used in metrology before gur and gu₂ (see figure 13 below). Thus, we can conclude that, in spite of the impression given by notations in lines 271 and 77, the sexagesimal numeration used in SKL is additive and not positional.

§5.2. Cardinal Numbers in Colophons

§5.2.1. Colophons provide us with further examples of cardinal numbers such as the number of lines (mu) or the number of sections (im-šu) written on the tablet. Figure 12 provides a few examples, taken from mathematical tablets (of unknown provenience).

§5.2.2. As in the case of the number of years, the units and the sixties are noted by means of the same sign. In ambiguous cases, the name of the sixties (šu-ši) is explicitly stated (examples 3 and 4). This principle is followed, to my knowledge, in all mathematical tablets. Thus, the system used to count lines and sections in the colophons is the same as the system used to count years in the SKL, i.e., a variant of system S. Though it looks positional, the number in example 1 of figure 12 belongs to an additive system (note the shape of the digit 4 in example 1, which illustrates the graphical variations mentioned in §4.3).

§5.2.3. The risk of confusion between the additive and positional systems occasionally arises for some numbers below 600 but never for numbers above or equal to 600, as shown by figure 13.

§5.3. Mathematical Texts

§5.3.1. The systems used in mathematical texts are generally the same as those of the metrological lists, though graphical variations may appear. This uniformity is not surprising, since the scholars who wrote the mathematical texts were former pupils of the scribal schools and had been taught how to write measures and numbers by learning metrological lists and tables. These school tab-

³⁵ I checked this by means of the tablets quoted as sources in ETCSL 2.1.1: the Nippur tablets CBS 13293+13484 (the first fragment published in Poebel 1914: 4, both fragments together in Hallo 1963: 54), CBS 14220 (Legrain 1922: 1), CBS 13981 and 13994 (Poebel 1914: 2-3); the Tell Leilan tablet L87+ (Vincente 1995: 244-245); the Tutub tablet UCBC 9-1819 (Finkelstein 1963: 40); and the Susa tablets (Scheil 1934).

lets were found almost everywhere in Mesopotamia, with identical contents. The wide diffusion of standard metrology by way of education explains why mathematical texts are relatively homogenous in their notations of numbers and measures. It is clear, however, that a detailed study of all numerical notations in cuneiform mathematical texts still needs to be carried out.

System S		← 10		← 6		← 10		← 6		← 10	
	šar'u		šar ₂		geš'u		geš ₂		u		aš
	36 000		3 600		600		60		10		1
System used in SKL		← 10		← 6		← 10		← 6		← 10	
	šar'u		šar ₂		geš'u		geš ₂		u		diš
	36 000		3 600		600		60		10		1
System used in colophons						← 6		← 10			
					geš ₂		u		diš		
					60		10		1		
Common system								← 10			
							u		diš		
							10		1		
Positional system		← 10		← 6		← 10		← 6		← 10	

§5.3.2. As a matter of fact, some graphical anomalies do occur. I will quote here a few examples related to the notation of measures in ninda.

YBC 4612, obv., 1 (MCT, S):



Usually, these notations are transliterated 3.45 ninda and 1.20 ninda, as if the notation were positional (see §6.5). However, if we consider the whole text, we find the following notations:

YBC 4612 rev. 6: (translation: 6 1/2 ninda)

YBC 4612 rev. 11: (translation: 17 1/2 ninda)

YBC 4612 rev. 13: (translation: 66 1/2 ninda)

§5.3.3. If the notations were positional, the klasmatogram (1/2) would not appear and the numerical notations would be as follows, respectively:

(6.30)

(17.30)

(1.6.30)

In fact, in lines 6, 11, and 13 of YBC 4612 quoted in §5.3.2, the notations of length measures follow the pattern of metrological lists. What is the matter in line 1? According to the metrological lists, the measures in ninda use values ranging from 1 to 59 noted by means of tens (u) and units (diš) repeated as many times as necessary. Beyond 59 ninda, a superior order of the units of measure (indicated by the metrogram UŠ) is

Figure 13: Comparison of sexagesimal numerations

introduced. One can imagine that in line 1 the mention of this larger unit was omitted. In other texts, the metrogram UŠ is restored, for instance in YBC 4666 obverse 13 (MCT, K):

pa₅-sig₅ 5(diš) UŠ uš 2(diš) kuš₃ dagal 1(diš) kuš₃ bur₃-bi
¹/₃ gin₂ eš₂-kar₃
 One canal. 5 UŠ its length, 2 kuš₃ its width, 1 kuš₃ its depth,
¹/₃ gin₂ the work norm.

§5.3.4. Note the presence in this example of the three components in the notation of length: the arithmogram 5, the metrogram UŠ, and the substantive grapheme uš. For the transliteration of line 1 of YBC 4612, we could restore the unit UŠ in the same way:

3(diš) <UŠ> 4(u) 5(diš) ninda
 1(diš) <UŠ> 2(u) ninda.

Or simply indicate that the first digit represents sixties:

3(geš₂) 4(u) 5(diš) ninda
 1(geš₂) 2(u) ninda

§5.3.5. To sum up, notation of the number of ninda may appear positional in some parts of a text, but this is an illusion, since the positional character disappears in other parts of the same text (in ambiguous cases or when fractions are used). As in the case of counting, the confusion disappears if we consider the system to which the signs belong.

§6. Some Suggestions for Transliteration

§6.1. Introduction

§6.1.1. The problems connected with the transliteration, transcription, and translation of mathematical texts are not new (see in particular Neugebauer 1933-1934). However, they resurfaced when projects aimed at digitizing cuneiform sources, such as the CDLI, began.

§6.1.1.1. The conventions developed for the CDLI were defined principally by Damerow, Englund and Tinney at the first CDLI technical meeting in Kinsey Hall (now Humanities Building), UCLA, in March 2001. The initial documentation provided guidance on the transliteration of graphemes and is included in the current ATF documentation, with few changes (Tinney 2009a). This was supplemented by a reference document consisting of a set of tables giving Ur III metrological systems and examples (Englund and Tinney n.d.). A subsequent white paper elaborated a classification of graphemes (count-grapheme, unit-grapheme, integral-value grapheme) and laid the foundations for an analytical framework to support computational processing of Mesopotamian metrology (Tinney 2004). Implementation of this processing is now under way (Tinney 2009b).

§6.1.1.2. The substance of the present paper was prepared in advance of a CDLI technical meeting held in Berlin in May 2008. Following that meeting, a preliminary version of the documentation of the CDLI ‘mathematical’ conventions was prepared that incorporated several of the points in the present paper, and was discussed via e-mail by the author, Peter Damerow, Bob Englund, Eleanor Robson and Steve Tinney, resulting in further refinements. The latest version of this document is available as part of the ATF documentation (Tinney 2009c).

§6.1.1.3. The results of these discussions allows the transliteration of metrological notations found in mathematical texts. However, some problems remain concerning place value notation. The digitization of mathematical texts is only just beginning, and conventions for place value notation in databases are still being debated.

§6.1.2. The following suggestions rely mostly on the principles elaborated by the CDLI, but I wish to add one more: it must be possible for a reader, provided only with the transliteration of a cuneiform numerical notation, to determine if this notation is positional or not. In other words, cuneiform positional notation should

be transliterated by modern positional notation, and cuneiform non-positional notation should be transliterated by modern non-positional notation.

§6.2. Transliteration of the Measures of Length in ninda

§6.2.1. As shown in §5.3, the numerical values used to express a measure of length in ninda are not positional, and this should be visible in the transliteration. Let us take the example of the tablet YBC 4612 already quoted above:

<i>Transliteration by Neugebauer (1945: 103)</i>	<i>Suggested transliteration</i>
3,45 GAR uš	3(geš ₂) 4(u) 5(diš) ninda uš
1,20 GAR sag	1(geš ₂) 2(u) ninda sag
2,30 GAR uš	2(geš ₂) 3(u) ninda uš
6 1/2 GAR 3 kuš ₃ sag	6(diš) 1/2 ninda 3(diš) kuš ₃ sag
1,6 1/2 GAR 2 kuš ₃ uš	1(geš ₂) 6(diš) 1/2 ninda 2(diš) kuš ₃ uš

§6.3. Transliteration of Measures of Surface Area Using the Sign GAN₂

§6.3.1. For the reasons explained in §3.5, from a functional point of view, the ideogram GAN₂ should be considered not as a determinative but as a metrogram. The pronunciation of GAN₂ is unknown for the Old Babylonian period, despite the improvements made by Powell for some 3rd millennium texts. So upper-case letters are still justified. Let us go back to the example of the tablet YBC 4612:

<i>Transliteration by Neugebauer (1945: 103)</i>	<i>Suggested transliteration</i>
2(bur'u)iku a-ša ₃	2(bur'u) GAN ₂ a-ša ₃

§6.4. Transliteration of the Measures of Capacity ban₂ and barig

§6.4.1. The cuneiform notation of capacities is not positional (see §3.4). Nonetheless, the custom is to use, for the transliteration of administrative texts, a “positional” notation described by Sollberger (1966: 7) as follows:

Quantities expressed in gur and its subdivisions (nigida and ban₂) are not transliterated as integers and fractions but as a set of three numbers: thus 1 gur is 1.0.0, 1 nigida is 0.1.0, 1 ban₂ is 0.0.1. When sila₃ are mentioned, a fourth number is added followed by the word sila₃. This, with a slight variation in the punctuation, is also the system adopted by Kraus in recent publications.

§6.4.2. This notation system is very convenient and widely used. However, “positional” transliteration is a source of confusion because it gives the reader the impression that the ancient system is positional, which is not the case. Moreover, it implies an anachronistic use of zeroes. The application of the CDLI conventions

solves all these difficulties (see the composite text of the capacity measures list in §8.1).

§6.5. Transliteration of Cardinal Numbers

§6.5.1. As shown in §5, cardinal numbers are noted in cuneiform texts using a variant of system S, i.e., a sexagesimal additive numeration system (as stated above, the only difference is the orientation of the units sign, which is vertical instead of horizontal). The transliteration of these numbers therefore need not be different from the one used for system S. For example, in line 77 of the SKL, cited above, the transliteration of the notation $\overline{\text{II}}\overline{\text{II}}\overline{\text{II}}$ should be 2(geš₂) 2(u) (and not 2.20; see below §6.6). The same holds true for the lines numbers; if we go back to the example of the colophon of tablet AO 8865, the transliteration of the notation $\overline{\text{IV}}\overline{\text{III}}$ should be 4(geš₂) 1(u) 3(diš) (and not 4.13).

§6.6. Transliteration of Positional Numbers

§6.6.1. As stated above, it is not possible to apply the CDLI conventions to positional numbers for two reasons: (1) the CDLI notation is not positional even though the cuneiform notation is; (2) place value notation was first of all a tool for calculation and it should remain such in modern notations.

§6.6.2. To illustrate point (1), we should consider the metrological tables. Since the numbers noted in the left and right columns belong to different systems and are additive and positional respectively, it is important that this difference in systems appears clearly in the transliteration. The following example shows how an item of the weights table should be transliterated:

1(geš₂) 2(u) gu₂ 1.20

In contrast, transliteration such as:

1(geš₂) 2(u) gu₂ 1(geš₂) 2(u)

or such as:

1.20 gu₂ 1.20

would obscure the fundamental distinction that scribes made themselves between the numerical systems displayed in the left and right columns of the metrological tables.

§6.6.3. To illustrate the point (2), it is sufficient to imagine what would become of a multiplication table if the CDLI encoding system for numerical notations were used without any further overlay annotation:

4(u) 4(diš) 2(u) 6(diš) 4(u) a-ra₂ 1(diš) 4(u) 4(diš) 2(u)
6(diš) 4(u)

2(diš) 1(diš) 2(u) 8(diš) 5(u) 3(diš) 2(u)
3(diš) 2(diš) 1(u) 3(diš) 2(u)
4(diš) 2(diš) 5(u) 7(diš) 4(u) 6(diš) 4(u)
5(diš) 3(diš) 4(u) 2(diš) 1(u) 3(diš) 2(u)
etc.

This raw notation is unworkable for calculation. More importantly, the sign $\overline{\text{I}}$ transliterated 1(diš) belongs to an additive numeration (e.g., in the common system). In additive numerations, each sign represents an absolute value, and the value of the whole number is the sum of the values of the signs that compose it. Thus the notation 1(diš) in transliterations indicates not only the grapheme $\overline{\text{I}}$, but also the value 1. In place value notation, generally the sign $\overline{\text{I}}$ does not represent the value 1, so we cannot translate it as 1(diš).

§6.6.4. Another point should be raised. Neugebauer and Thureau-Dangin were in the habit of separating sexagesimal digits by means of dots or commas. This punctuation also indicates the numerical strings, and the modern reader of the transliteration easily perceives the beginning and the end of the number. For example, the number



is transliterated 44.26.40 by Thureau-Dangin and 44,26,40 by Neugebauer. The dots or commas do not belong to the cuneiform text. For this reason, some scholars avoid these marks and prefer the following transliteration: 44 26 40. However, in my opinion, this choice raises as many problems as the notations of Thureau-Dangin and Neugebauer, for the following reasons.

§6.6.5. The first problem is legibility. Let us take for example the end of the multiplication table quoted above. The lack of identification of the numerical strings makes the reading difficult, even with the insertion of small spaces between digits and larger spaces between numbers, as follows:

44 26 40 44 26 40 33 55 18 31 6 40

The use of punctuation makes the reading noticeably easier:

44.26.40 44.26.40 33.55.18.31.6.40

In texts containing numerical algorithms, such punctuation is absolutely necessary to the understanding of the calculation (see for example the tablets CBS 1215 and UET 6/2, 222). It is for the same reason that Sumerologists indicate in the transliterations verbal and nominal strings by means of dashes or dots. These marks do not

exist in Sumerian cuneiform texts or in mathematical ones.

§6.6.6. The second problem is that a blank space in the transliteration is a “sign,” as much as a punctuation mark is. In other words, introduction of spaces in the transliteration (for example small spaces separating digits and large spaces separating numbers) is not more faithful to the cuneiform text than the introduction of punctuation marks. It can even be argued that this encoding by blank spaces interferes with rules used in cuneiform texts to manage space. For example, a space between cuneiform signs can have mathematical significance in relation to the performance of an algorithm. This signification can be distinct from that of separator of numerical strings (see Ist Ni 10241, reverse). Conversely, spaces between signs can be deprived of mathematical significance and simply be used to fill a complete line (the notation of the same numbers in Ist Ni 10241 obverse is “justified” in a typographic sense).

§6.6.7. A third problem is linked with the more general issue of representation of spatial elements. In mathematical texts, the layout is not reduced to a simple disposition of the data in columns. The layout adheres to rules that are sometimes complex, and it takes on a crucial importance in cases such as numerical texts in which meaning is conveyed partially by the two-dimensional disposition of the information (see Proust forthcoming). Diagrams (geometrical figures or cadastral maps) containing cuneiform notations raise another type of encoding problem, which needs further examination. These remarks draw attention to the fact that the representation of spaces organized by the scribes on the clay surface, either as linear lines of writing or two-dimensional layouts in the case of algorithms and diagrams, is a problem as such. This problem has not been treated and will, of course, not be solved in this paper, but the question of how spatial representations should be encoded is worth a specific and detailed examination.

§6.6.8. Neugebauer also insisted that the floating character of the notation of positional numbers must absolutely be preserved in the transliteration. For him, transliterations of positional sexagesimal numbers should not bear marks such as zeros or commas that specify the order of magnitude of the number, since such marks do not exist in cuneiform texts (Neugebauer 1932-1933: 221).³⁶

³⁶ In my opinion, this should be the case in the translation

§6.6.9. The use of Neugebauer’s or Thureau-Dangin’s notations for the transliteration of positional numbers in internationally accessed databases will thus find a large consensus among current specialists. Difficulties may nevertheless arise for numerical texts that we do not yet know how to interpret. In these cases, neither numerical strings nor sexagesimal digits can be identified. A neutral representation of the sequences of tens and units may be the best solution in such case. This is exactly what the “conform transliteration” system, elaborated by Friberg (1993: 386), attempts to do: units are represented by digits ranging from 1 to 9, and tens by numbers followed by the degree symbol “°” (1° represents a Winkelhaken, i.e. 10, 2° represents 2 Winkelhakens, i.e., 20, etc.).

§6.6.10. For example, the tablet HS 231 (*TMH* 8, no. 72) is not clear, so the distinction of digits and numbers is not certain. The text could be transliterated as follows:

1. 3 2°
 2. 3 4° 5 1° 6
 3. 1 5° 3 2°
- etc.

§6.7. Summary

§6.7.1. My suggestion is that the transliteration of numbers should rely on a minimal prior interpretation, including the distinction of additive and positional notations, and, in the latter case, the identification of digits and numerical strings. Count graphemes (used for measuring and counting) would be transliterated with the CDLI convention. Place value notations would be transliterated according to the Neugebauer or Thureau-Dangin system, by which sexagesimal digits (“1-59”) are represented with numbers noted in modern Arabic numerals, and digits are separated by means of dots or commas.³⁷ In the case of numerical strings that are not fully understood, Friberg’s “conform transliteration system” would be appropriate.

§6.7.2. One should apply to the arithmograms the principles of transliteration usually applied to other

and commentaries as well, since the addition of marks as zeros or degrees, minutes, etc., is more a trap than a help for the modern reader. But this opinion differs from Neugebauer’s position on the subject, and is far from being generally accepted (Proust 2008c).

³⁷ As I said above, this paper concerns only transliteration, not translation, for which notations by specialists are even more various and complex, and generally indicate the order of magnitude of the numbers.

cuneiform signs. A standard publication transliteration distinguishes, on the one hand, the phonetic notations of Akkadian (represented by lowercase italics) and the ideographic notations of Sumerian (represented in various unitalicised ways by specialists), while simultaneously identifying the nominal and verbal strings by means of dashes or dots. These differences in font and punctuation do not belong to the cuneiform text, but their presence in the transliteration results from an initial reading by the scholar, who renders the text intelligible to others using these visual aids. The same process for mathematical texts, i.e., distinguishing additive numerations from positional ones and indicating the numerical strings, results from a reading process that comprehends the ancient significations and attempts to make them accessible to the user of transliterations.

§7. Conclusion

§7.1. Through their organization and disposition of information on the tablets, metrological lists and tables allow us to grasp how scribes worked with the disparate ancient metrological material at their disposal in order to elaborate a new and coherent system, and how they developed a clear means of representation in their writing system. Moreover, the entirety of the school documentation from Nippur demonstrates the great impor-

tance that ancient teachers attached to a clear distinction between metrology and place value notation, i.e., quantifying versus computing.

§7.2. I have based my arguments mainly on sources from Nippur. Nevertheless, it must be recalled that school tablets from other Babylonian cities show no major differences when compared to the Nippur material. The same metrological lists and tables have been found in Mari, Susa, Assur, Ugarit, etc. This wide diffusion of metrological lists and tables indicates that, in the Old Babylonian period and later, schools and other teaching places were the main vector of standardization.

§7.3. Nonetheless, “school tablets” do not mean “school texts.” Metrological texts were written on very different types of tablets according to the place, the time, or the milieu: we often find brief extracts on round tablets, as in the schools of Mari or Ur, but sometimes whole series appear on great prisms or tablets. For example, the prism AO 8865, perhaps from Larsa, is probably not an exercise performed by a young pupil, but rather the work of an experienced scribe. Metrological lists and tables fulfil not only a pedagogical function, but also a normative one. They are the “white papers” of the scribes.

§8. Metrological lists (composite text based on sources from Nippur)

§8.1. Capacities (še)

1(diš) gin ₂ še	1(u) 4(diš) gin ₂	9(diš) sila ₃	1(barig) 3(ban ₂) še
1(diš) 1/3 gin ₂	1(u) 5(diš) gin ₂	1(ban ₂) še	1(barig) 4(ban ₂) še
1(diš) 1/2 gin ₂	1(u) 6(diš) gin ₂	1(ban ₂) 1(diš) sila ₃	1(barig) 5(ban ₂) še
1(diš) 2/3 gin ₂	1(u) 7(diš) gin ₂	1(ban ₂) 2(diš) sila ₃	2(barig) še
1(diš) 5/6 gin ₂	1(u) 8(diš) gin ₂	1(ban ₂) 3(diš) sila ₃	2(barig) 1(ban ₂) še
2(diš) gin ₂	1(u) 9(diš) gin ₂	1(ban ₂) 4(diš) sila ₃	2(barig) 2(ban ₂) še
2(diš) 1/3 gin ₂	1/3 sila ₃	1(ban ₂) 5(diš) sila ₃	2(barig) 3(ban ₂) še
2(diš) 1/2 gin ₂	1/2 sila ₃	1(ban ₂) 6(diš) sila ₃	2(barig) 4(ban ₂) še
2(diš) 2/3 gin ₂	2/3 sila ₃	1(ban ₂) 7(diš) sila ₃	2(barig) 5(ban ₂) še
2(diš) 5/6 gin ₂	5/6 sila ₃	1(ban ₂) 8(diš) sila ₃	3(barig) še
3(diš) gin ₂	1(diš) sila ₃	1(ban ₂) 9(diš) sila ₃	3(barig) 1(ban ₂) še
4(diš) gin ₂	1(diš) 1/3 sila ₃	2(ban ₂) še	3(barig) 2(ban ₂) še
5(diš) gin ₂	1(diš) 1/2 sila ₃	2(ban ₂) 5(diš) sila ₃	3(barig) 3(ban ₂) še
6(diš) gin ₂	1(diš) 2/3 sila ₃	3(ban ₂) še	3(barig) 4(ban ₂) še
7(diš) gin ₂	1(diš) 5/6 sila ₃	3(ban ₂) 5(diš) sila ₃	3(barig) 5(ban ₂) še
8(diš) gin ₂	2(diš) sila ₃	4(ban ₂) še	4(barig) še
9(diš) gin ₂	3(diš) sila ₃	4(ban ₂) 5(diš) sila ₃	4(barig) 1(ban ₂) še
1(u) gin ₂	4(diš) sila ₃	5(ban ₂) še	4(barig) 2(ban ₂) še
1(u) 1(diš) gin ₂	5(diš) sila ₃	5(ban ₂) 5(diš) sila ₃	4(barig) 3(ban ₂) še
1(u) 2(diš) gin ₂	6(diš) sila ₃	1(barig) še	4(barig) 4(ban ₂) še
1(u) 3(diš) gin ₂	7(diš) sila ₃	1(barig) 1(ban ₂) še	4(barig) 5(ban ₂) še
	8(diš) sila ₃	1(barig) 2(ban ₂) še	1(aš) gur

1(aš) 1(barig) gur	1(šar ₂) 2(geš ^u) gur	2(u) 3(diš) še	2(diš) ma-na
1(aš) 2(barig) gur	1(šar ₂) 3(geš ^u) gur	2(u) 4(diš) še	3(diš) ma-na
1(aš) 3(barig) gur	1(šar ₂) 4(geš ^u) gur	2(u) 5(diš) še	4(diš) ma-na
1(aš) 4(barig) gur	1(šar ₂) 5(geš ^u) gur	2(u) 6(diš) še	5(diš) ma-na
2(aš) gur	2(šar ₂) gur	2(u) 7(diš) še	6(diš) ma-na
3(aš) gur	3(šar ₂) gur	2(u) 8(diš) še	7(diš) ma-na
4(aš) gur	4(šar ₂) gur	2(u) 9(diš) še	8(diš) ma-na
5(aš) gur	5(šar ₂) gur	igi 6(diš)-gal ₂ gin ₂	9(diš) ma-na
6(aš) gur	6(šar ₂) gur	igi 6(diš)-gal ₂ gin ₂ 1(u) še	1(u) ma-na
7(diš) gur	7(šar ₂) gur	igi 4(diš)-gal ₂ gin ₂	1(u) 1(diš) ma-na
8(aš) gur	8(šar ₂) gur	igi 4(diš)-gal ₂ gin ₂ 5(diš) še	1(u) 2(diš) ma-na
9(aš) gur	9(šar ₂) gur	1/3 gin ₂	1(u) 3(diš) ma-na
1(u) gur	1(šar ^u) gur	1/2 gin ₂	1(u) 4(diš) ma-na
1(u) 1(aš) gur	1(šar ^u) 1(šar ₂) gur	1/2 gin ₂ 1(u) še	1(u) 5(diš) ma-na
1(u) 2(aš) gur	1(šar ^u) 2(šar ₂) gur	1/2 gin ₂ 1(u) 5(diš) še	1(u) 6(diš) ma-na
1(u) 3(aš) gur	1(šar ^u) 3(šar ₂) gur	1/2 gin ₂ 2(u) 5(diš) še	1(u) 7(diš) ma-na
1(u) 4(aš) gur	1(šar ^u) 4(šar ₂) gur	2/3 gin ₂	1(u) 8(diš) ma-na
1(u) 5(aš) gur	1(šar ^u) 5(šar ₂) gur	2/3 gin ₂ 1(u) še	1(u) 9(diš) ma-na
1(u) 6(aš) gur	1(šar ^u) 6(šar ₂) gur	2/3 gin ₂ 1(u) 5(diš) še	2(u) ma-na
1(u) 7(diš) gur	1(šar ^u) 7(šar ₂) gur	2/3 gin ₂ 2(u) 5(diš) še	2(u) 1(diš) ma-na
1(u) 8(aš) gur	1(šar ^u) 8(šar ₂) gur	5/6 gin ₂	2(u) 2(diš) ma-na
1(u) 9(aš) gur	1(šar ^u) 9(šar ₂) gur	5/6 gin ₂ 1(u) še	2(u) 3(diš) ma-na
2(u) gur	2(šar ^u) gur	5/6 gin ₂ 1(u) 5(diš) še	2(u) 4(diš) ma-na
3(u) gur	3(šar ^u) gur	5/6 gin ₂ 2(u) 5(diš) še	2(u) 5(diš) ma-na
4(u) gur	4(šar ^u) gur	1(diš) gin ₂	2(u) 6(diš) ma-na
5(u) gur	5(šar ^u) gur	1(diš) 1/3 gin ₂	2(u) 7(diš) ma-na
1(geš ₂) gur	1(šargal) ^{gal} gur	1(diš) 1/2 gin ₂	2(u) 8(diš) ma-na
1(geš ₂) 1(u) gur	1(šargal) ^{gal} šu-nu-tag gur	1(diš) 2/3 gin ₂	2(u) 9(diš) ma-na
1(geš ₂) 2(u) gur		1(diš) 5/6 gin ₂	3(u) ma-na
1(geš ₂) 3(u) gur		2(diš) gin ₂	4(u) ma-na
1(geš ₂) 4(u) gur		3(diš) gin ₂	5(u) ma-na
1(geš ₂) 5(u) gur		4(diš) gin ₂	1(aš) gu ₂ ku ₃ -babbar
2(geš ₂) gur	§8.2. Weights (ku ₃ - babbar)	5(diš) gin ₂	1(aš) gu ₂ 1(u) ma-na
3(geš ₂) gur	1/2 še ku ₃ -babbar	6(diš) gin ₂	1(aš) gu ₂ 2(u) ma-na
4(geš ₂) gur	1(diš) še	7(diš) gin ₂	1(aš) gu ₂ 3(u) ma-na
5(geš ₂) gur	1(diš) 1/2 še	8(diš) gin ₂	1(aš) gu ₂ 4(u) ma-na
6(geš ₂) gur	2(diš) še	9(diš) gin ₂	1(aš) gu ₂ 5(u) ma-na
7(geš ₂) gur	2(diš) 1/2 še	1(u) gin ₂	2(aš) gu ₂
8(geš ₂) gur	3(diš) še	1(u) 1(diš) gin ₂	3(aš) gu ₂
9(geš ₂) gur	4(diš) še	1(u) 2(diš) gin ₂	4(aš) gu ₂
1(geš ^u) gur	5(diš) še	1(u) 3(diš) gin ₂	5(aš) gu ₂
1(geš ^u) 1(geš ₂) gur	6(diš) še	1(u) 4(diš) gin ₂	6(aš) gu ₂
1(geš ^u) 2(geš ₂) gur	7(diš) še	1(u) 5(diš) gin ₂	7(aš) gu ₂
1(geš ^u) 3(geš ₂) gur	8(diš) še	1(u) 6(diš) gin ₂	8(aš) gu ₂
1(geš ^u) 4(geš ₂) gur	9(diš) še	1(u) 7(diš) gin ₂	9(aš) gu ₂
1(geš ^u) 5(geš ₂) gur	1(u) še	1(u) 8(diš) gin ₂	1(u) gu ₂
1(geš ^u) 6(geš ₂) gur	1(u) 1(diš) še	1(u) 9(diš) gin ₂	1(u) 1(aš) gu ₂
1(geš ^u) 7(geš ₂) gur	1(u) 2(diš) še	1/3 ma-na	1(u) 2(aš) gu ₂
1(geš ^u) 8(geš ₂) gur	1(u) 3(diš) še	1/2 ma-na	1(u) 3(aš) gu ₂
1(geš ^u) 9(geš ₂) gur	1(u) 4(diš) še	2/3 ma-na	1(u) 4(aš) gu ₂
2(geš ^u) gur	1(u) 5(diš) še	5/6 ma-na	1(u) 5(aš) gu ₂
3(geš ^u) gur	1(u) 6(diš) še	1(diš) ma-na	1(u) 6(aš) gu ₂
4(geš ^u) gur	1(u) 7(diš) še	1(diš) 1/3 ma-na	1(u) 7(aš) gu ₂
5(geš ^u) gur	1(u) 8(diš) še	1(diš) 1/2 ma-na	1(u) 8(aš) gu ₂
1(šar ₂) gur	1(u) 9(diš) še	1(diš) 2/3 ma-na	1(u) 9(aš) gu ₂
1(šar ₂) 1(geš ^u) gur	2(u) še	1(diš) 5/6 ma-na	2(u) gu ₂
	2(u) 1(diš) še		
	2(u) 2(diš) še		

3(u) gu₂
 4(u) gu₂
 5(u) gu₂
 1(geš₂) gu₂
 1(geš₂) 2(u) gu₂
 1(geš₂) 3(u) gu₂
 1(geš₂) 4(u) gu₂
 1(geš₂) 5(u) gu₂
 2(geš₂) gu₂
 3(geš₂) gu₂
 4(geš₂) gu₂
 5(geš₂) gu₂
 6(geš₂) gu₂
 7(geš₂) gu₂
 8(geš₂) gu₂
 9(geš₂) gu₂
 1(geš'u) gu₂
 2(geš'u) gu₂
 3(geš'u) gu₂
 4(geš'u) gu₂
 5(geš'u) gu₂
 1(šar₂) gu₂
 2(šar₂) gu₂
 3(šar₂) gu₂
 4(šar₂) gu₂
 5(šar₂) gu₂
 6(šar₂) gu₂
 7(šar₂) gu₂
 8(šar₂) gu₂
 9(šar₂) gu₂
 1(šar'u) gu₂
 2(šar'u) gu₂
 3(šar'u) gu₂
 4(šar'u) gu₂
 5(šar'u) gu₂
 1(šargal)^{gal} gu₂

§8.3. Surfaces (a-ša₃)

1/3 sar a-ša₃
 1/2 sar
 2/3 sar
 5/6 sar
 1(diš) sar
 1(diš) 1/3 sar
 1(diš) 1/2 sar
 1(diš) 2/3 sar
 1(diš) 5/6 sar
 2(diš) sar
 3(diš) sar
 4(diš) sar
 5(diš) sar
 6(aš) sar
 7(diš) sar
 8(diš) sar
 9(diš) sar
 1(u) sar

1(u) 1(diš) sar
 1(u) 2(diš) sar
 1(u) 3(diš) sar
 1(u) 4(diš) sar
 1(u) 5(diš) sar
 1(u) 6(aš) sar
 1(u) 7(diš) sar
 1(u) 8(diš) sar
 1(u) 9(diš) sar
 2(u) sar
 3(u) sar
 4(u) sar
 1(ubu) GAN₂
 1(ubu) GAN₂ 1(u) sar
 1(ubu) GAN₂ 2(u) sar
 1(ubu) GAN₂ 3(u) sar
 1(ubu) GAN₂ 4(u) sar
 1(iku) GAN₂
 1(iku) 1(ubu) GAN₂
 2(iku) GAN₂
 2(iku) 1(ubu) GAN₂
 3(iku) GAN₂
 3(iku) 1(ubu) GAN₂
 4(iku) GAN₂
 4(iku) 1(ubu) GAN₂
 5(iku) GAN₂
 5(iku) 1(ubu) GAN₂
 1(eše₃) GAN₂
 1(eše₃) 1(iku) GAN₂
 1(eše₃) 2(iku) GAN₂
 1(eše₃) 3(iku) GAN₂
 1(eše₃) 4(iku) GAN₂
 1(eše₃) 5(iku) GAN₂
 2(eše₃) GAN₂
 2(eše₃) 1(iku) GAN₂
 2(eše₃) 2(iku) GAN₂
 2(eše₃) 3(iku) GAN₂
 2(eše₃) 4(iku) GAN₂
 2(eše₃) 5(iku) GAN₂
 1(bur₃) GAN₂
 1(bur₃) 1(eše₃) GAN₂
 1(bur₃) 2(eše₃) GAN₂
 2(bur₃) GAN₂
 3(bur₃) GAN₂
 4(bur₃) GAN₂
 5(bur₃) GAN₂
 6(bur₃) GAN₂
 7(bur₃) GAN₂
 8(bur₃) GAN₂
 9(bur₃) GAN₂
 1(bur'u) GAN₂
 1(bur'u) 1(bur₃) GAN₂
 1(bur'u) 2(bur₃) GAN₂
 1(bur'u) 3(bur₃) GAN₂
 1(bur'u) 4(bur₃) GAN₂
 1(bur'u) 5(bur₃) GAN₂

1(bur'u) 6(bur₃) GAN₂
 1(bur'u) 7(bur₃) GAN₂
 1(bur'u) 8(bur₃) GAN₂
 1(bur'u) 9(bur₃) GAN₂
 2(bur'u) GAN₂
 3(bur'u) GAN₂
 4(bur'u) GAN₂
 5(bur'u) GAN₂
 1(šar₂) GAN₂
 1(šar₂) 1(bur'u) GAN₂
 1(šar₂) 2(bur'u) GAN₂
 1(šar₂) 3(bur'u) GAN₂
 1(šar₂) 4(bur'u) GAN₂
 1(šar₂) 5(bur'u) GAN₂
 2(šar₂) GAN₂
 3(šar₂) GAN₂
 4(šar₂) GAN₂
 5(šar₂) GAN₂
 6(šar₂) GAN₂
 7(šar₂) GAN₂
 8(šar₂) GAN₂
 9(šar₂) GAN₂
 1(šar'u) GAN₂
 1(šar'u) 1(šar₂) GAN₂
 1(šar'u) 2(šar₂) GAN₂
 1(šar'u) 3(šar₂) GAN₂
 1(šar'u) 4(šar₂) GAN₂
 1(šar'u) 5(šar₂) GAN₂
 1(šar'u) 6(šar₂) GAN₂
 1(šar'u) 7(šar₂) GAN₂
 1(šar'u) 8(šar₂) GAN₂
 1(šar'u) 9(šar₂) GAN₂
 2(šar'u) GAN₂
 3(šar'u) GAN₂
 4(šar'u) GAN₂
 5(šar'u) GAN₂
 1(šargal)^{gal} GAN₂
 1(šargal)^{gal} šu-nu-tag GAN₂

§8.4. Lengths (uš, sag, dagal)

1(diš) šu-si
 2(diš) šu-si
 3(diš) šu-si
 4(diš) šu-si
 5(diš) šu-si
 6(aš) šu-si
 7(diš) šu-si
 8(diš) šu-si
 9(diš) šu-si
 1/3 kuš₃
 1/3 kuš₃ 1(diš) šu-si
 1/3 kuš₃ 2(diš) šu-si
 1/3 kuš₃ 3(diš) šu-si
 1/3 kuš₃ 4(diš) šu-si
 1/2 kuš₃

1/2 kuš₃ 1(diš) šu-si
 1/2 kuš₃ 2(diš) šu-si
 1/2 kuš₃ 3(diš) šu-si
 1/2 kuš₃ 4(diš) šu-si
 2/3 kuš₃
 2/3 kuš₃ 1(diš) šu-si
 2/3 kuš₃ 2(diš) šu-si
 2/3 kuš₃ 3(diš) šu-si
 2/3 kuš₃ 4(diš) šu-si
 5/6 kuš₃
 5/6 kuš₃ 1(diš) šu-si
 5/6 kuš₃ 2(diš) šu-si
 5/6 kuš₃ 3(diš) šu-si
 5/6 kuš₃ 4(diš) šu-si
 1(diš) kuš₃
 1(diš) 1/3 kuš₃
 1(diš) 1/2 kuš₃
 1(diš) 2/3 kuš₃
 2(diš) kuš₃
 3(diš) kuš₃
 4(diš) kuš₃
 5(diš) kuš₃
 1/2 ninda
 1/2 ninda 1(diš) kuš₃
 1/2 ninda 2(diš) kuš₃
 1/2 ninda 3(diš) kuš₃
 1/2 ninda 4(diš) kuš₃
 1/2 ninda 5(diš) kuš₃
 1(diš) ninda
 1(diš) 1/2 ninda
 2(diš) ninda
 2(diš) 1/2 ninda
 3(diš) ninda
 3(diš) 1/2 ninda
 4(diš) ninda
 4(diš) 1/2 ninda
 5(diš) ninda
 5(diš) 1/2 ninda
 6(diš) ninda
 6(diš) 1/2 ninda
 7(diš) ninda
 7(diš) 1/2 ninda
 8(diš) ninda
 8(diš) 1/2 ninda
 9(diš) ninda
 9(diš) 1/2 ninda
 1(u) ninda
 2(u) ninda
 3(u) ninda
 4(u) ninda
 4(u) 5(diš) ninda
 5(u) ninda
 5(u) 5(diš) ninda
 1(diš) UŠ
 1(diš) UŠ 1(u) ninda
 1(diš) UŠ 2(u) ninda

1(diš) UŠ 3(u) ninda	2/3 danna 1(diš) UŠ	6(diš) 1/2 danna	1(u) 7(diš) 1/2 danna
1(diš) UŠ 4(u) ninda	2/3 danna 2(diš) UŠ	7(diš) danna	1(u) 8(diš) danna
1(diš) UŠ 5(u) ninda	2/3 danna 3(diš) UŠ	7(diš) 1/2 danna	1(u) 8(diš) 1/2 danna
2(diš) UŠ	2/3 danna 4(diš) UŠ	8(diš) danna	1(u) 9(diš) danna
3(diš) UŠ	5/6 danna	8(diš) 1/2 danna	1(u) 9(diš) 1/2 danna
4(diš) UŠ	5/6 danna 1(diš) UŠ	9(diš) danna	2(u) danna
5(diš) UŠ	5/6 danna 2(diš) UŠ	9(diš) 1/2 danna	2(u) 1(diš) danna
6(diš) UŠ	5/6 danna 3(diš) UŠ	1(u) danna	2(u) 2(diš) danna
7(diš) UŠ	5/6 danna 4(diš) UŠ	1(u) 1/2 danna	2(u) 3(diš) danna
8(diš) UŠ	1(diš) danna	1(u) 1(diš) danna	2(u) 4(diš) danna
9(diš) UŠ	1(diš) 1/2 danna	1(u) 1(diš) 1/2 danna	2(u) 5(diš) danna
1(u) UŠ	1(diš) 2/3 danna	1(u) 2(diš) danna	2(u) 6(diš) danna
1(u) 1(diš) UŠ	1(diš) 5/6 danna	1(u) 2(diš) 1/2 danna	2(u) 7(diš) danna
1(u) 2(diš) UŠ	2(diš) danna	1(u) 3(diš) danna	2(u) 8(diš) danna
1(u) 3(diš) UŠ	2(diš) 1/2 danna	1(u) 3(diš) 1/2 danna	2(u) 9(diš) danna
1(u) 4(diš) UŠ	3(diš) danna	1(u) 4(diš) danna	3(u) danna
1/2 danna	3(diš) 1/2 danna	1(u) 4(diš) 1/2 danna	3(u) 5(diš) danna
1/2 danna 1(diš) UŠ	4(diš) danna	1(u) 5(diš) danna	4(u) danna
1/2 danna 2(diš) UŠ	4(diš) 1/2 danna	1(u) 5(diš) 1/2 danna	4(u) 5(diš) danna
1/2 danna 3(diš) UŠ	5(diš) danna	1(u) 6(diš) danna	5(u) danna
1/2 danna 4(diš) UŠ	5(diš) 1/2 danna	1(u) 6(diš) 1/2 danna	1(geš ₂) danna
2/3 danna	6(diš) danna	1(u) 7(diš) danna	

§9. Metrological tables (composite text based on sources from Nippur)

§9.1. Capacities (še)	1/2 sila ₃	30	3(ban ₂) 5(diš) sila ₃	35	1(aš) gur	5	
1(diš) gin ₂ še	1	2/3 sila ₃	40	4(ban ₂) še	40	1(aš) 1(barig) gur	6
1(diš) 1/3 gin ₂	1.20	5/6 sila ₃	50	4(ban ₂) 5(diš) sila ₃	45	1(aš) 2(barig) gur	7
1(diš) 1/2 gin ₂	1.30	1(diš) sila ₃	1	5(ban ₂) še	50	1(aš) 3(barig) gur	8
1(diš) 2/3 gin ₂	1.40	1(diš) 1/3 sila ₃	1.20	5(ban ₂) 5(diš) sila ₃	55	1(aš) 4(barig) gur	9
1(diš) 5/6 gin ₂	1.50	1(diš) 1/2 sila ₃	1.30	1(barig) še	1	2(aš) gur	10
2(diš) gin ₂	2	1(diš) 2/3 sila ₃	1.40	1(barig) 1(ban ₂) še	1.10	3(aš) gur	15
2(diš) 1/3 gin ₂	2.20	1(diš) 5/6 sila ₃	1.50	1(barig) 2(ban ₂) še	1.20	4(aš) gur	20
2(diš) 1/2 gin ₂	2.30	2(diš) sila ₃	2	1(barig) 3(ban ₂) še	1.30	5(aš) gur	25
2(diš) 2/3 gin ₂	2.40	3(diš) sila ₃	3	1(barig) 4(ban ₂) še	1.40	6(aš) gur	30
2(diš) 5/6 gin ₂	2.50	4(diš) sila ₃	4	1(barig) 5(ban ₂) še	1.50	7(diš) gur	35
3(diš) gin ₂	3	5(diš) sila ₃	5	2(barig) še	2	8(aš) gur	40
4(diš) gin ₂	4	6(diš) sila ₃	6	2(barig) 1(ban ₂) še	2.10	9(aš) gur	45
5(diš) gin ₂	5	7(diš) sila ₃	7	2(barig) 2(ban ₂) še	2.20	1(u) gur	50
6(diš) gin ₂	6	8(diš) sila ₃	8	2(barig) 3(ban ₂) še	2.30	1(u) 1(aš) gur	55
7(diš) gin ₂	7	9(diš) sila ₃	9	2(barig) 4(ban ₂) še	2.40	1(u) 2(aš) gur	1
8(diš) gin ₂	8	1(ban ₂) še	10	2(barig) 5(ban ₂) še	2.50	1(u) 3(aš) gur	1.5
9(diš) gin ₂	9	1(ban ₂) 1(diš) sila ₃	11	3(barig) še	3	1(u) 4(aš) gur	1.10
1(u) gin ₂	10	1(ban ₂) 2(diš) sila ₃	12	3(barig) 1(ban ₂) še	3.10	1(u) 5(aš) gur	1.15
1(u) 1(diš) gin ₂	11	1(ban ₂) 3(diš) sila ₃	13	3(barig) 2(ban ₂) še	3.20	1(u) 6(aš) gur	1.20
1(u) 2(diš) gin ₂	12	1(ban ₂) 4(diš) sila ₃	14	3(barig) 3(ban ₂) še	3.30	1(u) 7(diš) gur	1.25
1(u) 3(diš) gin ₂	13	1(ban ₂) 5(diš) sila ₃	15	3(barig) 4(ban ₂) še	3.40	1(u) 8(aš) gur	1.30
1(u) 4(diš) gin ₂	14	1(ban ₂) 6(diš) sila ₃	16	3(barig) 5(ban ₂) še	3.50	1(u) 9(aš) gur	1.35
1(u) 5(diš) gin ₂	15	1(ban ₂) 7(diš) sila ₃	17	4(barig) še	4	2(u) gur	1.40
1(u) 6(diš) gin ₂	16	1(ban ₂) 8(diš) sila ₃	18	4(barig) 1(ban ₂) še	4.10	3(u) gur	2.30
1(u) 7(diš) gin ₂	17	1(ban ₂) 9(diš) sila ₃	19	4(barig) 2(ban ₂) še	4.20	4(u) gur	3.20
1(u) 8(diš) gin ₂	18	2(ban ₂) še	20	4(barig) 3(ban ₂) še	4.30	5(u) gur	4.10
1(u) 9(diš) gin ₂	19	2(ban ₂) 5(diš) sila ₃	25	4(barig) 4(ban ₂) še	4.40	1(geš ₂) gur	5
1/3 sila ₃	20	3(ban ₂) še	30	4(barig) 5(ban ₂) še	4.50	1(geš ₂) 1(u) gur	5.50

1(geš ₂) 2(u) gur	6.40	§9.2. Weights (ku₃-babbar)	1(diš) ² / ₃ gin ₂	1.40	2(u) 9(diš) ma-na	29
1(geš ₂) 3(u) gur	7.30		1(diš) ⁵ / ₆ gin ₂	1.50	3(u) ma-na	30
1(geš ₂) 4(u) gur	8.20	¹ / ₂ še ku ₃ -babbar	2(diš) gin ₂	2	4(u) ma-na	40
1(geš ₂) 5(u) gur	9.10	1(diš) še	3(diš) gin ₂	3	5(u) ma-na	50
2(geš ₂) gur	10	1(diš) ¹ / ₂ še	4(diš) gin ₂	4	1(aš) gu ₂ ku ₃ -babbar	1
3(geš ₂) gur	15	2(diš) še	5(diš) gin ₂	5	1(aš) gu ₂ 1(u) ma-na	1.10
4(geš ₂) gur	20	2(diš) ¹ / ₂ še	6(diš) gin ₂	6	1(aš) gu ₂ 2(u) ma-na	1.20
5(geš ₂) gur	25	3(diš) še	7(diš) gin ₂	7	1(aš) gu ₂ 3(u) ma-na	1.30
6(geš ₂) gur	30	4(diš) še	8(diš) gin ₂	8	1(aš) gu ₂ 4(u) ma-na	1.40
7(geš ₂) gur	35	5(diš) še	9(diš) gin ₂	9	1(aš) gu ₂ 5(u) ma-na	1.50
8(geš ₂) gur	40	6(diš) še	1(u) gin ₂	10	2(aš) gu ₂	2
9(geš ₂) gur	45	7(diš) še	1(u) 1(diš) gin ₂	11	3(aš) gu ₂	3
1(geš' u) gur	50	8(diš) še	1(u) 2(diš) gin ₂	12	4(aš) gu ₂	4
1(geš' u) 1(geš) gur	55	9(diš) še	1(u) 3(diš) gin ₂	13	5(aš) gu ₂	5
1(geš' u) 2(geš ₂) gur	1	1(u) še	1(u) 4(diš) gin ₂	14	6(aš) gu ₂	6
1(geš' u) 3(geš ₂) gur	1.5	1(u) 1(diš) še	1(u) 5(diš) gin ₂	15	7(aš) gu ₂	7
1(geš' u) 4(geš ₂) gur	1.10	1(u) 2(diš) še	1(u) 6(diš) gin ₂	16	8(aš) gu ₂	8
1(geš' u) 5(geš ₂) gur	1.15	1(u) 3(diš) še	1(u) 7(diš) gin ₂	17	9(aš) gu ₂	9
1(geš' u) 6(geš ₂) gur	1.20	1(u) 4(diš) še	1(u) 8(diš) gin ₂	18	1(u) gu ₂	10
1(geš' u) 7(geš ₂) gur	1.25	1(u) 5(diš) še	1(u) 9(diš) gin ₂	19	1(u) 1(aš) gu ₂	11
1(geš' u) 8(geš ₂) gur	1.30	1(u) 6(diš) še	¹ / ₃ ma-na	20	1(u) 2(aš) gu ₂	12
1(geš' u) 9(geš ₂) gur	1.35	1(u) 7(diš) še	¹ / ₂ ma-na	30	1(u) 3(aš) gu ₂	13
2(geš' u) gur	1.40	1(u) 8(diš) še	² / ₃ ma-na	40	1(u) 4(aš) gu ₂	14
3(geš' u) gur	2.30	1(u) 9(diš) še	⁵ / ₆ ma-na	50	1(u) 5(aš) gu ₂	15
4(geš' u) gur	3.20	2(u) še	1(diš) ma-na	1	1(u) 6(aš) gu ₂	16
5(geš' u) gur	4.10	2(u) 1(diš) še	1(diš) ¹ / ₃ ma-na	1.20	1(u) 7(aš) gu ₂	17
1(šar ₂) gur	5	2(u) 2(diš) še	1(diš) ¹ / ₂ ma-na	1.30	1(u) 8(aš) gu ₂	18
1(šar ₂) 1(geš' u) gur	5.50	2(u) 3(diš) še	1(diš) ² / ₃ ma-na	1.40	1(u) 9(aš) gu ₂	19
1(šar ₂) 2(geš' u) gur	6.40	2(u) 4(diš) še	1(diš) ⁵ / ₆ ma-na	1.50	2(u) gu ₂	20
1(šar ₂) 3(geš' u) gur	7.30	2(u) 5(diš) še	2(diš) ma-na	2	3(u) gu ₂	30
1(šar ₂) 4(geš' u) gur	8.20	2(u) 6(diš) še	3(diš) ma-na	3	4(u) gu ₂	40
1(šar ₂) 5(geš' u) gur	9.10	2(u) 7(diš) še	4(diš) ma-na	4	5(u) gu ₂	50
2(šar ₂) gur	10	2(u) 8(diš) še	5(diš) ma-na	5	1(geš ₂) gu ₂	1
3(šar ₂) gur	15	2(u) 9(diš) še	6(diš) ma-na	6	1(geš ₂) 2(u) gu ₂	1.20
4(šar ₂) gur	20	igi 6(diš)-gal ₂ gin ₂	7(diš) ma-na	7	1(geš ₂) 3(u) gu ₂	1.30
5(šar ₂) gur	25	igi 6(diš)-gal ₂ gin ₂	8(diš) ma-na	8	1(geš ₂) 4(u) gu ₂	1.40
6(šar ₂) gur	30	1(u) še	9(diš) ma-na	9	1(geš ₂) 5(u) gu ₂	1.50
7(šar ₂) gur	35	igi 4(diš)-gal ₂ gin ₂	1(u) ma-na	10	2(geš ₂) gu ₂	2
8(šar ₂) gur	40	igi 4(diš)-gal ₂ gin ₂	1(u) 1(diš) ma-na	11	3(geš ₂) gu ₂	3
9(šar ₂) gur	45	5(diš) še	1(u) 2(diš) ma-na	12	4(geš ₂) gu ₂	4
1(šar' u) gur	50	¹ / ₃ gin ₂	1(u) 3(diš) ma-na	13	5(geš ₂) gu ₂	5
1(šar' u) 1(šar ₂) gur	55	¹ / ₂ gin ₂	1(u) 4(diš) ma-na	14	6(geš ₂) gu ₂	6
1(šar' u) 2(šar ₂) gur	1	¹ / ₂ gin ₂ 1(u) še	1(u) 5(diš) ma-na	15	7(geš ₂) gu ₂	7
1(šar' u) 3(šar ₂) gur	1.5	¹ / ₂ gin ₂ 1(u) 5(diš) še	1(u) 6(diš) ma-na	16	8(geš ₂) gu ₂	8
1(šar' u) 4(šar ₂) gur	1.10	¹ / ₂ gin ₂ 2(u) 5(diš) še	1(u) 7(diš) ma-na	17	9(geš ₂) gu ₂	9
1(šar' u) 5(šar ₂) gur	1.15	² / ₃ gin ₂	1(u) 8(diš) ma-na	18	1(geš' u) gu ₂	10
1(šar' u) 6(šar ₂) gur	1.20	² / ₃ gin ₂ 1(u) še	1(u) 9(diš) ma-na	19	2(geš' u) gu ₂	20
1(šar' u) 7(šar ₂) gur	1.25	² / ₃ gin ₂ 1(u) 5(diš) še	2(u) ma-na	20	3(geš' u) gu ₂	30
1(šar' u) 8(šar ₂) gur	1.30	² / ₃ gin ₂ 2(u) 5(diš) še	2(u) 1(diš) ma-na	21	4(geš' u) gu ₂	40
1(šar' u) 9(šar ₂) gur	1.35	⁵ / ₆ gin ₂	2(u) 2(diš) ma-na	22	5(geš' u) gu ₂	50
2(šar' u) gur	1.40	⁵ / ₆ gin ₂ 1(u) še	2(u) 3(diš) ma-na	23	1(šar ₂) gu ₂	1
3(šar' u) gur	2.30	⁵ / ₆ gin ₂ 1(u) 5(diš) še	2(u) 4(diš) ma-na	24	2(šar ₂) gu ₂	2
4(šar' u) gur	3.20	⁵ / ₆ gin ₂ 2(u) 5(diš) še	2(u) 5(diš) ma-na	25	3(šar ₂) gu ₂	3
5(šar' u) gur	4.10	1(diš) gin ₂	2(u) 6(diš) ma-na	26	4(šar ₂) gu ₂	4
1(šargal) ^{gal} gur	5	1(diš) ¹ / ₃ gin ₂	2(u) 7(diš) ma-na	27	5(šar ₂) gu ₂	5
1(šargal) ^{gal} šu-nu-tag gur	5	1(diš) ¹ / ₂ gin ₂	2(u) 8(diš) ma-na	28	6(šar ₂) gu ₂	6

5/6 danna	25	1(u) 5(diš) 1/2 danna	7.45	1/3 kuš ₃	20	4(diš) ninda	48
5/6 danna 1(diš) UŠ	26	1(u) 6(diš) danna	8	1/2 kuš ₃	30	4(diš) 1/2 ninda	54
5/6 danna 2(diš) UŠ	27	1(u) 6(diš) 1/2 danna	8.15	1/2 kuš ₃ 1(diš) šu-si	32	5(diš) ninda	1
5/6 danna 3(diš) UŠ	28	1(u) 7(diš) danna	8.30	1/2 kuš ₃ 2(diš) šu-si	34	5(diš) 1/2 ninda	1.6
5/6 danna 4(diš) UŠ	29	1(u) 7(diš) 1/2 danna	8.45	1/2 kuš ₃ 3(diš) šu-si	36	6(diš) ninda	1.12
1(diš) danna	30	1(u) 8(diš) danna	9	1/2 kuš ₃ 4(diš) šu-si	38	6(diš) 1/2 ninda	1.18
1(diš) 1/2 danna	45	1(u) 8(diš) 1/2 danna	9.15	2/3 kuš ₃	40	7(diš) ninda	1.24
1(diš) 2/3 danna	50	1(u) 9(diš) danna	9.30	2/3 kuš ₃ 1(diš) šu-si	42	7(diš) 1/2 ninda	1.30
1(diš) 5/6 danna	55	1(u) 9(diš) 1/2 danna	9.45	2/3 kuš ₃ 2(diš) šu-si	44	8(diš) ninda	1.36
2(diš) danna	1	2(u) danna	10	2/3 kuš ₃ 3(diš) šu-si	46	8(diš) 1/2 ninda	1.42
2(diš) 1/2 danna	1.15	2(u) 1(diš) danna	10.30	2/3 kuš ₃ 4(diš) šu-si	48	9(diš) ninda	1.48
3(diš) danna	1.30	2(u) 2(diš) danna	11	5/6 kuš ₃	50	9(diš) 1/2 ninda	1.54
3(diš) 1/2 danna	1.45	2(u) 3(diš) danna	11.30	5/6 kuš ₃ 1(diš) šu-si	52	1(u) ninda	2
4(diš) danna	2	2(u) 4(diš) danna	12	5/6 kuš ₃ 1(diš) šu-si	54	2(u) ninda	4
4(diš) 1/2 danna	2.15	2(u) 5(diš) danna	12.30	5/6 kuš ₃ 1(diš) šu-si	56	3(u) ninda	6
5(diš) danna	2.30	2(u) 6(diš) danna	13	5/6 kuš ₃ 1(diš) šu-si	58	4(u) ninda	8
5(diš) 1/2 danna	2.45	2(u) 7(diš) danna	13.30	1(diš) kuš ₃	1	5(u) ninda	10
6(diš) danna	3	2(u) 8(diš) danna	14	1(diš) 1/3 kuš ₃	1.20	1(diš) UŠ	12
6(diš) 1/2 danna	3.15	2(u) 9(diš) danna	14.30	1(diš) 1/2 kuš ₃	1.30	2(diš) UŠ	24
7(diš) danna	3.30	3(u) danna	15	1(diš) 2/3 kuš ₃	1.40	3(diš) UŠ	36
7(diš) 1/2 danna	3.45	3(u) 5(diš) danna	17.30	2(diš) kuš ₃	2	4(diš) UŠ	48
8(diš) danna	4	4(u) danna	20	3(diš) kuš ₃	3	5(diš) UŠ	1
8(diš) 1/2 danna	4.15	4(u) 5(diš) danna	22.30	4(diš) kuš ₃	4	6(diš) UŠ	1.12
9(diš) danna	4.30	5(u) danna	25	5(diš) kuš ₃	5	7(diš) UŠ	1.24
9(diš) 1/2 danna	4.45			1/2 ninda	6	8(diš) UŠ	1.36
1(u) danna	5			1/2 ninda 1(diš) kuš ₃	7	9(diš) UŠ	1.48
1(u) 1/2 danna	5.15			1/2 ninda 2(diš) kuš ₃	8	1(u) UŠ	2
1(u) 1(diš) danna	5.30	1(diš) šu-si	2	1/2 ninda 3(diš) kuš ₃	9	1(u) 1(diš) UŠ	2.12
1(u) 1(diš) 1/2 danna	5.45	2(diš) šu-si	4	1/2 ninda 4(diš) kuš ₃	10	1(u) 2(diš) UŠ	2.24
1(u) 2(diš) danna	6	3(diš) šu-si	6	1/2 ninda 5(diš) kuš ₃	11	1(u) 3(diš) UŠ	2.36
1(u) 2(diš) 1/2 danna	6.15	4(diš) šu-si	8	1(diš) ninda	12	1(u) 4(diš) UŠ	2.48
1(u) 3(diš) danna	6.30	5(diš) šu-si	10	1(diš) 1/2 ninda	18	1/2 danna	3
1(u) 3(diš) 1/2 danna	6.45	6(diš) šu-si	12	2(diš) ninda	24	2/3 danna	4
1(u) 4(diš) danna	7	7(diš) šu-si	14	2(diš) 1/2 ninda	30	5/6 danna	5
1(u) 4(diš) 1/2 danna	7.15	8(diš) šu-si	16	3(diš) ninda	36	1(diš) danna	6
1(u) 5(diš) danna	7.30	9(diš) šu-si	18	3(diš) 1/2 ninda	42		

**§9.5. Heights, depths
(sukud, bur₃)**

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