Translating Writings of Early Scholars in the Ancient Near East, Egypt, Greece and Rome

Beiträge zur Altertumskunde

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Band 344

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Methodological Aspects with Examples

Edited by
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DE GRUYTER

ISBN 978-3-11-044704-0
e-ISBN (PDF) 978-3-11-044881-8
e-ISBN (EPUB) 978-3-11-044817-7
ISSN 1616-0452

Library of Congress Cataloging-in-Publication Data
A CIP catalog record for this book has been applied for at the Library of Congress.

Bibliografische Information der Deutschen Nationalbibliothek
Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über http://dnb.dnb.de abrufbar.

Druck und Bindung: Hubert & Co. GmbH & Co. KG, Göttingen
♾ Gedruckt auf säurefreiem Papier
Printed in Germany

www.degruyter.com

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Section III: **Ancient Greece and Rome**

Jim Ritter (Paris)

Translating Babylonian Mathematical Problem Texts

Every translation is a partial translation. This is not only because the semantic range of any given word in the source language is never coterminous with the semantic range of the translated word in the target language. Even more importantly, it is so because a text is never an isolated artifact; it is necessarily embedded in a cultural matrix which shapes its meaning in an essential manner.[1] If the first of these two difficulties lies beyond the translator's – or any individual's – capacity to change, the second can be dealt with to some extent through a series of choices.

Firstly, it goes without saying, the translator must attempt to understand as far as possible the actual context(s) in which his or her text would have been read at the time of its composition. In our case here of Old Babylonian mathematics it is difficult to go beyond the simple fact that it seems to have taken place in an educational setting.

Secondly, in the absence of empirical knowledge of the societal context, one may turn to the *formal* characteristics of the text in the search for an informative Mesopotamian context. It is this approach which we adopt here, since it is particularly adapted to the case of mathematical texts. Then, as now, this genre is marked by a very rigid word order, a stereotyped and reduced verbal structure and a restricted vocabulary. These tight constraints on the rhetoric are not an arbitrary choice; they exist because of the need for maximal clarity and minimal ambiguity in the reasoning or the procedure being presented.

But naturally, the specific choices of these simplified, repetitive, and stereotyped structures cannot be the same in Akkadian and in English. The point is not to translate from the original's fixed technical language into the target language's contemporary equivalent (a Babylonian problem text cannot and should not sound like a published article in a modern mathematical journal or even a contemporary mathematical schoolbook) but rather to adopt a translation style which captures those aspects of the original style which play functional roles in the comprehension and utilization of the genre.

1 For a discussion of these issues in general, see the various contributions in Chemla (2004) and for the particular class of texts under discussion in this chapter, see Ritter (2004) in that same volume.

1 Mathematics in Mesopotamia

In one sense, mathematics has existed in Mesopotamia as long as writing has; long before writing was adapted for recording the spoken language, the first texts counted, measured, added and divided the economic, social and religious wealth of the community.[2] For a thousand years, from the middle of the fourth millennium BCE to the middle of the third, these accounting texts form the overwhelming majority of extant texts.[3] But from the Early Dynastic III period, around the middle of the third millennium, we start to find occasional texts which may be classified as mathematical in a stricter sense: metrological-mathematical tables and student exercises. Towards the end of that millennium, the use of a positional base-60 numerical system is in force for calculational purposes, though metrological systems resist to some extent this change. Finally, the beginning of the second millennium marks the appearance of mathematical problem texts, those which we will be treating in this chapter.[4]

Nonetheless, it should be remembered that of the several hundreds of thousands of cuneiform tablets known from Mesopotamia, only a few hundred are of a mathematical nature in the sense used here, that is that present a problem (with or without an explicit solution) or a table of a mathematical nature (inverses, squares, square roots, constants, ...) or a mathematical exercise. The vast majority of these tablets date from the Old Babylonian Period (roughly between 2000 and 1600 BCE) and we shall here be focused exclusively on texts from this period. In particular, we shall be looking at and translating mathematical *solved-problem texts*, that is, texts which pose a mathematical problem involving numerical data and ask for a numerical answer. The solution to the problem is then developed as a step-by-step procedure, each step invoking either an *arithmetic operation* (an operation which transforms one number into another) or a *control command* (an operation which operates a change in the information flow of the solution algorithm),[5] with the answer to the problem given as the final result.

2 Advances in the understanding of the origins of writing and mathematics in Mesopotamia are resumed and developed in Nissen, Damerow, and Englund (1990/1993).

3 Indeed, accounting and administrative texts play a dominant role for all periods of Mesopotamian history.

4 For an overall view of Mesopotamian mathematical texts see the detailed review article in the *Reallexikon der Assyriologie und Vorderasiatischen Archäologie* by Jöran Friberg (Friberg 1987–90).

5 We shall treat the terms ‘command’ and ‘operation’ as interchangeable synonyms.

2 Mathematical solved-problem texts

These texts were first really understood thanks to the pioneering work of Otto Neugebauer and François Thureau-Dangin in the nineteen-thirties,[6] who located such texts in museum collections and systematically published a large number of them.[7] We now know roughly one hundred of these solved-problem texts and the nearly 500 problems – completely or incompletely preserved – they contain. An individual tablet can contain anything from a single problem to over forty related or unrelated problems.

This class of texts is as well-defined by its formal structure as by its content. Solved-problem texts have four parts; in their order of appearance:

1° a title, usually a single word (optional)
2° the presentation of the problem
3° the question(s) to be answered (occasionally absent)
4° the method of solution of the problem

These four parts, well distinguished by their differing content, are equally so by their formal, grammatical structure, in particular the verbal tenses used.[8] In particular, part 4, the solution algorithm uses the Imperative or the Durative, while in the problem presentation, parts 2 and 3, when a finite verb is used it is generally in the Preterite.

Our translations will use English tensed forms to capture these distinctions with the following equivalences:

Akkadian verb form	**English verb form**
Preterite	Preterite
Durative	Future[9]
Imperative	Imperative

6 For a historical study of this early period in Assyriology see Høyrup (1996) and the various contributions in the proceedings of the Neugebauer Conference: Jones, Proust, and Steele (2015).

7 These classical compendia are Neugebauer (1935–1937), Thureau-Dangin (1938) and Neugebauer and Sachs (1945).

8 I shall use the somewhat inexact expression ‘verbal tense’ to refer to all the finite forms of the Akkadian verb: the three indicative inflected forms – Preterite, Durative and Perfect – as well as the two injunctive forms – Imperative and Precative.

9 We choose the English future rather than, say, the present for the solution procedure because the former tense ‘you *will* do thus-and-so’ carries a remnant of its modal past, quite appropriate to its role here as an instruction or command.

If the solution algorithm in the mathematical texts causes few difficulties in determining the main verbal tense, the frequent absence of finite verbs in the presentation part of the problem – which generally employs verbless nominal phrases, corresponding to English phrases construed with forms of the verb 'to be' – poses a difficulty. In the absence of a copula in Akkadian, there is no intrinsic way of determining the equivalent English form of 'to be' in such cases; parallelism with other finite verbs must then determine the English tense of 'to be' that should be used.

We will take these grammatical distinctions all the more seriously in that precisely the same distinctions reappear in the same relative positions in texts of quite different genres: medicine, divination and jurisprudence (domains which, with mathematics, I have called those of "rational practice") of the same period.[10]

3 Abstract numbers

The numerical system used in the solution of the problems is, as is well-known, a sexagesimal (base 60) place-notational one. What is perhaps less appreciated is that this system is in many ways an artificial abstract one, distinct from, though related to, contemporary metrological systems.[11]

The convention used here for transcribing numbers in the abstract sytem is to use 2 decimal digits to represent each individual sexagesimal digit. A zero will be placed in front of sexagesimal 1 through 9, except when these sexagesimal digits occur in the first position of a number. Moreover, a blank space will be used to separate sexagesimal digits. Thus,

will be transcribed 3 30 56 04.

In the absence of an explicit zero, the actual value – I will call this the absolute value – of a given number written in cuneiform is ambiguous, thus, 1 30 might represent any of the following absolute values (here, the semicolon ';'

10 See Ritter (2010). Later, in the first millennium, such domains as numerical astronomy enter the picture with similar distinctions in their procedure texts; see the chapter by Ossendrijver in this volume.

11 For one discussion of the third-millennium development of this abstract system from the various metrological systems see Ritter (1999) and Ritter (2001).

represents the Mesopotamian decimal point, marking the division between the integer and fractional part of a number):

$$1\ 30;00 = (1 \times 60^1) + (30 \times 60^0) = 60 + 30 = 90$$

or

$$1;30 = (1 \times 60^0) + (30 \times 60^{-1}) = 1 + 30/60 = 1\tfrac{1}{2}$$

or

$$0;01\ 30 = (1 \times 60^{-1}) + (30 \times 60^{-2}) = 1/60 + 30/3600 = 1/40 = 0.025$$

or

$$1\ 30\ 00;00 = (1 \times 60^2) + (30 \times 60^1) = 3600 + 1800 = 5400$$

or any other pair of successive powers of 60. Note that non-contiguous powers of 60 are excluded here since something like

$$(1 \times 60^2) + (30 \times 60^0) = 3600 + 30 = 3630$$

would have been written 1 30 (𒁹 𒌍), with an empty space left between the two non-contiguous sexagesimal digits. Note that to indicate that a sexagesimal number is an integer, a sexagesimal zero (decimally indicated by 00) will be added after the semicolon: e.g., 27;00 is the sexagesimal integer 27.

Determining the absolute value of a number is deciding where in that number is to be found the unit's place, or in other words, where one should put the semicolon that divides the integer part of a number from the fractional part. There do exist, however, certain ways of signaling the absolute value of a number when necessary in the problem texts and we shall see one of them in our examples (Problem II).

Since often the absolute values can only be determined after the whole problem has been analyzed, we shall adopt the convention of writing in the transliteration and transcription only what is written on the tablet, e.g., in the case of the above number "1 30", leaving the question of the precise powers of 60 (i.e., the position of the semicolon) for the last step of the translation. The determination of absolute values will depend on the existence of certain arithmetic operators like addition, subtraction or square root in the problem, whose determination depends on the absolute values of its arguments whereas other operations, such as multiplication or inverse, are invariant under change of absolute value and thus of no help in this task.[12]

12 It should be borne in mind that in the absence of explicit metrological values in a given problem, a unique solution for the attribution of absolute values cannot be guaranteed (Problems I and III). The question of how the ancient readers of these texts understood the absolute values is unclear, perhaps through oral indications.

4 Metrological numbers and units

Unlike the abstract system used for performing the calculations in the solution-algorithm section of problem texts, the normal metrological systems were not in general sexagesimal, though there was a constant pressure throughout Mesopotamian history to reshape them in sexagesimal form.[13] As in American, Liberian and Burmese (Myanmar) elementary mathematical schoolbooks today, a not inconsiderable number of Mesopotamian problems involved the translation of metrological units given in the presentation of the problem into the abstract system for the calculational work in the algorithm, followed by the translation of the answer into real-life metrological units at the end of the calculation (see Problem II).

The situation is rendered still more complicated by the fact that the cuneiform writing of the quantitative metrological values depended on the particular metrological system in which one was working. The writing of 'numbers' for surfaces and volumes, for instance, differed from each other as well as from the writing of abstract numbers.

When dimensional values are used in mathematical problem texts there is a default choice of unit for each metrological system. For example, lengths are understood to be in *nindanum* and as such will normally not mention the unit explicitly. If another length unit is used the value will have the explicit unit name attached (Problem II). Similarly, the default units for area and ground-volume are *mušarum* and for weight *manûm*.

It is frequent in modern work to use the Sumerian names of metrological units, based, one supposes, on the fact that in the vast majority of cases it is the Sumerian logograms of these units which are used in the text. However, the occasional use of full writings of the Akkadian absolute form (Problem II) suggests that in fact these units were anciently read in that language. Thus, though we shall transliterate them following their Sumerian logographic form, we shall transcribe them by their Akkadian equivalents.

As for the question of the English version of these unit names, modern sources generally choose to translate them either into 1° contemporary metric system equivalents (e.g., **sìla**, *qûm* = "liter") or into 2° 'biblical' units (e.g., **kùs**, *ammatum* = "cubit") or into 3° literal translations of the Mesopotamian terms (e.g., **nindan**, *nindanum* = "rod"). The first choice is totally anachronistic and has the added disadvantage of the necessity of requiring a change in almost all

13 See Powell (1987–90) for a detailed study of these metrological systems.

numerical values as well; this is clearly unadvisable and is to be ruled out. The second choice is more defensible from a historic point of view (the units used in the Old Testament are after all historically derived from the Mesopotamian systems), but today, when an intimate knowledge of the Bible is no longer current among young Assyriologists, we face the situation where unknown Akkadian names are replaced by equally unknown anglicized Hebrew ones.[14] Finally, not all ancient metrological names are easily translatable or even known. Since the Old Babylonian metrological system must be learned in any case in order to understand Old Babylonian mathematical texts, we shall simply leave metrological terms in the original Akkadian, in an invariable nominative case.

The principal Old Babylonian units for length, area, volume, capacity and weight are given in the Metrological Diagram (see the Appendix, C., Fig. 1).

5 Termini technici

The question of the translation of ‘technical’ terms is to be decided on the basis of their functionality in the carrying-out of each step of the procedure. As we have already pointed out, every sentence of the solution procedure corresponds to a command (‘do this’ or ‘you will do this’) either arithmetic or control. The result of this command is then, in the arithmetic case, the determination (add, multiply, find the inverse, …) of a number or, in the control case, its disposition (write down, store, …). In all cases, what will interest us in our translations is strictly what is to be found on the tablet, that is the arithmetic determination of a number by an operation on one or more numerical arguments or the disposition of a value. Therefore, we shall ‘flatten out’ some of the distinctions made at times in the literature, translating for example the Akkadian verbs *kamārum* and *waṣābum* indifferently as “to add” (though of course these verbs will be distinguished by their distinctive use of *u* (“and”) or *ana* (“to”) respectively as the word linking their two arguments). We shall not try to go beyond the direct material evidence nor speculate on possible origins of the material.[15]

Enclitic particle -*ma*. The enclitic particle -*ma*, attached as suffix to finite verbs or to predicates in general, is normally to be translated as “and” or “then”,

14 Though the two cases of “cubit” in length measure and “mina” as a weight measure have slightly more currency, the advantage to be gained by introducing them as exceptions hardly outweighs the inconsistency.

15 See the discussion in the following section 6: “‘Translating’ solved-problem texts”.

occasionally “but”. In mathematical problem texts it can appear in such a role in the problem presentation, in which case it will generally have the meaning “then” and will be so translated. However, more specific to mathematical texts, it is also used to set off a number, either a datum in the problem presentation or the numerical result of effecting an arithmetic operation in the solution algorithm. We shall translate this latter use by a dash “—”.

Constants. The mathematical sciences of any culture use a series of constants which serve to aid in calculation or in conversion from one metrological system to another. Our own modern society uses a whole plethora of them: *mathematical* constants, such as the ratio of the circumference of a circle to its diameter, π (3.14159…) and the base of natural logarithms e (2.718281828…); as well as *physical* constants such as the speed of light, c (2.998×10^8 m/s) and the electron mass m_e (9.10938×10^{-31} kg); *chemical* constants such as Avogadro’s number N_A (6.022141×10^{23} molecules/mole) and the heat of vaporization of water (540 cal/g); etc. The Mesopotamians also constructed tables for their numerical constants, known as *igigubbûm*.[16] These, like ours, contained values for mathematical, physical and other constants, though they were not the same as ours; for example, there is a Mesopotamian constant for the circle – not our π (3.14159…) but the number 5 (absolute value 0;05), since where we use π to calculate the area of the circle by multiplying it by the square of the radius, in Mesopotamia, the same calculation was effected by multiplying 0;05 times the square of the circumference (Problem II).[17]

6 ‘Translating’ solved-problem texts

Mathematical problem texts pose a series of problems which, at first glance, seem specific to the domain and these must be faced before any decisions about the content of a translation can be taken. For what commonly counts as a ‘translation’ of a mathematical text differs considerably from that of say a royal inscription or a religious hymn or a personal letter. Unlike these last, a

16 The traditional translation of *igigubbûm* as “coefficient” is most unfortunate, as the various meanings of the English word all imply a specific relationship to algebra or algebraic formulations of physical laws, none of which applies to the Mesopotamian case. I shall only use the term “constant” for *igigubbûm*. For a detailed study of these constants see Robson (1999).

17 That is, the functional role of the Babylonian constant of the circle corresponds to our modern $(4\pi)^{-1}$.

straightforward replacement of the Akkadian words of a mathematical text by those of the target language is far from sufficient to render it comprehensible, something that can be seen by attempting to read one of the classics of the modern study of Babylonian mathematics: the *Textes mathématiques babyloniens*.[18] The author, arguably the greatest Assyriologist of his generation, had worked on and published solved-problem texts for almost a decade when he collected all the texts then available to him and published in book form their transliteration and French translation, but without any elaboration or commentary.[19] The problem lies in the fact that a purely verbal rendition remains essentially opaque to the modern reader, even a mathematically sophisticated one; standing alone such a translation is unusable.

Hence, in translating an ancient mathematical text it is necessary to go beyond a simple replacement of words. Some sort of 'interpretation' must be offered to the reader (and this includes the translator himself), a rewriting of the purely rhetorical text that will convey an understanding of the content of the text. Traditionally, since the earliest modern translation of a Babylonian solved-problem text – that of the then-unpublished Berlin text VAT 6598 by Ernst Weidner during the First World War[20] – it has been the recasting of this material in elementary algebraic form that has dominated the Assyriological literature. It was the 'natural' mathematical form for such an undertaking, be it by a professional mathematician like Otto Neugebauer or an amateur mathematician like François Thureau-Dangin. But there is no evidence that the manipulation of abstract symbols played any role in ancient Mesopotamian society and, although such a rewriting may be attractive for a contemporary audience, we shall pass it up here as overly anachronistic.

A second type of recasting has been put forward since 1984 by the Danish historian of mathematics, Jens Høyrup.[21] By means of a detailed analysis of the vocabulary used in the mathematical texts, he presented powerful arguments for understanding a certain class of these as geometrical 'cut-and-paste' constructions. Though compelling in a number of respects, we shall not adopt this approach here, in large part because it does not apply to all types of problems

18 Thureau-Dangin (1938).

19 Thureau-Dangin inserted references to his own detailed treatments of the problems, published from 1930 on in the *Revue d'Assyriologie et d'Archéologie orientale*, as well as to Otto Neugebauer's exhaustive analyses of these texts in his three-volume *Mathematische Keilschrift-Texte* that had appeared a few years earlier (Neugebauer 1935–37).

20 Weidner (1916).

21 See Høyrup (2002).

(our Problems I and II, for example, do not allow such an interpretation, though Problem III does).

A good interpretive level would be one which would use only the cognitive tools used in Ancient Mesopotamia, which would apply to all mathematical problem texts and which would remain as close as possible to the formal structure of the text. Such an approach was first suggested for Babylonian mathematical texts as early as 1972 by the pioneer of computer algorithmic, Donald Knuth,[22] an algorithmic approach based on sequences of arithmetic and control commands. Unfortunately, the paper was published in a computer science journal and so remained unknown to Assyriologists in general until this approach was independently rediscovered in the late 1980s.[23] The advantages in this approach are major. First of all, the problem texts as they have come down to us *are* algorithmic in nature. There is no need to move beyond what is actually written and the same approach applies to the whole corpus. Moreover, the dual nature of the commands in the text – calculational (arithmetic operations) and control (initialization, storage, parallel computing) – parallels exactly the dual nature of modern programming. Perhaps most importantly, this interpretation is minimal in the sense that it does not block an algebraic or geometric further development if the reader or translator so desires. It is therefore this algorithmic extension that will be used as an interpretive framework.

7 Presentation

In what follows we shall decompose our presentation of a mathematical text into six parts making up the *Principal Section*. The first three are conventional in all Assyriological publications:

22 Knuth (1972) and Knuth (1976).

23 Ritter (1989b). The algorithmic approach proposed there in the case of Egyptian and Babylonian mathematics was more consciously influenced by Karine Chemla (1987) and extended to Babylonian medical, divination and juridical texts in Ritter (1989a). Further work in this direction has been carried out by Annette Imhausen (2003) in Egyptian mathematics and Mathieu Ossendrijver (2013) in Babylonian mathematical astronomy. See also their chapters in this volume.

1° photograph
For those tablets in museum collections, a large (and increasing) number of excellent high resolution photographs are available online in the Cuneiform Digital Library Initiative (CDLI) project at http://cdli.ucla.edu.

2° hand copy
Such a copy should aid the reader in identifying signs, particularly for those which may be unclear in the photograph but which the copyist has been able to disambiguate by actual hands-on manipulation of the original tablet. For published texts, such a handwritten copy is normally available in the edition publication. A number of these are also available on the CDLI site mentioned above.

3° transliteration (see also the explanation in the Appendix, A.)
Since Old Babylonian solved-problem texts were written in Akkadian, this means that a phonetic sign is replaced by its Akkadian phonetic equivalent (in *italic*), and, in the rarer cases of a logogram or of a semantic determinative, by its Sumerian reading when known (in ***boldface italic***). For those few signs whose specific Akkadian or Sumerian reading is not known in the context in which it appears, the name of the sign will be given in SMALL CAPITALS.

Special Signlists should be consulted for the phonetic values. For these see the Appendix, D.

Special notation for indicating destroyed or illegible signs, scribal errors, etc., is listed in the Transliteration Conventions List, see the Appendix, B.

A fourth step, although commonly carried out by the modern scholar, is almost never actually included in the published edition of the text but is, it seems to me, an essential part of the translation process. Therefore, we will explicitly include:

4° transcription (see also the explanation in the Appendix, A.)
Here one reconstructs, as far as possible, the actual Akkadian words that the transliteration represents. Evidently, a good grasp of Akkadian grammar is necessary here. For the standard Akkadian grammars see the Appendix, D.

In the context of our algorithmic approach to mathematical problem texts an additional step is useful before coming to the translation proper:

5° algorithmic form
The Akkadian transcribed text is first divided up into its four parts – title, presentation, question, solution algorithm, and the last is then subdivided

into its constituent steps, normally one verbal sentence per step (including the result), guided by the grammar and sense of the problem.

The solution algorithm will generally, though not invariably [Problems I, III], be framed by an initial expression, generally *atta ina epēšīka* (“You, in your proceeding:”) and terminated by a final expression, *kīam nēpešum* (“Such is the procedure.”) or some parts of these phrases. Moreover, given the very stereotyped nature of mathematical texts, the division into steps is generally quite easy. A step will almost invariably be a complete sentence with the numerical result indicated following the sentence-final verb in the case of an arithmetic command or without a numerical final value in the case of a control command. Intermediate results are sometimes given names or commentaries and these can be accommodated in a given step by giving the command and the name or commentary, separated by a large spacing.

The numbering of the solution algorithm used here will be the symbol ► for the introductory *atta ina epēšīka* and ◄ for the final *kīam nēpešum*, and with boldfaced **1**, **2**, ..., **N** for the N steps of the algorithm itself (but see Problem III for modifications to this basic numbering schema). The original tablet lines will be indicated by raised numbers to allow easy reference back to the transliterated text as it occurs on the tablet.[24] Note that any special notation signs [ ], < >, etc., are dropped at this stage.

Finally, there is:

6° the translation of the problem into English, keeping the algorithmic structure

It is at this point that the numerical values occurring in the text, relative up to this point, will be replaced by their absolute values, or, where several sets of such values are possible, by a choice of the simplest such set. For translation, the use of a good modern Akkadian dictionary is essential (for these see the Appendix, D.).

Words and expressions not in the original Akkadian text but necessary in the target language can be added at this stage, enclosed in parentheses ‘( )’.

24 The ‘one sentence = one command’ identity is often reflected in a general coincidence between beginnings of lines on the tablet and beginnings of steps in the solution algorithm section. Compare this to the lack of such conformity in the discursive Presentation and Question sections.

This Principal Section will be followed by:

7° the Commentary
Here, organized by parts of the problem and by step numbers in the solution part of the problem, will be placed the grammatical and syntactic details of the choices made in the English translation. Here, too, will be placed the preliminary indicators of organizational questions arising in the text; normally, further development of these questions will be taken up in the following Discussion. Here, finally, will be found the argument for the choice of absolute numerical values that appear in the translation.

The third and final section for each text will be:

8° the Discussion of the solution algorithm
The minimalist algorithmic interpretation will be developed here by looking at the way that information flows in the algorithm and pursuing a double rewriting in which this flow will be indicated in an abstract form that keeps to the structure of the text while revealing the information flow.

Problem I: YBC 6295

This tablet, in the Yale Babylonian Collection, was purchased on the market and its provenance is unknown.[25] It was first published by Neugebauer and Sachs in 1945.[26]

In its disposition this is a typical single problem tablet, with just the obverse inscribed. The reverse is blank, except for a short inscription: *a-na* **ᵈ***Sîn*, “To (the moon-god) Sîn”.

This is one of the rare problem texts which concern details of computation practices, though its practical applicability is in fact severely limited.[27] The point of the problem would seem to be to provide practice in working with cube roots and their tables as well as in handling the abstract number system.

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25 I should like to thank Ulla Kasten of the Yale Babylonian Collection and its curator, Professor Ben Foster, for providing the new photograph of the tablet and permission to publish it here.

26 Neugebauer and Sachs (1945): 42 (Aa).

27 See the end of the Discussion for this tablet.

1. Photograph – YBC 6295

Fig. 1: Photograph – YBC 6295

2. Hand copy – YBC 6295

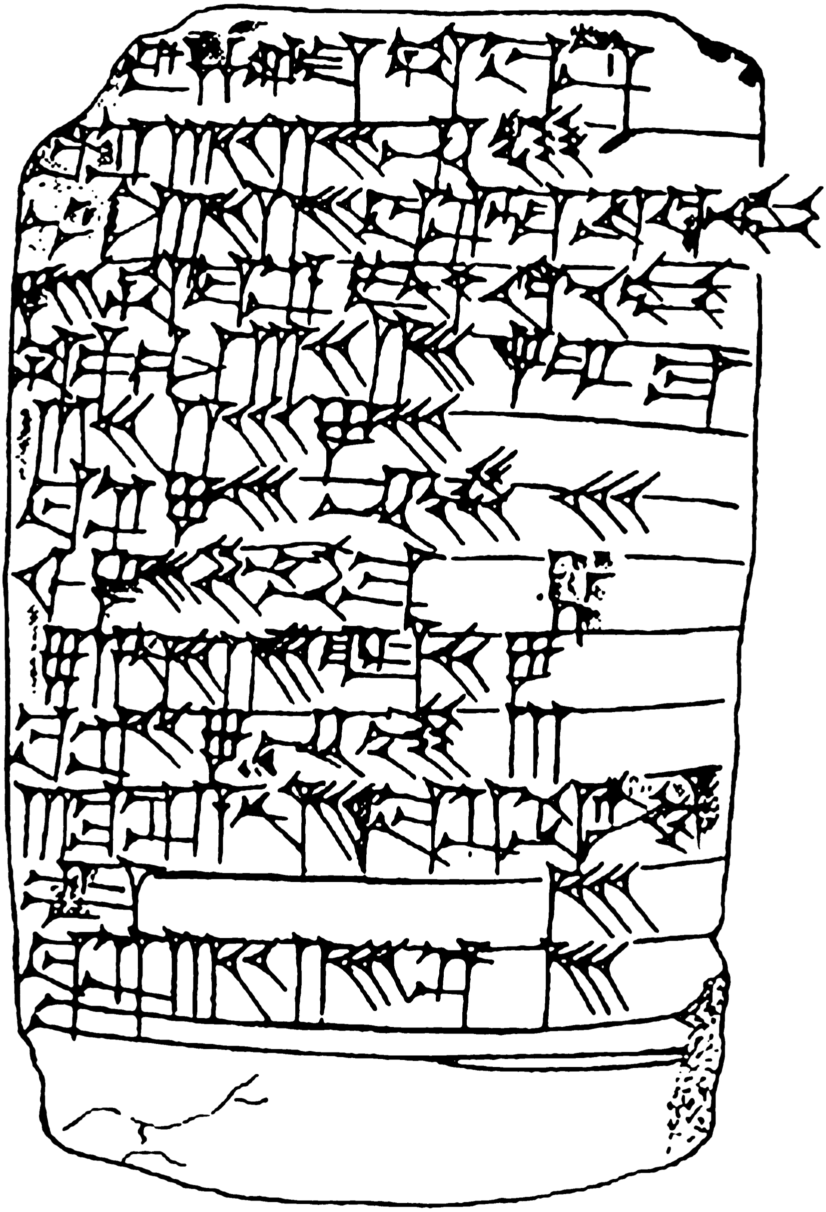

Fig. 2: Copy – YBC 6295

3. Transliteration – YBC 6295

obverse

(1) *[ma]-ak-ṣa-ru-um ša* ***ba-si***
ba-si *3 22 30* ***en-nam***
aš-šum 3 22 30 ***ba-si*** *la id-di-nu-kum*
7 30 ša ***ba-si*** *i-na-di-nu-kum*
(5) *ša-pa-al 3 22 30* ***gar-ra****-ma*
3 22 30 7 30
ba-si *7 30* ***en-nam*** *30*
igi *7 30 pu-ṭur-ma 8*
8 a-na <3> *22 30* ***íl*** *27*
(10) ***ba-si*** *27* ***en-nam*** *3*
3 ***ba-si*** *a-na 30* ***ba-si*** *ša-ni-im*
íl *1 30*
ba-si *3 22 30-****e*** *1 30*

4. Transcription – YBC 6295

obverse

(1) *[m]akṣarum ša basîm*
basi 3 22 30 mīnum
aššum 3 22 30 basâm la iddinūkum
7 30 ša basâm inaddinūkum
(5) *šapal 3 22 30 šukunma*
3 22 30 7 30
basi 7 30 mīnum 30
igi 7 30 puṭurma 8
8 ana <3> *22 30 iši 27*
(10) *basi 27 mīnum 3*
3 basâm ana 30 basîm šanim
iši 1 30
basi 3 22 30 1 30

5. Algorithm – YBC 6295

[1]*makṣarum ša basîm*

[2]*basi 3 22 30 mīnum*

	[3]*aššum 3 22 30 basâm la iddinūkum*
1	[4]*7 30 ša basâm inaddinūkum* [5]*šapal 3 22 30 šukunma*
	[6]*3 22 30 7 30*
2	[7]*basi 7 30 mīnum 30*
3	[8]*igi 7 30 puṭurma 8*
4	[9]*8 ana 3 22 30 iši 27*
5	[10]*basi 27 mīnum 3*
6	[11]*3 basâm ana 30 basîm šanim* [12]*iši 1 30*
	[13]*basi 3 22 30 1 30*

6. Translation – YBC 6295

The *makṣarum* of a (cube) root.

What is the cube root of 3 22 30?

►	As for 3 22 30, the root has not been given to you.
1	Put 7 30, for which the root is given, below 3 22 30. —
	3 22 30 0 07 30
2	What is the root of 7 30? 30.
3	Find the inverse of 7 30 — 8.
4	Multiply 8 by 3 22 30: 27.
5	What is the root of 27? 3.
6	Multiply 3, the root, by 30, the second root: 1 30.
	The root of 3 22 30: 1 30.

7. Commentary – YBC 6295

Title:

makṣarum. This term is a nominal formation meaning a "bundle" or "bale" (from the verbal root *kaṣārum*). A technical term in a few mathematical texts; judging from its use here, it has a technical meaning something like "coupling (one value with another)" in the sense of our 'interpolation'.

basîm. Written using the Sumerian logogram ***ba-si***. The Akkadian *basûm* is clearly a loanword from the Sumerian.

(Presentation) Question:

No presentation is necessary in that the question contains all the information necessary for its resolution.

► *aššum*. A non-standard 'begin algorithm' indicator, the *aššum* – normally "because, since; as for" – plays the role of indicating the *reason* for the algorithm which follows. This use corresponds to the use of 'given that' in contemporary mathematical writing. (Compare the use of *aššum* in Old Babylonian letters to indicate the start of a new subject).

iddinūkum. G-Pret 3rd person masculine plural of *nadānum* "to give" + *-kum* dative of 2nd person sg pronoun "to you". Literally "They have (not) given you". The 3rd person masculine plural of verbs is used for the impersonal (von Soden [1995]: §75 i): '*man*' (German) or '*on*' (French). In English, it is usually preferable to translate with a passive.

1 *inaddinūkum*. G-Durative 3rd person plural of *nadānum* + *-kum*. Note the change of tense with respect to the same construction in ►.

šapal. St. const. of *šaplum* "below". Though the meaning is "below", the positioning of the two signs resulting from this step is side by side. This use of a vertically oriented command and a horizontally oriented execution is standard in the mathematical texts and probably goes back to the third-millennium 90° shift in the direction of writing.

šukunma. G-Imp of *šakānum* "to put, place" + *-ma*. The verb is used as a control operation with the meaning of "assign the value", "initialize the algorithm with the following values: ...".

Note that the enclitic particle shows that the parallel writing down of the pair of values – 7 30 and 3 22 30 – is understood as the *result* of the operation of placement and thus, an enclitic *-ma* is used to introduce this result (compare the different situation in Step **1** of Problem II).

2 A nominal phrase in Sumerian logograms. Note the absence, as often, of *-ma* after the *mīnum* (***en-nam***) here and in the similarly constructed Step **5**.

3 *igi … puṭurma*. "Find the inverse of …" *puṭurma* = G-Imp of *paṭārum* "to loosen, release" + *-ma*. Used in mathematical texts, this is a technical expression for the calculation of an inverse $X^{-1} = 1/X$.

4 *iši* (***íl***). G-Imp of *našum* "to multiply". The construction is: X *ana* Y *našum* "to multiply X by Y" (X, Y numbers). Though *našum* usually has *-ma* attached when written phonetically, here the logographic writing does not.

5 See Step **2**.

6 See Step **4**. The two cube roots distinguished as the "(first) root", i.e., the most recently calculated root (Step **5**) and the "second root", that root calculated in a more distant step (Step **2**). The pair *basûm* and *basûm šanum* is an example of a common phenomenon in Akkadian, in which the first in an ordered sequence of objects is called simply by the name of the object; hence, simply *basûm* rather than a possible *basûm ištênum*. Similarly in Problem III, Steps **7a** and **7b**.

To determine the absolute values used in the problem, we focus our attention first on those arithmetic command steps which are sensitive to absolute value. Neither multiplication nor inverses are sensitive at all, but roots, addition and subtraction are. Now we have two steps with cube roots (**2** and **5**). Step **5** yields a set of possible pairs of values (27×60^{3n}, 3×60^{n}) for $n = \ldots, -2, -1, 0, 1, 2, \ldots$, and we adopt, as a first try, the simplest set (27;00, 3;00) ($n = 0$). Similarly in Step **2**, taking 0;30 (= ½) as the simplest possibility for the root yields $0;30 \times 0;30 \times 0;30 = (½)^3 = ⅛ = 7 \times 60^{-1} + 30 \times 60^{-2} = 0;07\ 30$ (as well of course as all other pairs of the form ($7 \times 60^{3n+2} + 30 \times 60^{3n+1}$, 30×60^{n}). $n = \ldots, -2, -1, 0, 1, 2, \ldots$). This fixes all the other steps:

3 $(0;07\ 30)^{-1} = \mathbf{8;00}$
4 $8;00 \times |3\ 22\ 30| = 27;00 \Rightarrow |3\ 22\ 30| = \mathbf{3;22\ 30}$.

6 $3;00 \times 0;30 = \mathbf{1;30}$

where we have used the convention that, in analogy with the symbolism for the (very different) mathematical sense of 'absolute value', we represent by |X| the absolute value (in our sense) of the relative number X. In addition, the determination of an absolute value of a number in a step is represented by putting that new absolute value in boldface.

We thus have one choice for absolute values for our problem, which we can now write as follows:

The *makṣarum* of a (cube) root.

What is the cube root of 3;22 30?

► As for 3;22 30, the root has not been given to you.

1 Put 0;07 30, for which the root is given to you, below 3;22 30.
3;22 30 0;07 30
2 What is the root of 0;07 30? 0;30.
3 Find the inverse of 0;07 30 — 8;00.
4 Multiply 8;00 by 3;22 30: 27;00.
5 What is the root of 27;00? 3;00.
6 Multiply 3;00, the root, by 0;30, the second root: 1;30.
The root of 3;22 30: 1;30.

Of course, it should be kept in mind that any other choice of absolute value in the permitted values in Steps **2** and **5** would have resulted in different absolute values in the above solution algorithm. For example, one might start with the hypothesis that the number whose cube root is demanded is an integer, namely 3 22 30;00 (= 12150 in decimal notation). Then, if 7 30 is also an integer by Step **2** it must be $30;00^3$ = 7 30 00;00 (= 27000 in decimal notation). Putting this into Step **3** gives the $(7\ 30\ 00;00)^{-1}$ = 0;00 00 08 and Step **4** implies the result 27;00 (just as we started with in our first fixing of the absolute values above). Finally, Step **6** has 1 30;00 as the final result. The whole algorithm would then look like this:

► As for 3 22 30;00, the root has not been given to you.
1 Put 7 30 00;00, for which the root is given to you, below 3 22 30;00.
3 22 30;00 7 30 00;00
2 What is the root of 7 30 00;00? 30;00.
3 Find the inverse of 7 30 00;00 — 0;00 00 08.
4 Multiply 0;00 00 08 by 3 22 30;00: 27;00.
5 What is the root of 27;00? 3;00.
6 Multiply 3;00, the root, by 30;00, the second root: 1 30;00.
The root of 3 22 30;00: 1 30;00.

And, of course, there are still other possibilities. In what follows we shall retain the first set of absolute values, with the number whose cube root is demanded as 3;22 30.

8. Discussion – YBC 6295

The question posed is to find the cube root of a certain number, 3;22 30, which "has not been given to you". The meaning of this rather enigmatic phrase is to be found in the way in which the arithmetic commands called for in solution algorithms were actually carried out: by the use of tables, the extensive use of which is a hallmark of Babylonian mathematical texts.[28] Among the mathematical tables we have from the Old Babylonian period are multiplication tables, tables of inverses, and tables of square and cube roots.[29] It is clearly one of this last category which was referenced here. Indeed, in the extant Old Babylonian cube root tables there is no entry for 3 22 30.

As has been pointed out in the Introduction, we adopt here a procedure in which, rather than seek a rewriting of the text in terms of elementary algebra or geometric manipulations, we attempt to follow as closely as possible the actual organization of the text; our understanding of the text will be advanced by following the flow of information within the text. To see this flow more clearly we will rewrite minimally the text in a two-step-procedure.

1° *Symbolic algorithm.* The first rewriting, which will allow us to see more clearly the nature of the commands which correspond to each step of the solution procedure, is simply to replace the verbal forms by a modern operational symbol, chosen among standard arithmetic and programming symbols:

Number = 3;22 30

▶

1	$\leftarrow$ 3;22 30	$\leftarrow$ 0;07 30	*Initialization*
2	$\sqrt[3]{0;07\ 30} = 0;30$		Cube root
3	$(0;07\ 30)\ )^{-1} = 8;00$		Inverse
4	$8;00 \times 3;22\ 30 = 27;00$		Multiplication
5	$\sqrt[3]{27;00} = 3;00$		Cube root
6	$3;00 \times 0;30 = 1;30$		Multiplication

28 As opposed to Egyptian mathematical practice, for example, in which much of the calculational work was actually done on the spot and makes up part of mathematical problem texts from that culture, see Imhausen (2016) and her chapter in this book.

29 For these tables and others consult Proust (2005).

In addition to the symbols of the operations we have added the names of the operations represented in each step. The symbolism used here is as follows:

$\leftarrow$ *initialization*

As opposed to the *arithmetic* commands of all the other steps in this problem, this is an example of a *control* command.[30] Control commands will be distinguished from arithmetic commands by the use of italics in their name.

Initialization serves to set a number or numbers which will serve as the point of departure of all the calculations to follow. Often a first step, it is not restricted to this position (see Problem III, Step **6**). Its result is simply the argument itself.

$\sqrt[3]{}$ cube root

$^{-1}$ inverse

$\times$ multiplication

2° *Abstract algorithm.* The second rewriting will be to indicate the flow of information in the algorithm by replacing each numerical value used in the text by its source from within or without the text itself. This is particularly easy in this case since everything is carefully labeled in the text, a situation which is not always present in Babylonian problem texts. We shall first add to our symbolic rewriting the identifications of the various numbers appearing in the algorithm there where the text explicitly mentions them, with the names of the *arguments* of each operation next to their value and the names of a *result* of a step in a column to the right:

Number = 3;22 30

▶

1	$\leftarrow$ 3;22 30 $\leftarrow$ 0;07 30 root given	
2	$\sqrt[3]{0;07\ 30} = 0;30$	
3	$(0;07\ 30)^{-1} = 8;00$	
4	$8;00 \times 3;22\ 30 = 27;00$	
5	$\sqrt[3]{27;00} = 3;00$	
6	3;00 root × 0;30 second root = 1;30	root of 3;22 30

30 There exists another control command, ‘parallel initialization’, with two or more arguments similar to the case here, but which introduces two or more series of independent calculations; that this is not an example of parallel initialization is clear from the fact that the calculations which follow use both arguments (3;22 30 and 0;07 30) right from the start. We will see this other type of initialization in Problem III.

In identifying the sources of our numbers we first consider those which are *not* explicitly labeled. These are by far the most numerous: they are numbers which simply repeat the result of the preceding step. Thus, the most general source for a value used in Step **N** is the result of the *preceding calculation* in Step **N**–**1** (and this default situation is the only one which is never explicitly mentioned). For example, in our text the 0;07 30 whose cube root is taken in Step **2** is simply the second of the assigned values in Step **1**, while the 27;00 whose cube root is taken in Step **5** is the result of the immediately preceding multiplication of Step **4**.

Our convention will be to replace each numerical value used in the algorithm (and its name if it has one) by its step number, written in boldface. Thus we will replace "$\sqrt[3]{27;00} = 3;00$" by "$\sqrt[3]{\mathbf{4}}$" for Step **5**, while for Step **2**, instead of "$\sqrt[3]{0;07\ 30} = 0;30$" we shall write "$\sqrt[3]{\mathbf{1b}}$" (the **b** here is to distinguish which of the two values serving as the result of **1** is in question, the other value in **1** being of course designated as **1a**). Moreover, the 3;00 of Step **6**, called "the root", is the immediately preceding result in Step **5**, and we shall replace both the value and its name in **6** by the step number from which it comes. There is no need to write down the numerical *result* of any calculation or control step since the symbol of the result is simply the step number itself. Our algorithm then looks like:

Number = 3;22 30

▶

1	← 3;22 30 ← 0;07 30 root given	
2	$\sqrt[3]{\mathbf{1b}}$	
3	$(0;07\ 30)^{-1} = 8;00$	
4	**3** × 3;22 30 = 27;00	
5	$\sqrt[3]{\mathbf{4}}$	
6	**5** × 0;30 second root = 1;30	root of 3;22 30

In such a way, we can account for all the numbers occurring in the single-argument Steps **2** and **5**, and one of the pair of numbers occurring in Step **4**. Another source of numbers inside the problem resides in the *datum* given in the preliminary Question section, the number whose cube root is to be computed; we see that it intervenes in Step **1**, where it is used as one of the initialization values.

Calling the data of a given problem **D** (with indexes if necessary, see Problem III), we rewrite the first part of Step **1** as "← **D**".

D		
▶		
1	← **D** ← 0;07 30 root given	
2	$\sqrt[3]{\mathbf{1b}}$	
3	(0;07 30)$^{-1}$ = 8;00	
4	**3** × 3;22 30 = 27;00	
5	$\sqrt[3]{\mathbf{4}}$	
6	**5** × 0;30 the second root = 1;30	the root of 3;22 30

Then, there are those values that are neither the result of the immediately preceding step nor the data of the problem. In our problem one of them bears the name of a result that is not its immediate predecessor, namely, Step **3**'s "(0;07 30)$^{-1}$ = 8;00", where, as in Step **2**, the value is clearly the earlier assigned value of Step **1b**. We will thus replace the value in Step **3** by that value's step number, i.e., **1b**. In the same way, the 3;22 30 of Step **4** is recognizably the datum; however, we will not replace it by **D** but rather by the step number by which it has been introduced into the algorithm, namely as the assigned value of Step **1a**. Finally, the 0;30 called the "second root" in Step **6** is to be replaced by its origin, the result in Step **2**:

D	
▶	
1	← **D** ← 0;07 30 root given
2	$\sqrt[3]{\mathbf{1b}}$
3	(**1b**)$^{-1}$
4	**3** × **1a**
5	$\sqrt[3]{\mathbf{4}}$
6	**5** × **2**

Finally, there is just one number remaining which does not come from within the problem, i.e., data or the result of some preceding step, namely the second initializing value 0;07 30, called the "root given (to you)". This is just a number that has been posited as an initializing value, whose explicit identification is that of a number whose cube root appears in the tables of cube roots (which is indeed the case). But why this cube root rather than any other in the table? As we shall see a bit later, given the way in which the problem is resolved, the number is subject to certain mathematical restrictions but otherwise can be freely chosen. We represent such constants and particularly the very important class known as *igigubbû* (Problem II), by the letter C in the preamble to the solution algorithm, with a brief identifying name in parentheses, and bring it in where it enters the algorithm. The final form of our abstract algorithm is thus:

D	C (tabulated cube root)	
▶		
1	← **D**	←C
2	$\sqrt[3]{\mathbf{1b}}$	
3	$(\mathbf{1b})^{-1}$	
4	$\mathbf{3} \times \mathbf{1a}$	
5	$\sqrt[3]{\mathbf{4}}$	
6	$\mathbf{5} \times \mathbf{2}$	

where by ‘tabulated’ we mean ‘exists as an entry in one of the standard tables’.

We can now make use of the information flow to see what is happening in the determination of the cube root of a number whose root is *not* in a table (but which is presumed to exist as a finite sexagesimal number). The unknown number (**D** = **1a**) is “coupled” (*makṣarum*) with another whose root *is* tabulated (C = **1b**) and the latter’s root exhibited (**2**). Steps **3** and **4** compute the inverse of a number X then multiply the result by another number Y. This common sequence of operations is equivalent to our division Y/X; the role of X is here played by the tabulated number (**1b**) and Y by the untabulated datum (**D** = **1a**). The cube root of this number is now calculated (which supposes that **4** is to be found in the table as well as C) in Step **5**. Finally, the cube root desired is found (**6**) by multiplying the cube root just calculated (**5**) by the cube root of the tabulated number (**2**).

Note that the practicality of this method for the determination of an unknown cube root is very limited. First, the inverse of the coupled tabulated number must exist (for Step **3**) and this will be the case only for what in the Assyriological literature is called a “regular number”, one whose prime factors are 2, 3 and 5 (the prime factors of 60, the base) and can thus be written in the form $2^a \times 3^b \times 5^c$ where a, b, c are (positive or negative) integers. Second, the cube root of **D**/C must be a tabulated number (for Step **5**). These are strong restrictions and few of the numbers generally met with in Old Babylonian mathematical texts would work.

Problem II: Haddad 104-1 (I 1–9)

During the Iraqi salvage excavations at Tell Haddād (Old Babylonian name: Mê-Turan)[31] in the early 1980s, an important solved-problem mathematical text was discovered and quickly published.[32] As in most of the texts from this region, the number of Sumerian logograms is reduced by comparison to the texts from further south, thus simplifying the task of recognizing their exact wording and grammatical structure.

The tablet contains ten problems, of which our selection is the first. Like this first problem, the majority treat the calculation of the capacity of granaries of various shapes and sizes, as well as other volume questions involving excavations and bricks.

A straightforward mathematical calculation, the point of the exercise lies rather in the transformation from standard metrological systems into the abstract number system and the inverse transformation at the end yielding an expression of an abstract result in terms of practical metrologically based units.

1. Photograph – Haddad 104-1

Fig. 3: Photograph – Haddad 104-1

31 In the Diyala Valley, to the north of the ancient major city of Ešnunna.

32 Al Rawi and Roaf (1987).

2. Hand copy – Haddad 104-1

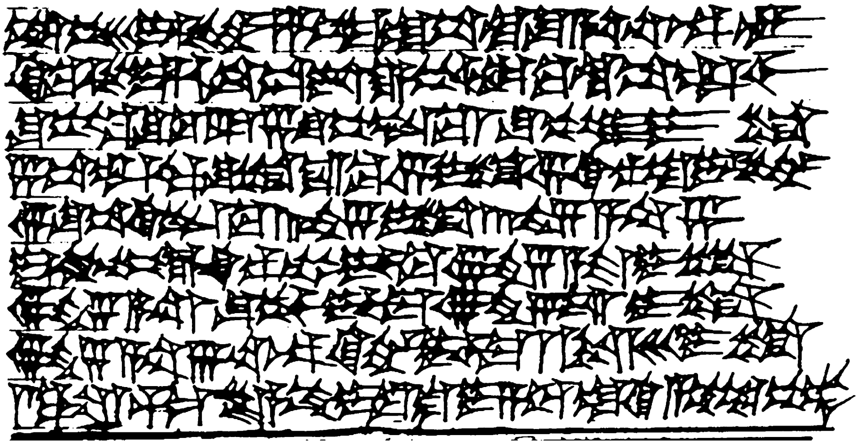

Fig. 4: Copy – Haddad 104-1

3. Transliteration – Haddad 104-1

obverse, col. I

(1) *ne-pé-eš i-ṣí-im 5 am-ma-at ta-la-šu a-na na-aš-pa-ak*
ki ma-ṣí i-re-ed-du i-na e-pé-ši-ka ma-la ta-al-lim
šu-up-lam šu-ku-un 5 šu-pí-il-ma 1 šu-up-lum i-li
5 ta-al-lam šu-li-iš-ma 15 i-li 15 ki-pa-at i-ṣí-im
(5) *15 šu-ta-ki-il-ma 3 45 i-li 3 45 a-na 5*
i-gi-gu-bé-e ki-pa-tim i-ši-ma 18 45 a-šà i-li
18 45 a-na 1 šu-up-lim i-ši-ma 18 45 ***saḫar*** *i-li*
18 45 a-na 6 na-aš-pa-ki-im i-ši-ma 1 52 30 i-li
*1 (**bariga**) 5 (**bán**) 2 ½* ***sìla*** *še-a-am i-ṣú-um i-ṣà-ba-at ki-a-am ne-pé-šum*

4. Transcription – Haddad 104-1

obverse, col. I

nēpeš iṣim 5 ammat tallašu ana našpak
kī maṣi ireddu ina epēšīka mala tallim
šuplam šukun 5 šuppilma 1 šuplum illi
5 tallam šullišma 15 illi 15 kippat iṣim
15 šutakīlma 3 45 illi 3 45 ana 5
igigubbê kippatim išīma 18 45 eqlum illi
18 45 ana 1 šuplim išīma 18 45 eperum illi
18 45 ana 6 našpakim išīma 1 52 30 illi
1 (pānum) 5 (sūtum) 2 ½ qûm še'am iṣum iṣabbat kīam nēpešum

5. Algorithm – Haddad 104-1

[1]*nēpeš iṣim*

5 ammat tallašu

ana našpak [2]*kī maṣi ireddu*

► *ina epēšīka*
1 *mala tallim* [3]*šuplam šukun*
2 *5 šuppilma 1 šuplum illi*
3 [4]*5 tallam šullišma 15 illi* *15 kippat iṣim*
4 [5]*15 šutakīlma 3 45 illi*
5 *3 45 ana 5* [6]*igigubbê kippatim išīma 18 45 eqlum illi*
6 [7]*18 45 ana 1 šuplim išīma 18 45 eperum illi*
7 [8]*18 45 ana 6 našpakim išīma 1 52 30 illi*
8 [9]*1 (pānum) 5 (sūtum) 2 ½ qûm še'am iṣum iṣabbat*
◄ *kīam nēpešum*

6. Translation – Haddad 104-1

Procedure for a log.

5, an *ammatum*, was its diameter.

For how much will it be suitable in capacity?

► In your proceeding:
1 Set down for the depth as much as the diameter.
2 Convert 5 — 1, the depth, will come up.
3 Triple 5, the diameter — 15 will come up. 15 will be the circumference of the log.
4 Square 15 — 3 45 will come up.
5 Multiply 3 45 by 5, the constant of the circle — 18 45, the area, will come up.
6 Multiply 18 45 by 1, the depth — 18 45, the ground-volume, will come up.
7 Multiply 18 45 by 6, the (constant of) capacity — 1 52 30 will come up.
8 The log will hold 1 *pānum* 5 *sūtum* 2 ½ *qûm* of grain.
◄ Such is the procedure.

7. Commentary – Haddad 104-1

Title:

nēpeš. The construct form of *nēpešum* (see line 9), this nominal form from the verb *epēšum* "to do" is the standard term for "procedure" in the sense of algorithm in the mathematical texts, but also is one of the terms for "prescription" in medical texts and (in its feminine form *nēpeštum*) for an "act of divination".

Presentation:

The use of a tenseless nominal phrase is typical of the pre-algorithmic part of a problem. For reasons mentioned in the Introduction, a preterite form of "to be" is used in the translation for this part.

5 ammat. Absolute form of *ammatum*, because a metrological unit. Not to be understood as "5 *ammatum*" but rather as "0;05 (*nindanum*, that is) an *ammatum*". The use of the phonetic writing of *ammat* rather than the logogram ***kùs*** is perhaps a way of signaling to the reader the correct reading. That this is the right way to read this expression is made clear in Step **2** below.

Question:

kī maṣi. Interrogative "how much?"; along with the pronoun *mīnum* "what? how much?", one of the two principal terms for introducing the question part of the problem.

našpak. Absolute form of *našpakum*. The word has the literal meaning of "granary" but has acquired the extended abstract meaning of "capacity", that is, a volume in units of grain. As such, it is assimilated to a metrological unit, whence the use of the absolute. See Steps **5** and **6** for similar technical terms.

ireddu. G-Durative of *redûm* "to be available for, suitable for". This is a rare use of a Durative finite verb form in a question.

► *ina epēšīka*. *ina* + G-Infinitive of *epēšum* + *-ka*, "in your doing, proceeding". Standard part of introductory phrase *attā ina epēšīka*, used either in its entirety or in the form *attā* alone or, as here *ina epēšīka* alone.

1 *šukun*. G-Imperative of *šakānum* "to place, set down". Again an initialization command (see Step **1** in Problem I above); here meaning to initialize the value of the depth *šuplum* as equal to the diameter *tallum*.

There is no numerical result, as in the case of an arithmetic command, nor a dispositional result, as in the similar control command in Problem I, so there is no need here of a final *-ma*.

2 *šuppilma*. Either Š-Imperative of *napālum* "to compensate, convert" or D-Imperative of the related verb *šupelûm* "to exchange, convert" + enclitic particle *-ma*. In either case, the meaning is roughly equivalent: "to convert (from one metrological unit to another)", here from *nindanum* to *ammatum*. This conversion of the depth is necessary since the unit of volume – the *mušarum* – is 1 *nindanum* × 1 *nindanum* × 1 *ammatum*.

illi. The verb *elûm* "to go up" is used to indicate the numerical result of each arithmetic operation in this text. The full structure of the marker of the numerical result of each step in this text is thus verb-*ma* X *illi*, where X is the numerical result.

The writing *i-li* is ambiguous between the G-Durative *illi* and the G-Preterite *īli* of the verb since consonant gemination is often not indicated in Old Babylonian cuneiform. That the Durative is meant here follows from the generalized use of the Durative in the solution procedure part of problem texts.

3 *šullišma*. D-Imperative of *šalāšum* "to triple" + *-ma*. Note that this is a specific arithmetic operation in Mesopotamia; the text does not say "multiply by 3", which would be *ana 3 iši*.

kippat. Construct form of *kippatum*, literally "circle"; also, as here, used to mean its essential attribute for Mesopotamians, its circumference.

15 kippat iṣim. An example of the marking of the meaning of an intermediate result. Here, the immediately preceding result, 0;15 (*nindanum*), is repeated with its significance: "the circumference of the log".

Note that the diameter, 0;15, is not converted from *nindanum* to *ammatum* since it is a horizontal measure and as such has *nindanum* as the default unit.

4 *šutakīlma*. Traditionally understood as the Št-Imperative of *akālum* "to eat" but more likely the Št-Imperative of *kullum* "to hold" + *-ma* (see the discussion in Høyrup [1996]: 48). The meaning in any case is clear: "to square". Note that this verb can take a single argument, as here: X *šutakūlum* "to square X", but also two identical arguments (see Problem III): X *u* X *šutakūlum*, "to multiply X by X".

5 *5 igigubbê kippatim*. The tables of constants list 5 (absolute value 0;05) as the constant (*igigubbûm*) of the circle, i.e., that constant by which the square of the circumference must be multiplied to yield the area. A similar use of these tables to be found in Step **7**.

išīma. G-Imperative *iši* of *našûm* "to multiply" + *-ma*. One finds the same verb and same operation in Steps **6** and **7**.

eqlum. The standard word for "area", the Akkadian word means literally "field".

6 *eperum*. This word for "dust, earth" has, by the same sort of extension as that undergone by *eqlum*, come to be used for "volume", principally of excavations, hence the specific translation of "ground-volume".

1 *šuplim*. "1 (*ammatum*), the depth". Recall that in the Babylonian volume system vertical measures are in *ammatum*, while horizontal measures are in *nindanum*.

7 *6 našpakim*. This sentence is parallel to the command in Step **5**, to be understood as 6 *igigubbê našpakim*, “6 (absolute value: 6 00 00;00, i.e., 6 × 60^3), the constant of capacity”. This very large constant converts the ground-volume unit of *mušarum* into the capacity unit *qûm*. Thus, the number 1 52;30 represents the number of *qûm* in the abstract number system in which the calculations are done.

8 *1 (pānum) 5 (sūtum) 2 ½ qûm še’am*. This step is a conversion, though one without any calculation necessary. Since the problem asks for the answer in capacity units, the abstract response must be translated into the capacity metrological system. By the Old Babylonian period, the two were sufficiently coherent that the metrological values could be read directly off the abstract number. Thus, here the abstract 1 52;30 can be read as 1 — 5 — 2;30 giving the number of *pānum* (1), the number of *sūtum* (5) and the number of *qûm* (2;30 = 2 ½).

Since the writing of numerical values within the metrological systems is non-sexagesimal and often distinct from that used in the abstract system, the Assyriological convention is to write the appropriate Sumerian logogram of the implicit unit within parentheses. Thus for example here the transcription ‘5 *sūtum*’ corresponds to the cuneiform , with no explicitly written unit and is transliterated ‘5 (***bán***)’. Note that the 2 ½ () *qûm* has an explicitly written unit, which is treated just as any other word: Sumerian ***sìla*** (in the transliteration) and Akkadian *qûm* (in the transcription) with no parentheses.

◀ *kīam nēpešum*. “Such is the procedure”, marking the end of the algorithm.

The absolute values in the translation of this problem are easily determined since the problem is embedded in metrological systems. We can thus fix some things directly. For example, consider the explicit statement in the problem presentation that the diameter is “5 (*nindanum*)”. Since this is stated to be 1 *ammatum*, and there are 12 *ammatum* in a *nindanum* (see the Metrological Diagram in the Appendix, C., Fig. 1), this yields 1;00 *ammatum* = 1/12 = 5/60 = 0;05 *nindanum*.

With this value and the absolute values of the two *igigubbûm*:

igigubbûm of the circle = 0;05
igigubbûm of capacity = 6 00 00:00

doubtlessly known by the ancient readers of the text, all the absolute values in the problem can then be fixed by simply following the calculation.

Procedure for a log.

0;05, an *ammatum*, was its diameter.

For how much will it be suitable in capacity?

► In your proceeding:
1 Set down for the depth as much as the diameter.
2 Convert 0;05 — 1;00, the depth, will come up.
3 Triple 0;05, the diameter — 0;15 will come up. 0;15 will be the circumference of the log.
4 Square 0;15 — 0;03 45 will come up.
5 Multiply 0;03 45 by 0;05, the constant of the circle — 0;00 18 45, the area, will come up.
6 Multiply 0;00 18 45 by 1;00, the depth — 0;00 18 45, the ground-volume, will come up.
7 Multiply 0;00 18 45 by 6 00 00;00, the (constant of) capacity — 1 52;30 will come up.
8 The log will hold 1 *pānum* 5 *sūtum* 2 ½ *qûm* of grain.
◄ Such is the procedure.

8. Discussion – Haddad 104-1

The problem concerns a cylinder, as high as it is wide, called a "log". The question posed is to find the volume of this cylinder in units of capacity, i.e., grain measure.

The procedure, as in most mathematical texts dealing with 'real-world' problems, has a triple structure. The problem, given in terms of real metrologically-embedded quantities must first have its data transformed into the abstract number system; the mathematical procedure is then carried out on these abstract values; and the final result then translated back into the correct metrological value.

As usual, we begin with the ***symbolic algorithm*** for Haddad 104-1:

Diameter = 0;05 *nindanum*

▶

1	← 0;05	*Initialization*
2	0;05 ▷ 1;00	Conversion
3	③ 0;05 = 0;15	Tripling
4	⊠ 0;15 = 0;03 45	Squaring
5	0;03 45 × 0;05 = 0;00 18 45	Multiplication
6	0;00 18 45 × 1;00 = 0;00 18 45	Multiplication
7	0;00 18 45 × 6 00 00;00 = 1 52;30	Multiplication
8	(1 52;30) ▷ 1 *pānum* 5 *sūtum* 2 ½ *qûm*	Conversion

◀

The symbolism used here which has not already been introduced in Problem I is as follows:

③ tripling (see the Commentary for the distinction with multiplication by 3)

⊠ squaring (see Commentary)
The sign is a combination of the two equivalent uses of the verb *šutakūlum*: "to square X", or "to multiply X by X"; hence ⊠, a combination of □ and ×.

▷ conversion of one unit into another (of the same metrological structure)
In Step **2** it refers to a change of *nindanum* into *ammatum* (by multiplication by 12) and in Step **8** to a change from *qûm* to *pānum sūtum* and *qûm* (by reading off powers of 60). In the latter case, no verb is used and the result is simply stated.

Note that Step **7** could also be considered a metrological conversion (from ground-volume *mušarum* to capacity *qûm*) but that functionally it is treated as a multiplication with the verb *našûm* "to multiply" rather than the verb *šuppulum* "to convert"; perhaps because it is a transformation *between* metrological systems rather than *within* one.

2° ***Abstract algorithm.*** The flow of information in this algorithm is particularly easy since a great deal is carefully labeled in the text, a situation which is not always present in Babylonian problem texts:

D		
▶		
1	← **D**	depth
2	**1** ▷	depth
3	3 **D**	circumference of the log
4	⊠ **3**	
5	**4** × 0;05 constant of the circle	area
6	**5** × 1;00 depth = 0;00 18 45	ground-volume
7	**6** × 6 00 00;00 (constant of) capacity = 1 52;30	
8	**7** ▷	grain
◀		

Of those values which are neither the result of the immediately preceding step nor the data of the problem, we are helped out by the fact one of them bears the name of a result that is not its immediate predecessor, namely, Step **6**'s "1;00, depth":

D		
▶		
1	← **D**	
2	**1** ▷	
3	3 **D**	
4	⊠ **3**	
5	**4** × 0;05 [constant of the circle] = 0;00 18 45	area
6	**5** × **2**	
7	**6** × 6 00 00;00 [(constant of) capacity] = 1 52;30	
8	**7** ▷	
◀		

Finally, we have just two numbers which do not come from within the problem, i.e., data or the result of some preceding step, with the stated names:

Step **5** – 0;05 is the "constant of the circle",
Step **7** – 6 00 00;00 is the "(constant of) capacity".

Each number is called a "constant" (*igigubbûm*); they appear on tables of constants and are identified explicitly – "0;05, constant of the circle" (Step **5**) – or implicitly – "6 00 00;00, capacity" with "constant of" understood (Step **7**).

With our standard notation for constants, including now *igigubbûm* constants (see Problem I):

5 **4** × C_1
7 **6** × C_2

Hence, our complete abstract algorithm for this problem will be the following:

D C_1 (constant of circle) C_2 (constant of grain)
▶
1 ← **D**
2 **1** ▷
3 ③ **D**
4 ⊠ **3**
5 **4** × C_1
6 **5** × **2**
7 **6** × C_2
8 **7** ▷
◀

The procedure followed is straightforward, all the more so in that the results of the steps are carefully named in the text; not only of that with the final answer (Step **8**) but also of most of the intermediate steps. The only exception is the squaring in Step **4** but this is an intermediate step in the calculation of the area of the cylinder's base and has no simple geometric referent.

After fixing the height in the right units (Steps **1**–**2**), that is, those which make up part of the abstract system, the area of the circular base is calculated in the standard Babylonian fashion; by calculating the circumference (Step **3**), then squaring it (Step **4**), and multiplying the result by the "constant of the circle" (Step **5**). This yields the area of the base and it is then multiplied by the height (Step **6**) to obtain the volume. But this will be in the ground-volume units employed in the abstract numerical system (*mušarum*) and the result must then be converted into units of grain (Steps **7**–**8**) using the "constant of capacity" (6 00 00;00 = 6×60^2 = 21600 *qûm*/*mušarum*).

The final answer, 1 *pānum* 5 *sūtum* 2 ½ *qûm,* is essentially directly read off the total number of *qûm* in the abstract sexagesimal system, 1 52;30, since the Old Babylonian capacity system is essentially sexagesimal itself (see the Metrological Diagram in the Appendix, C., Fig. 1). This explains the abbreviated form of the final conversion comprising Step **8**, with only a restatement of the abstract system result 1 52;30 in the preceding step.

Problem III: BM 13901-9 (II 3–10)

A large 4-column tablet of unknown provenance, purchased by the British Museum in the late nineteenth century. The purchase from a dealer without known place of origin is unfortunately typical of a large number of mathematical tablets.[33]

The tablet originally held 24 solved problems involving squares, very systematically and pedagogically arranged in increasing order of complexity. The question, though never explicitly stated, is always the same: to determine the side of one or more squares with the data consisting of the value of sums or differences of sides and areas, or of the difference or ratio of successive sides in problems treating more than one square.

This problem is a comparatively simple example of a large species of Old Babylonian mathematical problems involving areas and sides of squares. Their development and the algorithms developed for their solution mark one of the high points of sophistication in ancient mathematical literature.

1. Photograph – BM 13901-9

Fig. 5: Photograph– BM 13901-9

33 The original publications are to be found in Thureau-Dangin (1936) and Thureau-Dangin (1938): n° 1–21 and Neugebauer (1935–37): III 1–14.

2. Hand copy – BM 13901-9

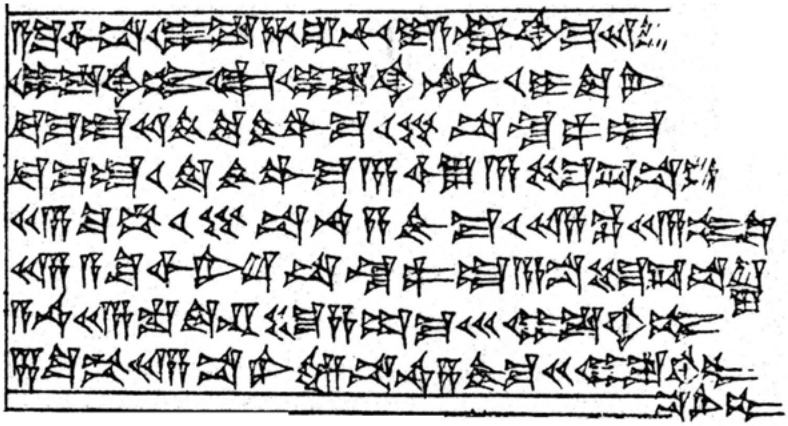

Fig. 6: Copy – BM 13901-9

3. Transliteration – BM 13901-9

obverse, col. II

(3) ***a-šà*** *ši-ta mi-it-ḫa-ra-ti-ia ak-mur-ma 21* ⸢*40*⸣
mi-it-ḫar-tum ***ugu*** *mi-it-ḫar-tim 10 i-te-er*
(5) *ba-ma-at 21 40 te-ḫe-pe-ma 10 50 ta-la-pa-at*
ba-ma-at 10 te-ḫe-pe-ma 5 ù 5 tu-uš-ta- ⸢*kal*⸣
25 ***šà-bi*** *10 50 ta-na-sà-aḫ-ma 10 25-****e*** *25* ***íb-si***$_8$
25 a-di ši-ni-šu ta-la-pa-at 5 ša tu-uš-ta-ki-lu
a-na 25 iš-te-en tu-ṣa-ab-ma 30 mi-it-ḫar-tum
(10) *5* ***šà-bi*** *25 ša-ni-im ta-na-sà-aḫ-ma 20 mi-it-ḫar-tum ša-ni-tum*

4. Transcription – BM 13901-9

obverse, col. II

(3) *eqlēt šitta mitḫarātīja akmurma 21 40*
mitḫartum eli mitḫartim 10 īter
(5) *bāmat 21 40 teḫeppēma 10 50 talappat*
bāmat 10 teḫeppēma 5 ù 5 tuštakāl
*25 ina libbi 10 50 tanassaḫma 10 25-***e** *25 basûm*
25 adi šinīšu talappat 5 ša tuštakīlu
ana 25 ištēn tuṣṣabma 30 mitḫartum
(10) *5 ina libbi 25 šanîm tanassaḫma 20 mitḫartum šanītum*

5. Algorithm – BM 13901-9

[3]*eqlēt šitta mitḫarātīja akmurma 21 40*
[4]*mitḫartum eli mitḫartim 10 īter*

1	[5]*bāmat 21 40 teḫeppēma 10 50 talappat*	
2	[6]*bāmat 10 teḫeppēma*	
3	*5 u 5 tuštakkal*	
4	[7]*25 ina libbi 10 50 tanassaḫma 10 25*	
5	*25 basûm*	
6	[8]*25 adi šinīšu talappat*	
7a	*5 ša tuštakīlu* [9]*ana 25 ištēn tuṣṣabma*	*30 mitḫartum*
7b	[10]*5 ina libbi 25 šanîm tanassaḫma*	*20 mitḫartum šanītum*

6. Translation – BM 13901-9

I added the surface of my two sides — 21 40.
One side exceeded the (other) side by 10.

1 You will fractionalize the half of 21 40 — you will write down 10 50.
2 You will fractionalize the half of 10 — (5).
3 You will multiply 5 by 5 (— 25).
4 You will subtract 25 from 10 50 — 10 25.
5 25 will be the square root.
6 You will write down 25 twice.
7a You will add the 5 that you multiplied to the first 25 — 30 will be the (first) side.
7b You will subtract 5 from the second 25 — 20 will be the second side.

7. Commentary – BM 13901-9

(Title):

No title is present in this or any other of the problems on this tablet.

Presentation:

The verb *akmur* (1st person singular of *kamārum*, "to add") is unambiguously the G-Preterite form while *īter* (3rd person singular of *watārum*, "to exceed, be greater than"), whose writing *i-te-er* might correspond to either *īter* (G-Preterite) or *itter* (G-Durative), is chosen to be the former by parallelism.

The two data values, one of area (21 40) and one of length (10), have no metrological units attached and thus should be interpreted as measured in *mušarum* and *nindanum* respectively. But the abstract nature of this and the other problems on this tablet indicate that such measures are not to be taken too seriously. This is really a problem concerned fundamentally with the abstract number system, like our Problem I.

(Question):

As indicated in the introduction to this text, there is no explicitly formulated question asked, though the answers provided by the solution algorithm, as in the other problems on this tablet, indicate clearly its nature: find the sides of the two squares.

(►, ◄):

No explicit introductions or ends of the solution algorithms are marked anywhere on this tablet.

1 *bāmat* X *teḫeppēma*, literally "you will break the half of X", with the 2nd person singular of the G-Durative of *ḫepûm* + *-ma*. The expression 1/p X *ḫepûm* is the standard manner of taking a fraction 1/p of a number X, with the fractional part 1/p indicated by either a phonetic spelling (as here), or by one of the special status fractions (½, ⅓, ¼, ⅙)[34], or by a general fraction ***igi-***p***-gâl***. To allow for this generalized use, the translation "to fractionalize the half, third, ... of X" is to be preferred to the customary "to halve X, to take a third of X, etc.".

talappat. 2nd person singular of G-Durative of *lapātum* "to write down". The use of the enclitic particle *-ma* followed by a verb announcing a result, standard in Problem I, is the only instance of this formulation in this Problem. Much more common in all the problems of this tablet is the formulation used in Step **2** *q.v.*

2 This second fractionalization employs the same verb as in Step **1** but is not followed by a phrase introducing the result of the operation. Instead, an abbreviated formulation is used in which for a Step **N** whose result is used as the first argument in the following Step **N+1**, no result is stated; the first argument of the Step **N+1** serves *simultaneously* as the result of Step **N**. This is possible in Akkadian since the verb of a sentence is sentence final. However, this is not the case in English; an explicit result must be supplied, hence the frequent use of "(X)" in the translation of this text to indicate the result of a step.

3 *tuštakkal*. 2nd person singular Št-Durative of *kullum* + *-ma* (see the commentary to Step **4** of Haddad 104-1 above). Note that this same verb used with a single argument (as in Haddad 104-1) must be translated "to square" whereas here, with two (identical) arguments, the English requires "to multiply".

Note that the operative verb here lacks not only the numerical result, as in Step **2** and for the same reason, but also the *-ma* enclitic particle that is generally present here.

4 X_1 *ina libbi* X_2 *tanassaḫma*. "you will subtract X_1 from X_2" with the 2nd person singular of the G-Durative of *nasāḫum*.

*10 25-**e***. The answer to the operation in this step has a suffixed ***-e*** (𒂊); this sign serves here, as elsewhere on this tablet, to separate two numbers

34 See Benoit, Chemla, and Ritter (1992): 8.

written next to each other, obviating the risk of them being read as a single, long number. This danger exists here because of the particular abbreviated manner of writing results (see Discussion).

5 *basûm*. This Akkadian reading of the Sumerian logogram ***íb-si***$_8$ is the conventional one, found in the dictionaries. It has been proposed that when used as a noun as here the logogram is to be read *mitḫartum* and translated "equalside".[35] The functional meaning in any case is "square root". Taking this step as a nominal phrase, our translation convention requires a future form of the (unexpressed) copula.

6 *talappat*. 2nd person singular of G-Durative of *lapātum* "to write down, record". The use is similar to that of *šakānum* as in Problem II, Step **1**, i.e., an initialization command, though here the preceding *adi šinīšu* indicates that the 25 is to be written 'twice'. This indicates the beginning of a computation *in parallel*.

7a *5 ša tuštakīlu*. "the 5 that you multiplied" with *tuštakīlu* in the 3rd person singular of the Št-Preterite, of *kullum* + *-u* for the subjunctive mood (as required after *ša*), that is, the 5 that served in the earlier multiplication (Step **3**). This is an example of a functional reference (see the Discussion) and is always of the form X *ša* Verb (Preterite + *-u*) "the X that you/was ...ed", in contrast to the use of the Durative tense everywhere else in a solution algorithm.

mitḫartum. "Side of the square", serving as the predicate of a nominal sentence: "will be the side of the square". For *mitḫartum* here and *mitḫartum šanītum* in the parallel Step **7b** see the Commentary to Problem I, Step **6**.

The numbering of this step as **7a** and the following as **7b** rather than the expected **7** and **8** respectively is explained in the Discussion.

7b X_1 *ina libbi* X_2 *tanassaḫma* "you will subtract X_1 from X_2" with 2nd person singular G-Durative of *nasāḫum* + *-ma*.

Absolute values are more difficult to fix here in the absence of both metrological values and of *igigubbûm*. As in Problem I, the only steps helpful here are those involving roots, addition and subtraction: Steps **4**, **5**, **7a**, **7b**. For the result of Step **1**, let us limit ourselves, at least at first, to one of the three 'simplest' choices: 10 25;00, 10;25 and 0;10 25. Then the root, Step **5**, rules out 10;25 as a

35 Høyrup (1990): 49–50 and Høyrup (2002): 26–27.

possibility for the result of Step **4**, since the square root of 10;25 would be 3;13.... This leaves 10 25;00 and 0;10 25. If we assume the first choice, the subtraction in Step **3** forces 25;00 as the absolute value for Step **2** which in turn yields 5;00 for the result of Step **2** which gives 10;00 for the second datum of the problem. Now Step **1** gives 21 40;00 for the value of the first datum. Using the calculated values for 10;25, the answers to the problem will be 30;00 (Step **7a**) and 20;00 (Step **7b**) and these two do indeed yield 10 25;00 for the sums of their areas. Thus, 10 25;00 fits the whole scheme of the problem and we adopt this choice. But note that, as is often the case, there are alternatives; indeed, our third choice, 0;10 25, is equally consistent with all the operations of the problem.

I added the surface of my two sides — 21 40;00.
One side exceeded the (other) side by 10;00.

1 You will fractionalize the half of 21 40;00 — you will write down 10 50;00.
2 You will fractionalize the half of 10;00 — (5;00).
3 You will multiply 5;00 by 5;00 (— 25;00).
4 You will subtract 25;00 from 10 50;00 — 10 25;00.
5 25;00 will be the square root.
6 You will write down 25;00 twice.
7a You will add the 5;00 that you multiplied to the first 25;00 — 30;00 will be the (first) side.
7b You will subtract 5;00 from the second 25;00 — 20;00 will be the second side.

8. Discussion – BM 13901-9

Here, the problem concerns the determination of the sides of two squares, given that the sum of their areas is 21 40 (1300 in decimal notation) and the difference of their sides is 10:

1° *Symbolic algorithm.*

	$\text{Surface}_1 + \text{Surface}_2 = 21\ 40;00$	$\text{Side}_1 - \text{Side}_2 = 10;00$
(▶)		
1	½ 21 40;00 = 10 50;00	Fractionalization by ½
2	½ 10;00 = 5;00	Fractionalization by ½
3	5;00 ⊠ 5;00 = 25;00	Multiplication
4	10 50;00 – 25;00 = 10 25;00	Subtraction
5	√ 10 25;00 = 25;00	Square root
6	← 25;00 ← 25;00	*Parallel initialization*
7a	25;00 + 5;00 = 30;00	Addition
7b	25;00 – 5;00 = 20;00	Subtraction
(◀)		

New symbols introduced here are:

½ fractionalization (see Commentary)

– subtraction

√ square root

← ← *parallel initialization.*
This is a variant of the command operation of (simple) initialization seen in the previous Problems. Here two or more values are set as the initial values in a set of calculations to be carried out 'in parallel' in the following steps, that is, in a functionally *independent* manner. To distinguish the two values in what follows, we shall refer to them as **6a** and **6b**.

Here, the following two steps are an addition, resp. subtraction, of the result of Step **3** to, resp. from, one of the two values (of course, writing being a linear medium, the two calculations must be expressed in sequential form in the text). The fact that the two steps are in parallel leads to our (re)numbering them with the same value for **N**, distinguished by the letters **a**, **b**,...

2° ***Abstract algorithm.*** If Problem II used a large number of names for its various arguments and results as befits its straightforward metrological and elementary status, this problem on the other hand uses very few indeed. As always, however, the bulk of the numbers appearing in the text are results coming from the immediately preceding step and by simple inspection we can replace these numbers by the preceding step number:

$\mathbf{D}_1$ (21 40;00)		$\mathbf{D}_2$ (10;00)	
(▶)			
1	½ 21 40;00 = 10 50;00		
2	½ 10;00 = 5;00		
3	**2** $\boxtimes$ **2**		
4	10 50;00 – **3**		
5	√ **4**		
6	**5**	**5**	
7a	**6a** + 5;00 that you multiplied		(first) side
7b	**6b** – 5;00		second side
(◀)			

We have distinguished the dual values of the parallel initialization Step **6** by calling the first result **6a** and the second **6b**. The use of the data in the problem is clear from their values (21 40;00 and 10;00), even if, unlike Problems I and II, no explicit naming is used in the text to identify them; we substitute in the standard way:

$\mathbf{D}_1$	$\mathbf{D}_2$		
(▶)			
1	½ $\mathbf{D}_1$ = 10 50;00		
2	½ $\mathbf{D}_2$ = 5;00		
3	**2** $\boxtimes$ **2**		
4	10 50;00 – **3**		
5	√ **4**		
6	**5**	**5**	
7a	**6a** + 5;00 that you multiplied		(first) side
7b	**6b** – 5;00		second side
(◀)			

Now, the only names used in this problem are for the two final results (Steps **7a** and **7b**), called "(first) side" and "second side", and for one argument (Step **7a**) in which the added number 5;00 is referred to as "the 5;00 that you multiplied". This last introduces another way of referring to a non-immediately preceding result, by creating a *functional reference*: if a name has not been attributed to a number when it is created as a *result* (as was the case for all the numbers in Problem II), the text can refer to the step in which the number was first *used*. Here, the phrase following 5;00 in Step **7** – which had no name attached when it was created in Step **2** – refers to the use of that number in being multiplied by itself in Step **3**. Note that our conventions call for the step number to be that in which it was *formed* (thus, we substitute **2**, not **3** in this case).

The first argument in Step **4** is of a similar kind but as the 10 50;00 – which is the result of Step **1** – has not been used as an argument in a calculation before this step, there is no way to give it a functional reference. Nevertheless, it is quite recognizable from its value and once again we can substitute in the standard way:

$\mathbf{D_1}$	$\mathbf{D_2}$	
(▶)		
1	½ $\mathbf{D_1}$	
2	½ $\mathbf{D_2}$	
3	**2** ⊠ **2**	
4	**1** – **3**	
5	√ **4**	
6	**5**	**5**
7a	**6a** + **2**	
7b	**6b** – **2**	
(◀)		

As a final step in our abstract rewriting, we show more clearly the parallel nature of Steps **7a** and **7b** by writing them on the same line. This yields the final form of our abstract algorithm for Problem III:

$\mathbf{D_1}$	$\mathbf{D_2}$	
(▶)		
1	½ $\mathbf{D_1}$	
2	½ $\mathbf{D_2}$	
3	**2** ⊠ **2**	
4	**1** – **3**	
5	√ **4**	
6	**5**	**5**
7	**6a** + **2**	**6b** – **2**
(◀)		

One can now follow the procedure through the steps in the following way. Step **1** computes a sort of average area, that of a square whose area would lie halfway between the two given squares, $\bar{A}$. Step **2** computes one-half of the difference between the lengths of the sides of the two squares ½ ΔS. Step **3** then gives the area of a square whose side would have the length of this half-difference and **4** calculates the difference between the area of the average square $\bar{A}$ and the square built on the half-difference; this is in fact the area of a square $A_{1/2}$ the length of whose side is halfway between the lengths of the two given squares. It is then sufficient to find the side of this new square by taking the square root of its area (Step **5**) and, in two parallel calculations **6**–**7**, add and subtract the half-length difference ½ ΔS from this side, which establishes the lengths of the original large square (**7a**) and small square (**7b**).

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Figures

Fig. 1: Courtesy Yale Babylonian Collection.
Fig. 2: Neugebauer and Sachs (1945): pl. 22.
Fig. 3: Al-Rawi and Roaf (1987): 217.
Fig. 4: Al-Rawi and Roaf (1987): 213.
Fig. 5: © Trustees of the British Museum.
Fig. 6: Thureau-Dangin (1936): 29.