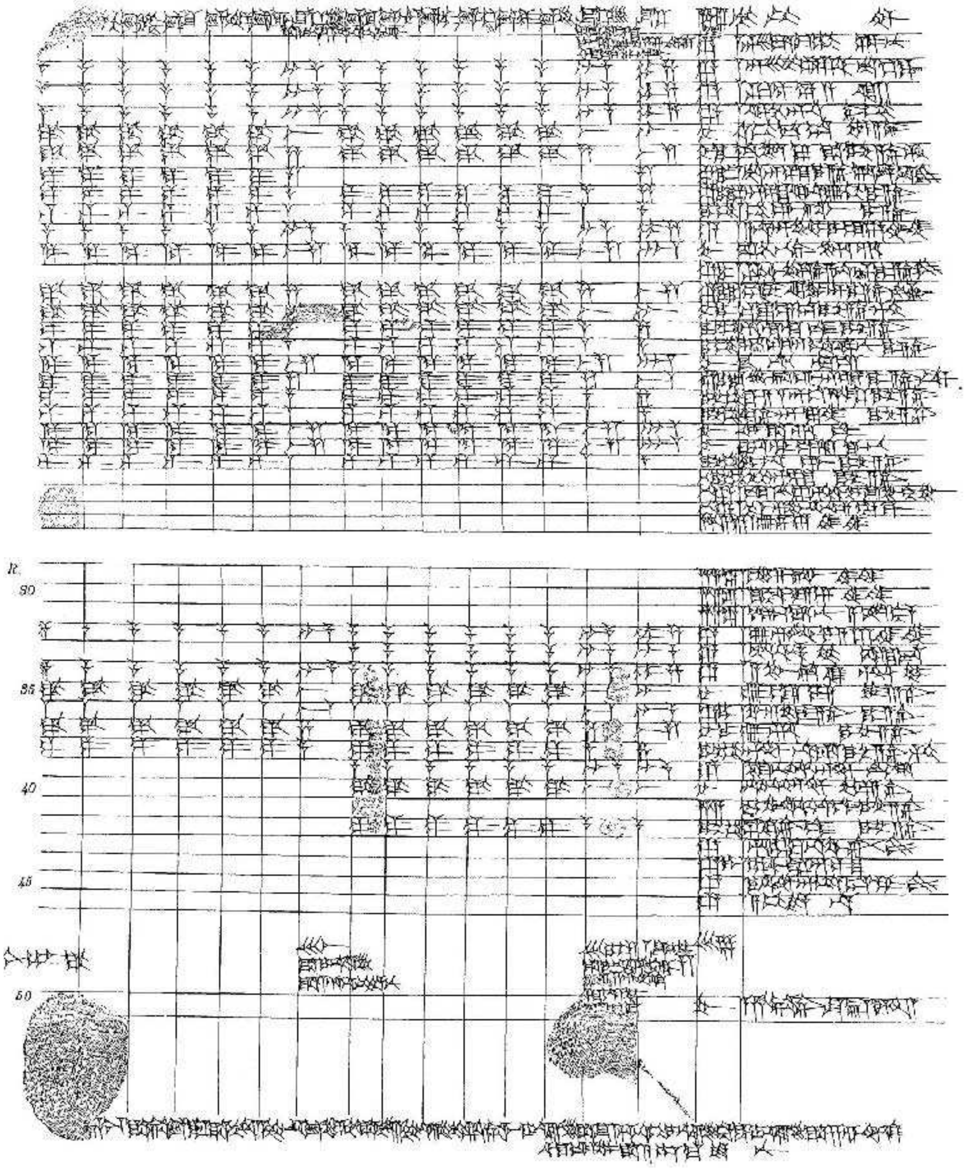
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Tables and tabular formatting in Sumer, Babylonia, and Assyria, 2500 BCE–50 CE

ELEANOR ROBSON

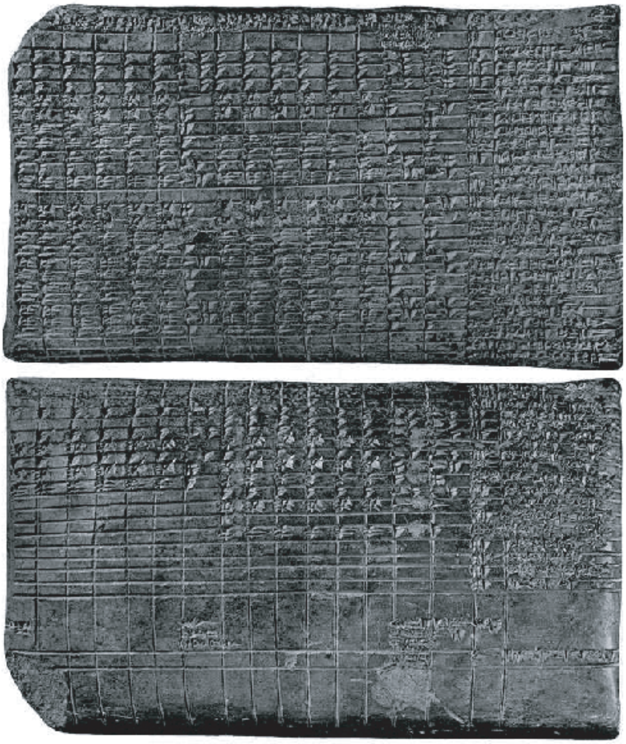


Fig. 1.1 Month-by-month wage account for the temple of Enlil at Nippur, for the year 1295 BCE. This tablet, recording the monthly salaries of forty-five temple personnel, exhibits most of the classic features of Mesopotamian tables. Column headings at the top of the table give the month names. There are also subtotals for each person every six months and a yearly total. Names and professions are given in the final column. Eighteen of the individuals listed get no payment for all or half the year; most of these are classified (in the penultimate column) as dead or absconded. There are explanatory interpolations under two headings and under their totals, and a summary at the end. (CBS 3323, from Nippur; now in the University Museum, University of Pennsylvania. A. T. Clay, *Documents from the temple archives of Nippur dated in the reigns of the Cassite rulers*, 3 vols., The University Museum, Philadelphia, 1906, pls. 25–6.)

While most of this book is concerned with the development and production of standardized mathematical tables for use as reference tools, this chapter will focus more on the invention and evolution of the numerical table as an information storage device. This was not a one-off event, with clearly traceable consequences across an ever-widening arena of functions, contexts, and cultures. Rather, even within the single cultural milieu of ancient Mesopotamia we shall see a fitful pattern of invention, partial adoption, disappearance, and re-invention time after time over the course of some two and a half millennia. Broadly speaking, documents with tabular formatting could be found in three distinct Mesopotamian locales: in the large institutional administrative archives of Sumer and Babylonia; amongst the detritus of scribal schooling, especially in mathematics and metrology; and, later, in the scholarly libraries attached to the great temples of Assyria and Babylonia. We shall examine all three in turn.

But first, if we are to talk about tables clearly and effectively we need a consistent terminology to describe them. I shall define a *formal* table as having both vertical and horizontal rulings to separate categories of information; *informal* tables, on the other hand, separate quantitative and qualitative data by spatial arrangement only, without explicit delimiters. Documents with no tabular formatting at all we might call *prose-like* or *prosaic. Headed* tables have columnar headings, while *unheaded* tables do not. Some tables are preceded by *titles* or introductory *preambles*; others are followed by *summaries* and/or *colophons*. In Mesopotamian tables, any qualitative or descriptive information is almost invariably contained in the final right-hand column (*row labels*), or interrupts the table as an *explanatory interpolation*. Mesopotamian tables have at most two *axes of organization*: the horizontal axis, along which different types of numerical information are categorized, and the vertical axis, down which the data is attributed to different individuals or areas. While some tables have two *axes of calculation* others exhibit just one, usually vertical, axis of calculation, or none at all. Both calculation and organization run, as a rule, left to right and top to bottom, following the direction of cuneiform script.

Tabular formatting in administrative records

There are no tables to be found in the earliest written records of Sumer. The 5600 documents from late fourth-millennium Uruk and its neighbours 'serv[ed] the accounting needs of a complex administration including offices of the fisheries, of herded animals and animal products, of field management, grain production and processing, and of labor'.[1] They were formatted not into lines of ordered text but into boxes or 'cases', each of which represented one sense unit or accounting unit. The spatial organization of the cases on the surface of the tablet could be quite complex,[2] reflecting often sophisticated accounting procedures,[3] but nowhere do we see the separation of quantitative and qualitative data into separate cases. This earliest writing comprised just number signs and ideograms, representing whole words or ideas, which could be displayed in any order within each case. But once writing evolved the capacity to represent syllables and thereby approximate the sounds of speech, it was no longer determined solely by the needs of accounting. As literacy spread into other social domains—for recording legal decisions, royal deeds, and even poetry—it became increasingly imperative to order the written symbols to follow the structure of Sumerian as a spoken language. The visual logic of the earliest accounts was thus irrevocably lost in the shift to linear organization of writing during the first half of the third millennium BCE.

Nevertheless, accounting continued to dominate the written record: a recent estimate has put the total administrative output at some 97 per cent of all surviving tablets from the third millennium.[4] The bureaucracy of the twenty-first century BCE kingdom of Ur was particularly prolific: some 45 500 administrative records have so far been published from its archives in the ancient cities of southern Iraq.[5] The accounts are almost exclusively prosaic, in both senses of the word: they record the day to day transfers of goods, livestock, and personnel from the responsibility of one official to another, and those records are made linearly across the surface of the tablet. This, for instance, is an extract from a monthly summary of four different types of ovids under the responsibility of the central livestock depot at Puzrish-Dagan:

7 nanny goats 3 billy goats: day 10
5 sheep, 5 kids, 5 billy goats
7 nanny goats: day 13
4 kids, 1 nanny goat 1 billy goat: day 14
2 sheep, 3 kids 2 nanny goats
4 billy goats: day 15
3 sheep, 15 kids 3 billy goats, 9 nanny goats: day 16
3 sheep, 5 kids, 1 billy goat
6 nanny goats: day 18
12 sheep, 11 billy goats, 7 nanny goats:
day 20[6]

Early Mesopotamia: Sumer and Babylonia

Mesopotamia is the classical Greek name for the Land Between Two Rivers, the Tigris and Euphrates, corresponding more or less to the modern state of Iraq. Ancient national, regional, and cultural boundaries were very fluid, however, and at times Mesopotamia incorporated neighbouring lands which now belong to modern Turkey, Syria, and Iran.

Mesopotamia was first settled some eight thousand years ago, as the Gulf receded and people moved south into the gradually more habitable marshlands it left behind. The shift towards urban living was a long and complex process, but by the middle of the fourth millennium BCE the cities of Mesopotamia were the largest, wealthiest, and most sophisticated in the world. More or less politically autonomous, and each centred around the temple of an anthropomorphic deity, the cities drew their strength from local religious power and from long-distance trade of agricultural and animal products in exchange for metals and luxury goods.

Driven by the need to manage complex economies and large populations of people and livestock, the temple administrators of the city of Uruk developed an accounting system which, by about 3300 BCE, had matured into the world's first writing. It encoded the Sumerian language—related to no other known language, living or dead—with marks in the surface of hand-held clay tablets. Originally impressed number signs and incised pictograms or more abstract symbols, representing whole words, were arranged in boxes, or *cases*. During the Early Dynastic period (*c.* 3000–2400 BCE) the incised script developed into wedge-shaped or *cuneiform* characters, while retaining the ancient impressed notation for numerals. At the same time, cases evolved into ordered lines of characters, which evolved the capacity to represent syllables and thereby approximate the sounds of speech. This enabled the scribes of Sumer to record a wide variety of documents and texts: they were no longer restricted to keeping accounts, although this was still a core activity. A second language also came into widespread use in the course of the third millennium. Akkadian is a Semitic language, indirectly related to Hebrew and Arabic, and bears no resemblance at all to Sumerian.

The last third of the third millennium BCE saw some major shifts in Mesopotamian socio-political structure. For the first time, large territorial states came into being, centred on one city but aiming to unify the whole land. The kingdoms of Akkad (*c.* 2350–2250 BCE) and Ur (*c.* 2100–2000 BCE) were each created by the ambitions and abilities of individual, long-lived and charismatic kings, Sargon and Shulgi, who seem to have constructed their empires through sheer force of personality. Inherently unstable, these kingdoms both have histories of rapid unification, centralization, and standardization followed by unsteady but inexorable decline over the next two or three generations.

Early Mesopotamia: Sumer and Babylonia *cont.*

The fall of the kingdom of Ur at the end of the third millennium saw the political landscape of Mesopotamia revert to its earlier configuration of small city states vying for land, water rights, and trade routes. The city of Nippur, religious centre of Enlil, the ‘father of the gods’, changed hands between the southern kingdoms of Isin and Larsa at least half a dozen times in two centuries. In the early eighteenth century Hammurabi, king of the hitherto unimportant city of Babylon, managed to outwit and outmanoeuvre his rival dynasts in this real-life game of Monopoly, often playing one ruler off against another. By the end of his forty-year reign (*c.* 1792–1750 BCE) vast swathes of southern Mesopotamia had come under Babylonian influence or control. Hammurabi also made sweeping religious changes, culminating in the ‘retirement’ of Enlil in favour of his personal deity Marduk as head of the pantheon. However, none of Hammurabi’s successors appears to have had his longevity, intelligence, or stamina, so that as before Mesopotamia was united only for a matter of decades before a series of contractions and retrenchments. By the end of the sixteenth century Babylon was little more than a city state again.

For the city of Nippur under Hammurabi’s successor Samsu-iluna we can closely identify the curriculum used in three or four different scribal schools, and the variations between them. Scribal schools were for the most part very small indeed, housed in the home of the teacher, whose primary profession was often that of a temple administrator or priest. They catered for up to half a dozen boys at any one time, each at different stages of their education. Much of the learning was through repeated copying and rote memorization.

It is not an easy task to total each category of animal, or to see at a glance which days there are no entries for: given that this document would have been compiled from at least thirty daily records and might itself have been one of up to five hundred tablets from which an annual account had to be compiled, this non-tabular format was gravely inefficient.

A tiny number of tables are known from the bureaucratic archives of Ur, one of which is also from Puzrish-Dagan.[7] It too is an account of sheep and goats:

3	3	3	2	1	lambs
93	93	93	6[2]	31	first-rate sheep
6	6	6	4	2	billy goats
102	102	102	68	[3]4	

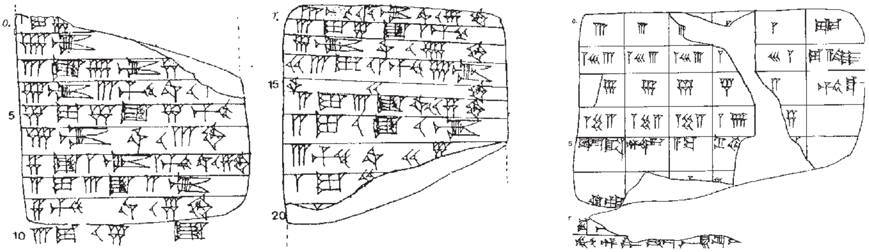


Fig. 1.2 Compare these two accounts of sheep and goats. On the left the non-tabular account, *c.* 2050–2025 BCE, makes no visual separation between numerical and descriptive data, nor between different categories of data. On the right is the earliest known tabular account, *c.* 2028 BCE. Row labels giving the type of animal are on the far right; columnar totals are in line 4. Line 5 appears to contain column labels in the form of abbreviated personal names. Summary information at the bottom of the tablet is largely missing. The date is on the top of the reverse. (AUAM 73.0639 and AUAM 73.0400, from Puzrish-Dagan; now in the Horn Archaeological Museum, Berrien Springs, Michigan. M. Sigrist, *Neo-Sumerian account texts in the Horn Archaeological Museum*, vol. 1, Andrews University Press, Berrien Springs, Michigan, 1984, no. 63, no. 56.)

In the fifth row the columns are labelled with the abbreviated names of the officials responsible; at the bottom is the date on which it was drawn up. It has most of the features of later administrative tables: two axes of organization, one, vertical axis of calculation, row labels to the right of the table—but no headings, so that we cannot tell why the three varieties of livestock were further categorized.

This appears to be a reasonably successful attempt at table making—its structure is clear, the additions are correct—so why didn't the idea catch on? We can put forward a tentative hypothesis, based on its probable archival context. Puzrish-Dagan, a purpose-built depot near Nippur,[8] functioned as the central livestock management centre for the kingdom of Ur for over thirty years (*c.* 2056–2020 BCE). Almost 12 000 documents from its archives have been published, all of them excavated illicitly and therefore without context. Hundreds of tablets from each year of its operation have survived. Our table comes from the penultimate active year of Puzrish-Dagan, after which date the number of surviving records declines dramatically, from an annual average of 220 in the preceding decade to single figures. Maybe the scribes were simply under too much pressure preventing institutional collapse—part of the general 'relatively sudden retreat of the central administration'[9] at that time—to experiment further with new data storage formats.

It was not until the mid-nineteenth century BCE that tables began to be used with any consistency or continuity.[10] The first phase in their development is attested by a group of around two hundred tabular lists from the temple of the god Ninurta in Nippur. Over four hundred tablets record regular food offerings (bread, flour, and beer) made to Ninurta and other deities and divine objects which were then redistributed to temple personnel.[11] The ancient dates on the documents themselves span some eighty years and six changes of political allegiance between the kingdoms of Isin and Larsa between 1871 and 1795 BCE. At least three different methods were used to record the offerings and redistributions, the most successful and enduring of which spanned at least twenty years (*c.* 1855–1836) and one change of government. The archive's scribe ruled the obverse of the tablet vertically into six columns, heading them 'bread', 'shortbread', 'fine flour', '*utu*-flour', 'beer', but leaving the final column blank. In this final column he listed the names of the divine recipients and in the others the quantities of goods they received. He made no calculations, horizontal or vertical, and on the reverse he never tabulated the redistributions but simply listed them prosaically, aligned with the names of the human beneficiaries.

Outside the Nippur offerings archive we can detect three phases in the development of administrative tables,[12] the first of which was a period of experiment and evolution (*c.* 1850–1795). The tables from the immediately following decade (the mid-1790s to 80s) exhibit by contrast a relatively rigorous standardization. Almost all of them are in landscape format, and all are headed uniformly, with the final heading (over the row labels) reading MU.BI.IM 'its name'. None has an introductory title or preamble but almost all have two axes of calculation and interlinear explanatory interpolations. Several groups of related tables can be detected, in which the scribes sometimes experiment with the format and layout of the documents. There is then a twenty-five year gap in the documentary record, until after the conquest of Larsa by Babylon. At this point (*c.* 1758 BCE) the tables begin to be preceded by general introductory matter, and the tablets are predominantly in portrait format.

Very few tables are attested after the mid-1720s for over 70 years until the last half of the seventeenth century BCE. All are much simpler organizationally than their precursors, with at most one axis of calculation, usually simple columnar additions. Almost all are in portrait orientation, reflecting the fact that they have no more than three data columns—whereas the earlier administrative tables regularly had eight or nine. These late tables are almost all from Sippar and the region around Babylon, whereas their predecessors are mostly from Larsa and the south.

After the mid-seventeenth century BCE the Mesopotamian historical record falls silent for about three hundred years. Our next evidence for tabular documentation comes once again from Nippur, and shows a remarkable similarity to what had gone before.[13] An administrative archive of some 600 tablets, spanning the years *c.* 1360–1225, comprises 'records of the receipts of taxes or rents from outlying districts about Nippur; of commercial transactions conducted with this property; and the payment of the salaries of the storehouse officials as well as of the priests, and others in the temple service. In other words they refer to the handling and disposition of the taxes after they had been collected.'[14] Some 33 percent of the records in that archive are tabular. They exhibit all the featural complexity of the eighteenth-century tables—headings, totals, subtotals, two axes of calculation, titles and preambles, explanatory interpolations, even the final column heading MU.BI.IM 'its name'—some four hundred years after the last datable table from eighteenth-century Nippur. We can only speculate how this extraordinary stability was maintained, as although drawings of all the documents in the archive were published nearly a century ago, no analytical work has yet been done on them. Nevertheless, we can get a sense of the strength of the written tradition, and a hint of the huge gaps in all aspects of our knowledge of Mesopotamian history.

So far we have had nothing to say about northern Iraq: that is simply because there is no evidence for administrative tables there before the first millennium BCE. Even in the period of Assyrian world domination, and in the face of massive documentation, the evidence for tables is very slim. Administrators in the successive capital cities of Ashur, Kalhu, and Nineveh used tables simply as columnar lists, in order to separate numerical data of different types: wine from beer, present from absent.[15] Most were headed, with columnar totals at the end of the table. No surviving exemplars exhibit horizontal calculations or any operations more complex than simple addition. Just eighteen of the 450 published administrative records from Nineveh are tables, some 4 per cent of the total. The evidence for first-millennium Babylonia is even scarcer.

Tempting though it is to attribute the lack of tables in the first millennium BCE to some ill-defined decline in bureaucratic ability, a more satisfying explanation is at hand. From the late second millennium onwards, Aramaic became increasingly common as a written language. For whereas the Assyrian and Babylonian dialects of the Akkadian language were still recorded with the increasingly cumbersome and recondite cuneiform script on clay tablets, Aramaic used an alphabet of just 22 letters, written freehand with ink on parchment or papyrus—neither of which organic media survives at all in the archaeological conditions of Iraq. All three languages, however, were also written on waxed wooden or ivory writing boards, several specimens of which have survived,[16] and for which there is ample documentation in the cuneiform record. The durability, tradition and tamper-proof qualities of tablets ensured their continuing use for legal documents and religious literature, whereas writing boards and ink-based media were particularly suited to keeping accounts. Cuneiform tablets had to be inscribed while the clay was fresh; the new media, on the other hand, could be drawn up, corrected and added to over time, like modern-day ledgers. It is highly likely, then, that the vast majority of tabular accounts from the first millennium BCE perished long ago with the boards on which they were written.

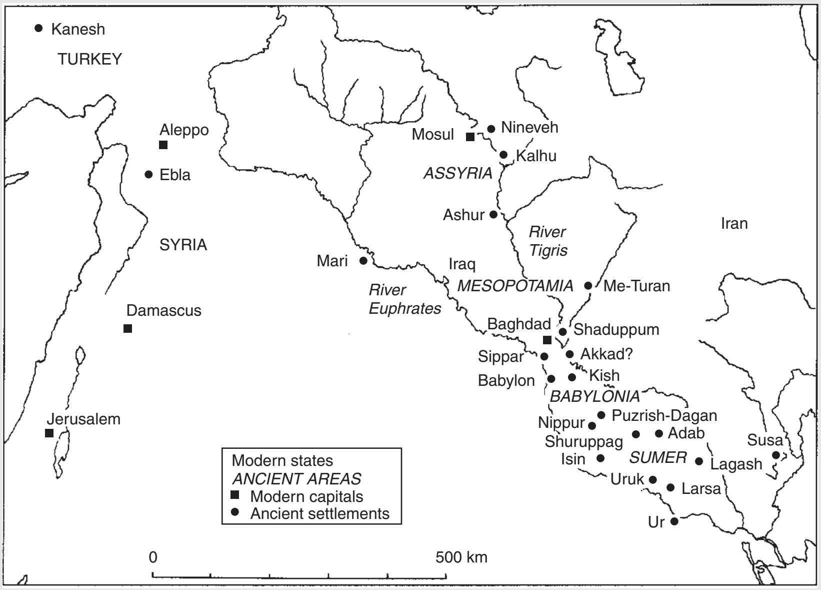
Formal and informal tables in school mathematics

The first securely datable mathematical table in world history comes from the Sumerian city of Shuruppag, *c.* 2600 BCE. The tablet is ruled into three columns on each side with ten rows on the front or obverse side. The first two columns of the obverse list length measures from *c.* 3.6 km to 360 m in descending units of 360 m, followed by the Sumerian word sá ('equal' and/or 'opposite'), while the final column gives their products in area measure. Only six rows are extant or partially preserved on the reverse. They continue the table in smaller units, from 300 to 60 m in 60 m steps, and then perhaps (in the damaged and missing lower half) from 56 to 6 m in 6 m steps. While the table is organized along two axes, there is just one axis of calculation, namely, the horizontal multiplications. Around a thousand tablets were excavated from Shuruppag, almost all of them from houses and buildings which burned down in a city-wide fire in about 2600 BCE,[17] but sadly we have no detailed context for this tablet because its excavation number was lost or never recorded.[18]

While at first sight this table appears simply to list the areas of square fields, in fact it is akin to much more abstract multiplication tables. Prior to the invention of the sexagesimal place value system in the twenty-first century BCE there was no concept of abstract numeration: numbers were not thought of as independent entities but as attributes of concrete objects—the length of a line, for

Later Mesopotamia: Babylonia and Assyria

The disintegration of Babylonian kingdom in the late seventeenth century BCE marked the end of the old order. Major new communication, transport, and military technologies were developing that would henceforth enable huge empires to grow and flourish over centuries. Horses enabled people and ideas to travel faster than ever before, while glass, and later iron, became luxury must-have commodities that stimulated long-distance trade and contact between elites all over the Middle East from Turkey to Iran to Egypt.



Map of the Middle East, showing places mentioned in this chapter

In Babylon the Kassite dynasty (*c.* 1550–1150) had taken power. Although the ruling family had its origins somewhere outside Mesopotamia they quickly adopted the languages, religion, and customs of their native predecessors. The old city temples retained their traditional wealth and status too, remaining significant employers, producers, and consumers in the Babylonian economy. It was a time of literary and scholarly innovation, in which traditional texts were revised, expanded, and interpreted anew. While the Kassites ruled over southern Iraq, the north of the land belonged to the Syrian empire of Mitanni. From the mid-fourteenth century onwards, though, the city of Ashur on the Tigris began to assert its independence, and gradually built up

Later Mesopotamia: Babylonia and Assyria *cont.*

a kingdom which by *c.* 1200 BCE was large enough to attempt a take-over of Kassite Babylonia. By the early first millennium Assyria held vast tracts of the Middle East in its power, extracting heavy tribute from those lands that were not yet under its direct control. In the heyday of the Assyrian empire (*c.* 900– 650 BCE) its capital moved up the Tigris, first to Kalhu and then to Nineveh, where Sargon and his successors Sennacherib, Esarhaddon, and Ashurbanipal ruled the Middle East from enormous palaces built at the expense of their subject peoples, decorated with detailed depictions of their conquests and filled with booty from their annual campaigns. Cuneiform survived the coming of alphabetic Aramaic in the early first millennium, in large part because of its continued high status. Assyrian attempts to usurp Babylon as the cultural centre of Mesopotamia were never entirely successful, but Ashurbanipal managed to create a definitive library of traditional Mesopotamian works in his palace at Nineveh, in part through the theft of tablets from Babylonia and the employment of captive Babylonian scribes to write for him.

Nineveh fell to the Babylonians and Medes in 612 BCE. Babylon enjoyed a brief period of independence and prosperity under Nebuchadnezzar and his successors, but in 539 BCE it became subsumed into the unprecedentedly enormous Persian empire. Although this event marked the definitive loss of Mesopotamian political autonomy it was by no means the end of its cultural history: the last datable cuneiform tablet is an astronomical record from Babylon written in 75 CE, some 600 years later. It had survived political and cultural conquest by Persians, Greek-speaking Macedonians, and the later Parthians. By the end, though, the temple of Marduk in Babylon was the last refuge of a small handful of traditionalist priests and scholars, the guardians of a late flowering of the cuneiform culture that had blossomed for over three thousand years.

instance, or the quantity of sheep in a flock. Therefore in the third millennium BCE to square a number meant, at some level, to construct a square area from two equal lengths. The table was not the only means available for expressing numerical relationships, though: a linear version has survived from the city of Adab.[19] It too is in three columns, but these function more like newspaper columns than the columns of a table. Odd lines of the text give length measurements from 1 to 12 cubits (*c.* 0.5 m–6 m), followed, as expected, by the word sá. The even lines give the areas that result from squaring the lengths. Once again, we have no exact information about its original context, but the style of its handwriting suggests that it is perhaps a few generations younger

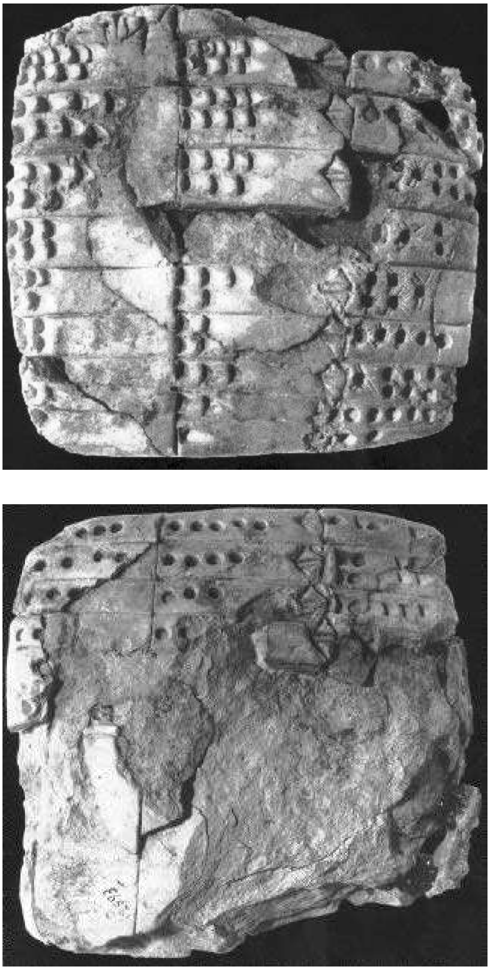


Fig. 1.3 The world's oldest datable mathematical table, from Shuruppag, *c.* 2600 BCE. The first two columns contain identical lengths in descending order from 600 to 60 rods (*c.* 3600–360 m) and the final column contains the square area of their product. The sequence continues on the reverse, and probably finished at 1 rod (6 m). (VAT 12593, from Shuruppag, now in the Vorderasiatisches Museum, Berlin. H. Nissen, P. Damerow, and R. K. Englund, *Archaic bookkeeping*, University of Chicago Press, London and Chigago, 1993, fig. 119.)

than the tablet from Shuruppag (*c.* 2550 BCE). It is signed by someone called Nammah—about whom, frustratingly, we know nothing else at all.[20]

Oddly enough, we find a strong disinclination towards the truly tabular format in later school arithmetic, where one might expect to find tables galore. While there are many thousands of published and unpublished documents from the early second millennium BCE that are normally characterized as mathematical tables, on closer inspection most of them turn out to be prosaic lists of equivalences rather than formally laid out tables.

As usual, the evidence is patchy and difficult to date accurately. The earliest exemplars of the second-millennium arithmetical tradition are actually from the tail end of the third millennium BCE, at the time of the first sexagesimal numeration. We have just one or two administrative tablets with the sexagesimal workings still showing,[21] and a handful of reciprocal tables, or lists of inverses, which are highly likely to have originated in the administrative archives of the twenty-first century kingdom of Ur.[22] They list sexagesimally regular numbers up to 60, or sometimes beyond, with their inverses. Their informally tabular appearance derives from the fact that each line is left–right justified, as are the lines on all well written cuneiform tablets. Little attempt was made to align the numbers themselves.

The same type of reciprocal list is very well attested in later centuries, when it belonged at the head of the standard series of multiplications taught in scribal schools. The students were required to memorize it, like all other elementary school subjects, through repeated copying and revision, first as individual sections and then as long extracts from the whole list. In general, more verbose forms of the list were copied on first exposure to a new section, while a terser—and often much more untidy—version, consisting of the numbers only, was repeated for revision. The whole sequence could comprise up to forty lists, each of up to 25 lines long—some 1000 lines in total. Different multiplications were omitted in different schools, it appears, but the sequence was universally the same, with head numbers from 50 down to 1;15. The multiplicands were always 1–20, 30, 40, and 50, sometimes followed by the square and square root of the head number. At around the same point in the curriculum, students copied out a similar sequence of metrological units, from the capacity, weight, area, and length systems respectively. The series ran from tiny units to enormous ones, and at its maximum extent had about 600 entries.[23]

Less frequently extra-curricular arithmetical lists were copied, such as reciprocal pairs halved and doubled from an initial pair 2 05 ~ 28 48;[24] squares and inverse squares of integers and half-integers to 60;[25] and inverse cubes from 1 to 30.[26] The lists are usually informally tabular, like the multiplication lists, although occasionally formal columnar layouts are adopted, but never with column headings. The so-called coefficient lists, or lists of geometrical, metrological, and practical calculation constants, are similarly

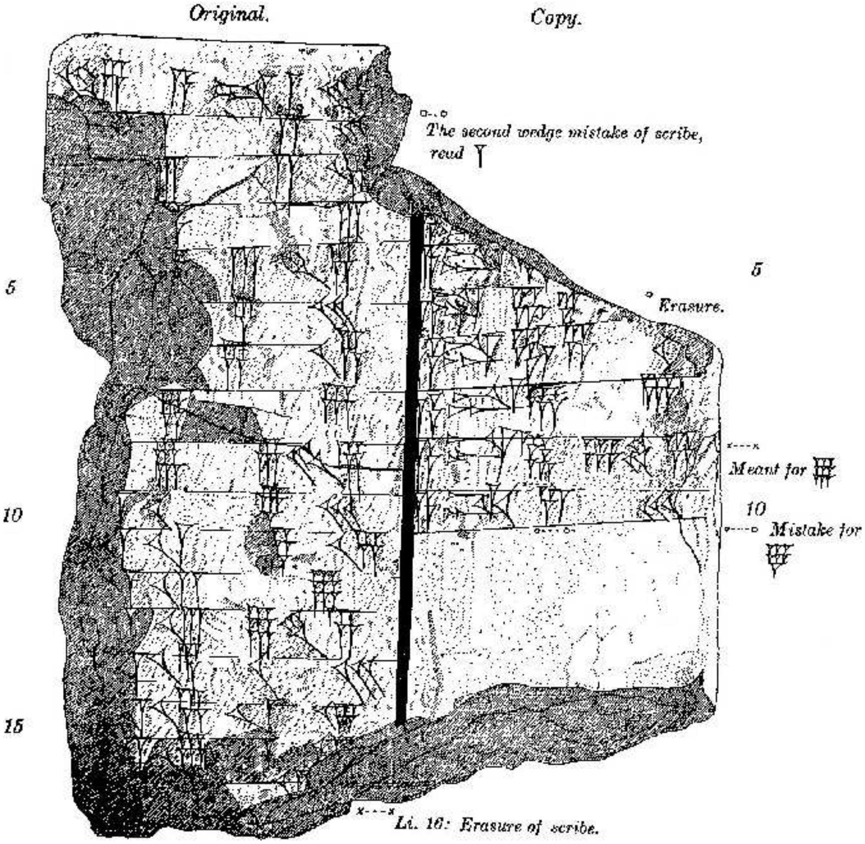


Fig. 1.4 A fifteen times multiplication 'table' for multiplicands 1–20, 30, 40, and 50, from early second-millennium Nippur. The teacher's copy is in the left hand column, the more poorly executed student's copy in the right. The student's copy is unfinished. The tablet broke in antiquity, and parts of it are now missing. (CBS 2142, from Nippur, now in the Collection of the Babylonian Section, University Museum, Philadelphia. H. Hilprecht, *Mathematical, metrological and chronological tablets from the Temple library of Nippur*, Philadelphia: Department of Archaeology, University of Pennsylvania, 1906, no. 94.)

informal, making a visual separation between the values and the names of the constants, but breaking columnar boundaries whenever necessitated by long names or explanations.[27] Only one is ruled into more than two columns, and only one other is headed.

Scribal students were expected not only to memorize arithmetical facts but also to put them into practice. Around a hundred published or partially published rough calculations and diagrams, mostly on small round or square tablets from Ur and Nippur, show the students' numerical solutions to mathematical problems.[28] As one might expect, certain standard layouts were used for particular problem types and students were encouraged to use columnar rulings to group data types.

Relatively little contextual detail is known about the metrological and arithmetical lists and tables, despite their superabundance in archaeological museums. Because there is a general (mis-)perception that they are all identical they are heavily under-represented in publications, so that it is very difficult to quantify the corpus, or give a good account of its chronological and geographical range. Nevertheless a few general statements can be made: around 430 exemplars have been published (*c.* 300 standard arithmetical lists, *c.* 50 non-standard ones, *c.* 50 metrological lists, *c.* 20 tabular calculations, *c.* 10 coefficient lists), representing perhaps 5 or 10 per cent of such material in accessible museums. They are known from almost all early second millennium urban centres in and around southern Iraq. We can estimate the date of several groups of these tablets, either based on direct archaeological information, or from clues gleaned from other tablets in the same museum accession lots. The earliest datable tablets are probably from the 1790s–80s (Larsa, Ur, and Uruk) but there are also groups from the 1750s–40s (Isin, Larsa, Mari, Nippur, Shaduppum, Ur) and the 1640s (Sippar, Susa).

Extraordinarily, the sophisticated bureaucratic ability to display and manipulate numerical data was almost never exploited in the elementary training of future scribes. The arithmetical and metrological lists could easily have been organized into multi-columnar headed tables, with two axes of organization and calculation, given the current clerical facility with tabular layouts. Yet there is only one known mathematical cuneiform tablet which is conspicuously indebted to administrative tabular practice. Plimpton 322 has achieved such iconic status as the Mesopotamian mathematical tablet *par excellence* that it comes as quite a shock to realise how odd it is.[29] Its fame derives from its mathematical content: fifteen rows of four extant columns containing sophisticated data relating to Pythagoras' theorem. The fact that this data is laid out in a landscape-oriented headed table, with a final heading MU.BI.IM ('its name') for the non-numerical data, has gone completely unremarked. These, of course, are formal features of administrative tables from Larsa during the period of rigorous standardization in the 1790s–80s BCE. Clearly, then, there was potential for innovations in data-management to cross over from administration into mathematics; it is puzzling that it happened so rarely.

At the moment we cannot even begin to date Babylonian tablets written in the later second millennium BCE, unless the scribes dated them for us, so badly do we understand the changes in writing conventions at that time.

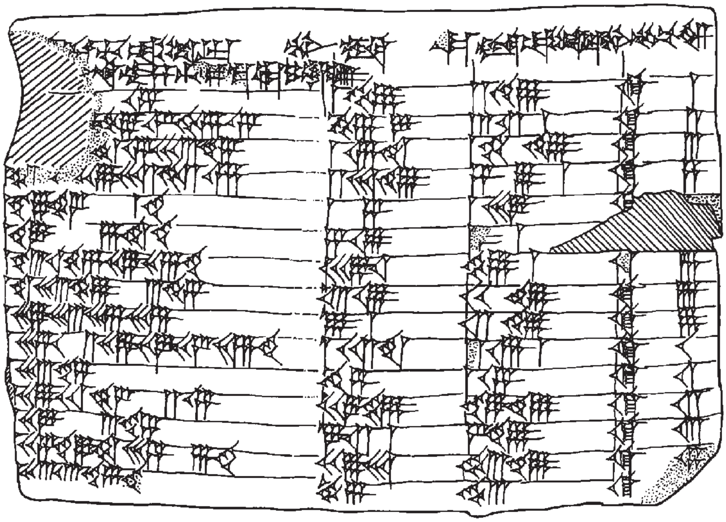


Fig. 1.5 Plimpton 322, the only true table in Old Babylonian mathematics, from Larsa *c.* 1800 BCE. The second and third columns of the table list the diagonals and short sides of 15 right triangles, while the final column is simply a line count. The heading on the first surviving column, which reads, 'The holding-square on the diagonal from which 1 is torn out, so that the short side comes up', gives important information about how the table was computed. There is still much speculation about the what the missing column(s) on the left of the tablet might have contained. (Plimpton 322, from Larsa, now in the University of Columbia Rare Book and Manuscript Library, New York. O. Neugebauer and A. Sachs, *Mathematical cuneiform texts*, American Oriental Society, New Haven, 1945, pp. 38–41. Drawing by E. Robson.)

Thus, in the absence of secure archaeological evidence, we cannot say with any confidence at all whether any mathematical tables have survived from Kassite Babylonia. Early publications often ascribe a Kassite date to multiplication lists that we would now confidently date to the eighteenth century,[30] while one of the coefficients lists may be from any time between fourteenth and the sixth century BCE.[31]

Assyrian metrological and arithmetical lists and tables are equally thin on the ground. A large metrological list has been discovered at Kanesh, an Assyrian trading post in eastern Turkey, *c.* 1850 BCE. It is now rather damaged but originally ran from 1 shekel (*c.* 8 g) to 100 talents (*c.* 3 metric tonnes).[32] The earliest tablet from Ashur itself, dating to *c.* 1200 BCE, gives the standard reciprocals, the complete set of multiplications from 50 to 1;15, and squares of integers to 30, in the classic early second-millennium format.[33] There is only one list of Assyrian capacity and weight measures, the extant part of which runs from 1 ban (*c.* 10 litres) to 70 000 'ass-loads' (*c.* 7 million litres) and 2 shekels to just over

1 mina (*c.* 16–500 g).[34] A small table gives the ratios between length units from 6 grains = 1 finger (*c.* 1.7 cm) to 30 USH = 1 double-mile (*c.* 10.8 km).[35] There is even less from Nineveh, the later Assyrian capital: one enigmatic table of 3-place reciprocals in base 70,[36] and a few lexical tables giving the names of unit fractions in Sumerian and Akkadian, all from Ashurbanipal's palace library.[37]

Several thousand school tablets of the first millennium BCE are also known from the Babylonian cities of Kish, Nippur, Sippar, Ur, Uruk, and Babylon itself, of which a tiny number are mathematical or metrological.[38] The earliest are from a group of fifteen school tablets found in the archive of the city governor of Nippur, *c.* 755–730 BCE. Both list capacity measures from 1 ban to 1 gur (*c.* 10–300 litres); the first is on the back of a letter, while the second has been disguised as a ration list by attaching a fictitious name to each entry and giving an (erroneous) total.[39]

About a century later are the two hundred or so tablets from a badly preserved building in Kish, where 'almost every room contained tablets which had been stored in large jars, arranged round the room according to contents, primarily syllabaries and religious texts.'[40] Sadly the exact disposition of the tablets within the jars was never recorded. Thirteen of those tablets contain extracts from the standard list of weights and measures, none of which is truly tabular, inscribed on the same tablets as snippets from elementary syllabaries and word lists.[41] At the other end of the scholastic spectrum is a beautifully inscribed mathematical table. Whereas most mathematical lists and tables are known to us from several dozens, hundreds, or even thousands of exemplars, this document is unique. It is the only identified table that combines both squares and square roots in a single entry. Each line reads 'n times n is n^2, whose square root is n', where $n = 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, \ldots, 59, 59\frac{1}{2}$. The vertical lines of the tabular formatting appear to have served more as aids to producing a beautifully written document than as conceptual separators of different classes of data. This suggests it was written by a professional scribe and designed as a reference document for a library. The tablet is signed at the bottom by one Bêl-bani-apli ('the lord [god Marduk] is creator of an heir').[42]

Tables for scholarship and astronomy

While little use was made of tabular formatting for mathematical training, from the early second millennium BCE onwards it became ubiquitous in other aspects of schooling and scholarship. The table's horizontal axis of organization enabled inter-columnar relationships to be expressed between words, phrases, or longer texts. One of its first uses was in the school sign-lists now called Proto-Ea and Proto-Diri, attested from at least the eighteenth century BCE, which enumerate the different Sumerian readings each cuneiform sign or sign-combination can take. Each has a simple bi-columnar structure, with no headings. The first five lines of Proto-Ea go as follows:

a2-a	A	'The sign A can be read as "aya"
ia	A	The sign A can be read as "iya"
du-ru	A	The sign A can be read "duru"
e	A	The sign A can be read "e"
a	A	The sign A can be read "a" '.[43]

An optional further column to the right gave one or more Akkadian translations for each Sumerian sign combination. In the course of time the standard lists of Sumerian nouns also acquired columns of Akkadian translations; then and now they are known by their first line, UR$_5$.RA = *hubullu* (a type of loan). Outside Mesopotamia proper, such lists also accrued Egyptian, Hittite, Hurrian, or Ugaritic translations and could comprise up to five columns of text.[44] Headings were never used, however. It is a real irony—and testament to our collective blindness to document formatting—that these intrinsically tabular documents are nowadays called 'lexical *lists*' while their list-like arithmetical counterparts are always known as 'multiplication *tables*'.

A tiny percentage of urban Mesopotamians trained as scribes, and a tiny percentage of scribes became neither bureaucrats nor amanuenses but scholarly experts, Akkadian *ummânu*. From the early second millennium BCE onwards, Mesopotamian scholars carried one or more of the following titles:

Tupsharru	'scribe/celestial diviner'. Experts in interpreting celestial (and other) portents.
Barû	'haruspex/extispicer/diviner'. Experts in extispicy [divination by the entrails of sacrificed goats] and lecanomancy [divination by the patterns of oil on water].
Ashipu	'exorcist/healer-seer'. Experts in magical manipulation of the supernatural.
Asû	'physician'. Experts in curing diseases by drugs and physical remedies.
Kalû	'lamentation chanter'. Experts in soothing angered gods.[45]

Each scholarly discipline had its own corpus of specialist literature, running to a hundred or more standard tablets of omens, prescriptions, or laments. Indeed, the scholarly corpus is so vast, complex, and esoteric that only a tiny proportion of it has been published and studied thoroughly. Tabular documents were used sporadically throughout the corpus, whether, for instance, for adding commentary to core works or for drawing up tables of propitious and unpropitious days of the month. We, however, will focus on the works of the celestial diviners—more commonly if anachronistically referred to in the modern literature as astronomers or astrologers.

Most of our contextualized knowledge of the men themselves comes from three separate locales. At the Assyrian court at Nineveh, *c.* 750–612 BCE, a select inner circle of scholars wrote regularly to the king about what the future portended for matters of state and the well-being of the royal family. A great many of their letters and reports survive, which directly cite the scholarly reference works, thereby enabling us to see how they worked in practise.[46] We also have tablets by and about the celestial diviners attached to two later Babylonian temples: the Resh of Anu in Uruk (*c.* 400–100 BCE and perhaps later) and the Esangila of Marduk in Babylon (*c.* 200 BCE–50 CE and probably earlier).[47] By this late stage in Mesopotamian history, Babylon was ruled by a succession of outsiders (Persians, Greeks, Parthians) with their own belief systems; the professional locus of the scholars, who could no longer rely on state patronage, had thus become the temples.

Until at least the second century BCE the full Akkadian title of the celestial diviner was *tupshar Enuma Anu Ellil*, 'scribe of "When [the gods] Anu [and] Ellil" ', after the first line of the huge compendium of celestial omens that constituted their core reference work:

> When Anu, Ellil, and Ea, the great gods, in their sure counsel had fixed the designs of heaven and earth, they assigned to the hands of the great gods the duty to form the day well and to renew the month of mankind to behold.[48]

The series runs to 68 or 70 tablets (it exists in different versions) which were divided into four sections: lunar omens (tablets 1–22), solar omens (23–29), weather omens (30–49), and omens from stars and planets (50–70). There is strong evidence to suggest that parts of it at least go back to the early second millennium, but the earliest surviving sources come from the Assyrian city of Kalḫu, *c.* 720 BCE—including a 'deluxe' edition written on 16 ivory writing boards for Sargon, king of Assyria, which was found down a palace well with only a few fragments of its wax writing surviving.[49] Fortunately the title and destination of the work had been inscribed in cuneiform on the front cover! Other copies found in Assyria had been looted from Babylonian libraries in 647 BCE.

Only one of *Enuma Anu Ellil's* 70-odd tablets is tabular. Tablet 14 in fact comprises four tables, all related to lunar motion:

A. Duration of lunar visibility in the equinoctial month, [according to] the tradition of Nippur
B. Duration of lunar visibility in the equinoctial month, [according to] the tradition of Babylon
C. Seasonal variation in length of day and night
D. Monthly variation in lunar visibility and new moon; and in lunar visibility at full moon.[50]

The first two tables have 30 entries each, the last two 24. They each end with one-line summary 'headings' but are otherwise very informal, with verbose, list-like entries that are broken into sense units on the surface of the tablet. Here, for instance, is the third entry from each of the four tables:

(A)	On day 3	the moon	is present for 15 (USH).
(B)	On day 3:	1/2 mina	6 shekels.
(C)	In month Ayyar, day 15:		the day watch is $2\frac{2}{3}$ mina, the night watch is $2\frac{1}{3}$ mina.
(D)	In month Ayyar, day 1:		the moon's period of visibility is 10 USH.[51]

All four model the moon's visibility using what are now called 'linear zigzag functions' whereby the variable increases or decreases by a constant amount over fixed intervals, and attains fixed maxima and minima. Plotted on a graph over time such functions trace regular zigzags, hence their modern name. Although these are the earliest known *tabular* representations of linear zigzag functions, the schemes on which they are based can be traced right back to much shorter descriptions in mathematical and astronomical texts of the early second millennium BCE, about a thousand years earlier.[52]

At around the same time that *Enuma Anu Ellil* was being copied for the king of Assyria, scholars began to make systematic records of celestial phenomena without always taking omens from them. These observations were collected into daily, monthly, and annual Diaries, as they are now called, from at least 652 BCE. They formed a continuous tradition, much of which is now lost, that would last for around seven hundred years and eventually (indirectly, and in translation) comprised much of the observational basis for Ptolemy's *Almagest* of the second century CE. The monthly Diaries typically contain the following data:

> A statement of the length of the preceding month; the time interval between sunset and moonset on the first day of the month; time intervals between sun/moonrise/set in the middle of the month; the time interval between moonrise and sunrise on the morning of the moon's last visibility; the dates on which the moon approached the various Normal Stars and the watch of the night in which this occurred; and the date and description of lunar and solar eclipses. For the planets they record dates and position among the stars of first and last visibility, direct and retrograde motion and stationary points, and conjunctions with Normal Stars. [...] The dates of the solstices and equinoxes and of the heliacal rising of Sirius are recorded. [...] Weather conditions are regularly reported, and occasional astronomical phenomena such as meteors and comets (including Halley's Comet in 164 and 87 BCE).[53]

These diaries were key to the development of mathematical astronomy, some time around 500 BCE. They enabled scholars 'to devise consistent and powerful sets of rules (embodied in "Procedure Texts") that allowed them to generate tables (called by modern scholars "Ephemerides") permitting one to foretell the longitudes and times of the occurrences of lunar and planetary phenomena and the longitudes of these bodies in the intervals between these occurrences.'[54] Alongside those Procedure Texts and Ephemerides increasingly accurate and sophisticated mathematical tables were developed, presumably to help scholars with their calculations. Not surprisingly, many of them come from Uruk and Babylon, where the celestial scholars were most active.

Some collections of lunar and planetary data are also set out in a ruled tabular format. Each cell of the table contains a description (often on more than one line) of a particular observed (or sometimes predicted) planetary or lunar event. Successive events are given in the cells going down a column, and going along each row we get events separated by the characteristic planetary or lunar periods (e.g. 8 years for Venus, 18 years for the Moon). The similarities between events separated by these periods can then be easily viewed. There are never any headings to these tables. Frequently, columns continue from one side of the tablet to the other. Sometimes, however, the tablets turn sideways so that the rows continue from one side to the other.[55]

In many ways the dozen or so extant metrological lists and tables of the first millennium BCE are the inverse of their second-millennium counterparts: they now followed the order of capacity, weight, area, and length but more often reversed that order. Tables tended to put the sexagesimal (or more

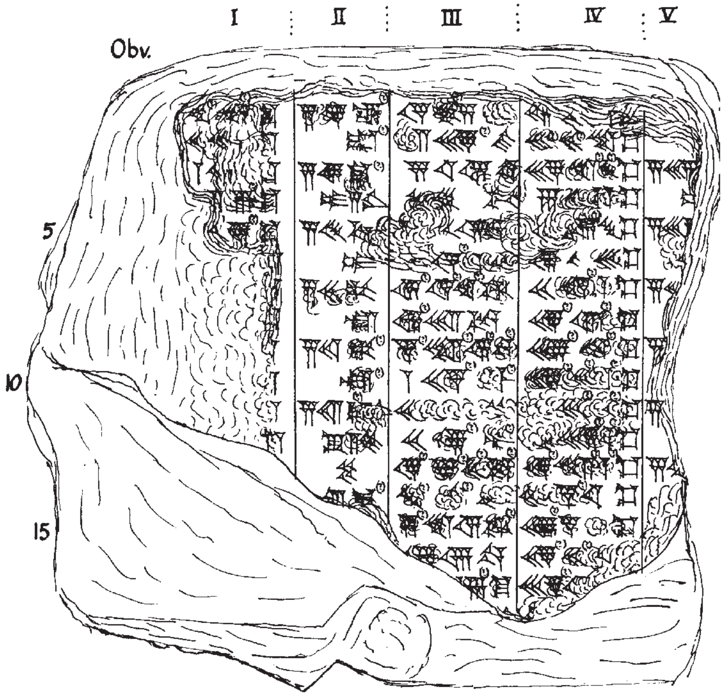


Fig. 1.6 The latest datable Ephemeris, from Babylon, recording lunar and solar eclipse possibilities for the years 12 BCE–43 CE. It is a fragment of a larger tablet in which columns are grouped into threes: date (here cols. II, V), longitude of Moon (col. III), and magnitude of eclipse (cols. I, IV). (BM 34083 = Sp. 181, from Babylon; now housed in the British Museum. T. G. Pinches and J. N. Strassmaier, *Late Babylonian astronomical and related texts*, (ed. A.J. Sachs), Brown University Press, Providence, Rhode Island, 1955, no. 49; studied by O. Neugeubauer, *Astronomical cuneiform texts*, 3 vols., reprint edition, Springer, Berlin, 1955, no. 53, pp. 115–16.)

often decimal) equivalent to the left of the metrological units instead of the right.[56] Very few multiplication tables survive; instead the small but growing corpus consists primarily of tables of many-place regular reciprocals between 1 and 3, and many-place squares and square roots of integers and half-integers.[57] It has yet to be shown whether these tables would be any use in calculating Ephemerides using the methods of the Procedure Texts.

Ephemerides, always tabular, were drawn up from at least 300 BCE until about 50 CE. They fall into two distinct groups: those for the Moon, and those for the planets. A typical lunar Ephemeris tabulates twelve to eighteen functions of the Moon over a whole year, at new and full moon; about a third of known examples record the data for the new moon on the front of the tablet and the full moon on the reverse. Two different calculational methods are used: System A, which is based on step functions for calculating longitudes (in which a variable jumps discontinuously between constant values at fixed intervals); and System B, which uses linear zigzag functions of the sort used in *Enuma Anu Ellil* tablet 14.[68] Despite the complexity of the data they record, the columns of Ephemerides are never headed. Planetary Ephemerides were much less sophisticated, predicting just five or six key moments in the planets' journeys across the night skies. The latest datable Ephemeris is from Babylon, predicting lunar and solar eclipse possibilities for the period 12 BCE–43 CE. The latest datable cuneiform record of them all is also from Babylon, an astronomical almanac predicting the main celestial events for the year 75 CE.

Conclusions

Only rarely in Mesopotamian history were tables a mainstream document format. They took a long time to catch hold, first appearing over half a millennium after the invention of writing and only establishing themselves in the nineteenth century BCE. At that point their potential as powerful tools for the management of quantitative data was fulfilled remarkably rapidly, over a period of a few decades in the early eighteenth century, and this potential continued to be exploited (allowing for huge gaps in the evidential record) for fully five hundred years, at least in and around the city of Nippur. Perhaps it is no coincidence that tables took off only after the invention of the sexagesimal place value system and the concomitant conceptual separation of quantifier and quantified, for Mesopotamian tables made manifest this distinction between quantitative and qualitative, by drawing physical dividing lines between them. In its turn, the new format enabled numerical data and relationships to be seen and explored in ways hitherto unimaginable. The material objects themselves facilitated conceptual advances in quantitative thinking. But even in the heyday of Mesopotamian tables in the first half of the eighteenth century BCE, they account for only 1 or 2 per cent of all administrative documents and scribes continued to prefer simpler linear or prosaic methods of managing information.

Paradoxically, even as tabular formatting began to infiltrate other textual types, cuneiform scribes became less ambitious in using tables to manipulate and check numerical data as the writing board took over many of the functions of the tablet. It was only with the development of mathematical astronomy in the latter half of the first millennium BCE that tables came into their own again to record and calculate the multiple, complex, and lengthy variables of lunar and planetary theory.

Even more surprising is that Mesopotamian scholastic mathematics employed tables very rarely, preferring to express arithmetical and metrological equivalences as lists. Where we do see truly tabular mathematical documents, their debt to administrative or astronomical practice is transparent. While the inner workings of Mesopotamian mathematical lists are generally well understood there is still much we do not grasp about their functioning within the larger educational system. We understand even less of the history of truly tabular tables in Mesopotamia, and their place in the history of ideas.

Further reading

H. Nissen, P. Damerow, and R. K. Englund, *Archaic bookkeeping*, University of Chicago Press, London and Chicago, 1993, is an excellent overview of the development of writing and accounting in third-millennium Mesopotamia. It also has lots of good photos, including the table of squares from Shuruppag. The classic account of early second millennium mathematical tables is O. Neugebauer, *The exact sciences in antiquity,* 2nd edn, Dover, New York, 1969, pp. 3–52. A more recent summary is K. Nemet-Nejat, 'Systems for learning mathematics in Mesopotamian scribal schools', *Journal of Near Eastern Studies* 54 (1995), 241–60. Almost nothing has been written on administrative tables; see for the moment E. Robson, 'Accounting for change: problematising the development of tabular accounts in early Mesopotamia', in *The social and economic implications of accounting in the ancient world: a colloquium held at the British Museum, November 2000*, ed. M. Hudson and M. Van De Mieroop, to appear, and E. Robson, 'Words and pictures: new light on Plimpton 322', *American Mathematical Monthly* 109 (2002), 105–20. A survey of mathematical tables from later Mesopotamia is given in J. Friberg, 'On the structure of cuneiform mathematical table texts from the –1st millennium', in *Die Rolle der Astronomie in den Kulturen Mesopotamiens* (ed. H. D. Galter), RM- Druck- & Verlagsgesellschaft, Graz, 1993, pp. 383–405.

For more general introductions to Mesopotamian mathematics, see J. Høyrup, 'Mesopotamian mathematics', in *Companion encyclopedia of the history and philosophy of the mathematical sciences* (ed. I. Grattan-Guiness), Routledge, London, 1994, pp. 21–9 and E. Robson, 'The uses of mathematics in ancient Iraq, 6000–600 BCE', in *Mathematics across cultures: the history of non-Western mathematics* (ed. H. Selin), Kluwer, Dordrecht, 2000, pp. 93–113. Astronomy is discussed by J. Britton and C. B. F. Walker, 'Astronomy and astrology in Mesopotamia', in *Astronomy before the telescope* (ed. C. B. F. Walker), British Museum Press, London, 1996, pp. 42–67 and by A. Aaboe, 'Babylonian arithmetical astronomy', in *Episodes in the early history of astronomy*, Springer Verlag, Heidelberg, 2001, pp. 24–65. The first chapter of the same book, 'What every young person ought to know about naked-eye astronomy' (pp. 1–23) sets out the observational basics. M. Roaf's *Cultural atlas of Mesopotamia and the ancient Near East*, Facts on File, New York, 1990, is an excellent overview of Mesopotamian history, while C. B. F. Walker's *Cuneiform*, British Museum Press, London, 1987, is a clear explanation of the writing and numeration systems of ancient Mesopotamia.

Notes

1. Discussed in detail by R. K. Englund, 'Texts from the Uruk period', in *Späturuk-Zeit und Frühdynastische Zeit* (ed. P. Attinger and C. Uelinger), Freiburg and Göttingen, 1998, pp. 15–233; quote from p. 215.
2. R. K. Englund, 'Texts from the Uruk period', in *Späturuk-Zeit und Frühdynastische Zeit* (ed. P. Attinger and C. Uelinger), Freiburg and Göttingen, 1998, pp. 56–64, esp. fig. 17 on pp. 58–9; H. Nissen, P. Damerow, and R. K. Englund, *Archaic bookkeeping*, University of Chicago Press, London and Chigago, 1993, p. 30.
3. See, for instance the accounts of the 'Kushim' brewing archive described in H. Nissen, P. Damerow, and R. K. Englund, *Archaic bookkeeping*, University of Chicago Press, London and Chigago, 1993, pp. 36–46.
4. R. K. Englund, 'CDLI proposal', *The cuneiform digital library initiative*, <http://www.cdli.ucla.edu/proposal.pdf>, UCLA, 2001.
5. R. K. Englund, M. Fitzgerald, *et al.*, 'Ur III catalogue', *The cuneiform digital library initiative*, <http://www.cdli.ucla.edu/progress.html>, UCLA, 2001.
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7. AUAM 73.0400, probably from Puzrish-Dagan, now in the Horn Archaeological Museum, Michigan. M. Sigrist, *Neo-Sumerian account texts in the Horn Archaeological Museum*, Andrews University Press, Berrien Springs, 1984, no. 56.
8. M. Sigrist, *Drehem*, CDL Press, Bethesda, 1992.
9. M. Civil, 'Ur III bureaucracy: quantitative aspects', in *The organization of power: aspects of bureaucracy in the ancient Near East,* 2nd edn (ed. McG. Gibson and R. D. Biggs), University of Chicago Press, Chicago, 1992, pp. 35–44, especially p. 44.

10. A more detailed presentation of this discussion is in E. Robson, 'Accounting for change: problematising the development of tabular accounts in early Mesopotamia', in *The social and economic implications of accounting in the ancient world: a colloquium held at the British Museum, November 2000* (ed. M. Hudson and M. Van De Mieroop), to appear.
11. R. M. Sigrist, *Les* sattukku *dans l'Eshumesha durant la période d'Isin et Larsa*, Undena, Malibu, 1984.
12. E. Robson, 'Accounting for change: problematising the development of tabular accounts in early Mesopotamia', in *The social and economic implications of accounting in the ancient world: a colloquium held at the British Museum, November 2000* (ed. M. Hudson and M. Van De Mieroop), to appear.
13. A.T. Clay, *Documents from the temple archives of Nippur dated in the reigns of the Cassite rulers*, 3 vols., The University Museum, Philadelphia, 1906–1912; I. Bernhardt, *Sozial-ökonomische Texte und Rechtsurkunden aus Nippur zur Kassitenzeit*, Akademie Verlag, Berlin, 1976.
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19. A 681, from Adab, now in the Oriental Institute, Chicago. D. D. Luckenbill, *Inscriptions from Adab*, University of Chicago Press, Chicago, 1930, no. 70; D. O. Edzard , 'Eine altsumerische Rechentafel (OIP 14, 70)', in *Lishan mithurti, Festschrift Wolfram Freiherr von Soden* (ed. W. Röllig), Verlag Butzon, Neukirchen-Vluyn, 1969, pp. 101–4.
20. Yang Zhi, *Sargonic inscriptions from Adab*, IHAC, Changchun, 1989, p. 14.
21. M. A. Powell, 'The antecedents of Old Babylonian place notation and the early history of Babylonian mathematics', *Historia Mathematica* 3 (1976), 417–39.
22. Ist T 7375, from Girsu, now in the Istanbul Archaeological Museum: O. Neugebauer, *Mathematische Keilschrift-texte*, vol. 1, Springer, Berlin, 1935, p. 10. HS 201, from Nippur, now in the Hilprecht Sammlung, University of Jena: J. Oelsner, 'HS 201—Eine Reziproken-tabelle der Ur III-Zeit', in *Changing views on ancient Near Eastern mathematics* (ed. J. Høyrup and P. Damerow), Dietrich Reimer Verlag, Berlin, 2001, pp. 53–60; BM 106425 and BM 106444 from Umma, now in the British Museum; and other tablets from Umma Nippur, and Girsu (unpublished, pers. comm. N. Koslova, B. Lafont, and C. Proust).
23. O. Neugebauer, *Mathematische Keilschrift-texte*, vol. 1, Springer, Berlin, 1935, pp. 8–14 (reciprocals), pp. 36–42 (multiplications), pp. 68–72 (metrology); vol. 2, p. 36, vol. 3, pp. 49–50 (reciprocals and multiplications); O. Neugebauer and A. Sachs, *Mathematical cuneiform texts*, American Oriental Society, New Haven,

1945, pp. 4–6 (metrology), pp. 11–12 (reciprocals), pp. 19–33 (multiplications); J. Friberg, 'Mathematik', in *Reallexikon der Assyriologie*, vol. 7 (ed. D. O. Edzard), Walther de Gruyter, Berlin, 1990, pp. 542–6 [in English].

24. J. Friberg, 'Mathematik', in *Reallexikon der Assyriologie*, vol. 7 (ed. D. O. Edzard), Walther de Gruyter, Berlin, 1990, pp. 549–50 [in English].
25. O. Neugebauer, *Mathematische Keilschrift-texte*, vol. 1, Springer, Berlin, 1935, pp. 68–72; O. Neugebauer and A. Sachs, *Mathematical cuneiform texts*, American Oriental Society, New Haven, 1945, pp. 33–5; J. Friberg, 'Mathematik', in *Reallexikon der Assyriologie*, vol. 7 (ed. D. O. Edzard), Walther de Gruyter, Berlin, 1990, pp. 546–7 [in English].
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29. E. Robson, 'Neither Sherlock Holmes nor Babylon: a reassessment of Plimpton 322', *Historia Mathematica*, 28 (2001), 1–40; E. Robson, 'Words and pictures: new light on Plimpton 322', *American Mathematical Monthly* 109 (2002), 105–20.
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33. Ist A 20 + VAT 9734, from Ashur, now in the Istanbul Archaeological Museum and the Vorderasiatisches Museum, Berlin. O. Neugebauer, *Mathematische Keilschrift-texte*, vol. 1, Springer, Berlin, 1935, pp. 11, 46–7, 70.
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